Kinetic physics in NIMROD using a continuum approach
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Electron, ion and hot particle DKEs implemented in NIMROD

- Hazeltine's form for the first-order drift kinetic equation in energy, ε, and magnetic moment, μ, variables:

$$\frac{\partial f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f + \left(\mu \frac{\partial B}{\partial t} + q(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} \right) \frac{\partial f}{\partial \varepsilon} = C(f)$$

- Transforming to $\xi = v_{||}/v$ and $s = v/v_0$ yields

$$\frac{\partial f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f - s \frac{\partial f}{\partial s} \left[\frac{\partial}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \right] \ln v_0 +$$
$$\frac{1-\xi^2}{2\xi} \left[-\xi^2 \frac{\partial \ln B}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_c) \cdot \left[\frac{q\mathbf{E}}{s^2 T_0} - \nabla \ln B \right] + \xi^2 v_{\mathrm{ExB}} \cdot \nabla \ln B \right] \frac{\partial f}{\partial \xi} +$$
$$\frac{s}{2} \left[(1-\xi^2) \frac{\partial \ln B}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_c) \cdot \frac{q\mathbf{E}}{s^2 T_0} + (1+\xi^2) v_{\mathrm{ExB}} \cdot \nabla \ln B \right] \frac{\partial f}{\partial s} = C(f)$$

where $\mathbf{v}_D = v_{\mathrm{ExB}} + \frac{s^2 T_0}{qB} \left(1+\xi^2\right) \mathbf{b} \times \nabla \ln B + \mathbf{v}_{\mathrm{c}}$

and $\mathbf{v}_{\mathrm{c}} = \frac{\mu_0 s^2 T_0}{qB^2} \left[2\xi^2 \mathbf{J}_\perp + (1-\xi^2) \mathbf{J}_{||} \right] + \frac{m v_0 s \xi}{qB^2} \mathbf{b} \times \frac{\partial \mathbf{B}}{\partial t}$.

Adding magnetic moment terms leads to cancellations.

- Magnetic moment terms are proportional to the "twist" function ($\tau = \mu_0 J_\| / B$, Ramos, *Phys Plasmas* **15**, 082106 (2006)):

$$\dot{\mu}\frac{\partial f}{\partial \mu} = -\frac{1-\xi^2}{2\xi}\left[\frac{\mu_0 \mathbf{J}_\|}{B^2}\cdot\left(\mathbf{E} - \frac{s^2 T_0}{q}(1-\xi^2)\nabla \ln B\right) + \frac{2T_0 s^2 \xi^2}{q}\mathbf{b}\cdot\nabla\frac{\mu_0 J_\|}{B^2}\right]\frac{\partial}{\partial \xi}$$

- Terms involving $J_\|$ lead to further cancellation:

$$\text{from} \quad \dot{\mu}\frac{\partial f}{\partial \mu} = -g(\xi)\frac{\mu_0 \mathbf{J}_\|}{B^2}\cdot\left(\mathbf{E} - \frac{s^2 T_0}{q}(1-\xi^2)\nabla \ln B\right) + \ldots$$

$$\text{and} \quad g(\xi)\mathbf{v}_c(J_\|)\cdot\left[\frac{q\mathbf{E}}{s^2 T_0} - \nabla \ln B\right] = g(\xi)\frac{s^2 T_0}{qB^2}(1-\xi^2)\mu_0 \mathbf{J}_\| \cdot\left[\frac{q\mathbf{E}}{s^2 T_0} - \nabla \ln B\right]$$

$$\text{we are left with} \quad -\xi(1-\xi^2)\left[\frac{\mu_0 \mathbf{J}_\|}{2B^2}\cdot\mathbf{E} + \frac{T_0 s^2}{q}\mathbf{b}\cdot\nabla\frac{\mu_0 J_\|}{B^2}\right]\frac{\partial}{\partial \xi}$$

which we will refer to as magnetic moment correction terms keeping in mind that they provide important cancellations with terms proportional to the parallel drift ($J_\|$) which were previously kept.

Modified electron, ion and hot particle DKEs in NIMROD

- With the magnetic moment derivative terms the DKE becomes

$$\frac{\partial f}{\partial t}+(\mathbf{v}_{||}+\mathbf{v}_D)\cdot\nabla f - s\frac{\partial f}{\partial s}\left[\frac{\partial}{\partial t}+(\mathbf{v}_{||}+\mathbf{v}_D)\cdot\nabla\right]\ln v_0 +$$

$$\frac{1-\xi^2}{2\xi}\left[-\xi^2\frac{\partial \ln B}{\partial t}+(\mathbf{v}_{||}+\mathbf{v}_c^*)\cdot\left[\frac{q\mathbf{E}}{s^2 T_0}-\nabla\ln B\right]+\xi^2 v_{\mathrm{ExB}}\cdot\nabla\ln B\right]\frac{\partial f}{\partial \xi}+$$

$$\frac{s}{2}\left[(1-\xi^2)\frac{\partial \ln B}{\partial t}+(\mathbf{v}_{||}+\mathbf{v}_c)\cdot\frac{q\mathbf{E}}{s^2 T_0}+(1+\xi^2)v_{\mathrm{ExB}}\cdot\nabla\ln B\right]\frac{\partial f}{\partial s}-$$

$$\xi(1-\xi^2)\left[\frac{\mu_0\mathbf{J}_{||}}{2B^2}\cdot\mathbf{E}+\frac{T_0 s^2}{q}\mathbf{b}\cdot\nabla\frac{\mu_0 J_{||}}{B^2}\right]\frac{\partial f}{\partial \xi} = C(f)$$

where $\mathbf{v}_c^* = \frac{s^2 T_0}{qB^2}2\xi^2\mu_0\mathbf{J}_\perp + \frac{m v_0 s\xi}{qB^2}\mathbf{b}\times\frac{\partial \mathbf{B}}{\partial t}$.

This fixes problems at $\xi=0$ with terms involving $J_{||}$ times $\mathbf{E}$ or $\nabla \ln B$ multiplying $(1/\xi)\partial/\partial\xi$.

Linearized form relatively simple

- Assuming $\mathbf{E}_{\rm eq} = 0$ and v_0 independent of time (only in order to fit linearized equation on one slide):

$$\frac{\partial \delta f}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D)_{\rm eq} \cdot \nabla \delta f - s \frac{\partial \delta f}{\partial s} (\mathbf{v}_{||} + \mathbf{v}_D)_{\rm eq} \cdot \nabla \ln v_0 -$$

$$\left[\frac{1-\xi^2}{2\xi} (\mathbf{v}_{||} + \frac{s^2 T_0}{qB^2} 2\xi^2 \mu_0 \mathbf{J}_\perp) \cdot \nabla \ln B + \xi(1-\xi^2) \frac{T_0 s^2}{q} \mathbf{b} \cdot \nabla \frac{\mu_0 J_{\|}}{B^2} \right]_{\rm eq} \frac{\partial \delta f}{\partial \xi} =$$

$$- \delta(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f_{\rm eq} + s \frac{\partial f_{\rm eq}}{\partial s} \delta(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln v_0 -$$

$$\frac{1-\xi^2}{2\xi} \left[-\xi^2 \frac{\partial \delta \ln B}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_c^*)_{\rm eq} \cdot \left[\frac{q \delta \mathbf{E}}{s^2 T_0} - \nabla \delta \ln B \right] - \right.$$

$$\left. \delta(\mathbf{v}_{||} + \mathbf{v}_c^*) \cdot \nabla \ln B_{\rm eq} + \xi^2 \delta v_{\rm ExB} \cdot \nabla \ln B_{\rm eq} \right] \frac{\partial f_{\rm eq}}{\partial \xi} -$$

$$\frac{s}{2} \left[(1-\xi^2) \frac{\partial \delta \ln B}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_c)_{\rm eq} \cdot \frac{q \delta \mathbf{E}}{s^2 T_0} + (1+\xi^2) \delta v_{\rm ExB} \cdot \nabla \ln B_{\rm eq} \right] \frac{\partial f_{\rm eq}}{\partial s} -$$

$$\xi(1-\xi^2) \left[(\frac{\mu_0 \mathbf{J}_{\|}}{2B^2})_{\rm eq} \cdot \delta \mathbf{E} + \frac{T_0 s^2}{q} \delta(\mathbf{b} \cdot \nabla \frac{\mu_0 J_{\|}}{B^2}) \right] \frac{\partial f_{\rm eq}}{\partial \xi}$$

Use NIMROD's spatial representation and efficient 2D velocity space representation.

Distribution functions expanded as

$$\delta f(R,Z,\phi,\xi,s,t) = \sum_i f_{i,n=0}(\xi,s,t)\alpha_{i,n=0} + \sum_{i,n>0} f_{i,n}(\xi,s,t)\alpha_{i,n} + f^*_{i,n}(\xi,s,t)\alpha^*_{i,n},$$

where $\alpha_{i,n} \equiv \psi_i(x,y)\exp(in\phi)$ and for 2D velocity space

$$f_{i,n}(\xi,s,t) = \sum_l \sum_{k=0}^{N_s-1} f_{i,n,l,k}(t) P_l(\xi)\delta(s-s_k).$$

$P_l(\xi)$ are 1D FE in a pitch-angle type variable, ξ, and DKEs are solved at N_s collocation points in normalized speed, s.

Couple energetic particle moment into NIMROD's fluid model.

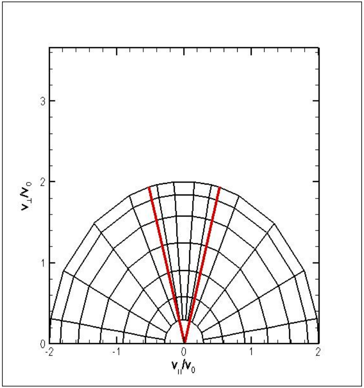
- Energetic particle pressure tensor couples to NIMROD's plasma flow evolution equation.

$$\rho \frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{P}_{\mathrm{h}}$$

$$= \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \left[\pi_{\| h} (\mathbf{bb} - \frac{\mathbf{I}}{3}) + p_{\mathrm{h}} \mathbf{I} \right],$$

where the CGL form for the energetic particle stress tensor is

$$\pi_{\| h} = \frac{m_{\mathrm{h}}}{2} \int d\mathbf{v} \left[3(\mathbf{b} \cdot (\mathbf{v} - \mathbf{V}))^2 - |(\mathbf{v} - \mathbf{V})|^2 \right] \delta f$$



Iterate to effect a simulaneous advance of $\mathbf{V}$ and δf.

- Figuring out how to center fluid and kinetic variables in time is important.

$$(\mathbf{V}, \delta f, \mathbf{P}_h(\mathbf{V}, \delta f))^j \qquad\qquad (n, \mathbf{B}, T_e, T_i, \mathbf{E}(\mathbf{V}))^{j+1/2}$$

$$\rho^{j+1/2} \frac{\Delta \mathbf{V}}{\Delta t} + \nabla \cdot \mathbf{P}_h(\Delta \mathbf{V}, \mathbf{V}^j, \Delta \delta f, \delta f^j) = \mathbf{J}^{j+1/2} \times \mathbf{B}^{j+1/2} - \nabla p^{j+1/2}$$

$$\frac{\Delta \delta f}{\Delta t} + ... = -\delta(\mathbf{v}_{||} + \mathbf{v}_D)^{j+1/2} \cdot \nabla f_{\mathrm{eq}}$$
$$\frac{1-\xi^2}{2\xi}\left[\xi^2 \mathbf{b}_{\mathrm{eq}} \cdot \nabla \times \delta \mathbf{E}(\Delta \mathbf{V}, \mathbf{V}^j) + (\mathbf{v}_{||} + \mathbf{v}_c^*)_{\mathrm{eq}} \cdot \left[\frac{q \delta \mathbf{E}(\Delta \mathbf{V}, \mathbf{V}^j)}{s^2 T_0} - \nabla \delta \ln B^{j+1/2}\right] - \right.$$
$$\left. \delta(\mathbf{v}_{||} + \mathbf{v}_c^*)^{j+1/2} \cdot \nabla \ln B_{\mathrm{eq}} + \xi^2 \delta v_{\mathbf{E}\times\mathbf{B}}(\Delta \mathbf{V}, \mathbf{V}^j) \cdot \nabla \ln B_{\mathrm{eq}}\right] \frac{\partial f_{\mathrm{eq}}}{\partial \xi} + ...$$

Conclusions

- Corrections due to $\dot{\mu}$ important in the formulation of delta-f DKEs in NIMROD.
- Simultaneous advance of $\mathbf{V}$ and δf for energetic particles permits larger time steps.