Nonlinear simulations of locking in the presence of tearing layers with real frequencies

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For tearing regimes with real frequencies, the Maxwell torque induced by a static error field locks the plasma to the tearing mode phase velocity ($v_{ph}$)[1]

- Real frequencies for tearing modes are due to diamagnetic effects or the Glasser effect[2] in the resistive-inertial regime and as seen recently in the viscoresistive (VR) regime[3,4].✓
- In the presence of a small error field and plasma rotation, the maximum perturbation occurs at the $v_{ph}$ of the tearing mode, and this response peaks when the stable tearing mode ($\gamma \leq 0$) is close to marginal stability ⇒ Reconnection driven by an error field ✓
- For large island width the sound wave can flatten the pressure gradient, slowing down the propagation (Scott effect for $\omega_*$[5]) ✓
- This decrease in propagation frequency might reduce the effect of locking to the phase velocity and allow locking to zero velocity. ⊠
- We perform simulations with NIMROD to investigate all of the above in a large-aspect ratio periodic cylinder with a hollow pressure profile.

[4]. Poster P2.020 by A. Cole on Monday Apr 23
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2 Theoretical Background
   - Glasser effect
   - Quasilinear theory for locking
   - The Scott effect

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4 Results
   - Linear Simulations
   - Nonlinear Simulations
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Review of the Glasser effect

Dispersion relation in resistive-inertial (RI) regime\cite{2}

\[
\Delta' = \Delta(\gamma) = (\gamma \tau)^{5/4} - \frac{D_m}{(\gamma \tau)^{1/4}}
\]  

(1)

where \( D_m = (1 - q(r_t)^2) \) \( D_s \) is the Mercier parameter and \( D_s = -\frac{2r_t p'(r_t)}{B^2 R^2 q'(r_t)^2} \) the Suydam parameter.

- For \( \Delta_{cr} < \Delta' < \Delta_{min} \), a pair of complex roots, i.e., \( \gamma = \gamma_r \pm i\omega_r \).
- For \( 0 < \Delta' < \Delta_{cr} \), stabilization/damping, i.e., \( \gamma_r \leq 0 \).

Dispersion relation for \( D_m \neq 0 \)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{dispersion_plot.png}
\caption{Locus of roots}
\end{figure}
Backward propagating wave \((\omega_r < 0)\) with a zero frequency in the lab frame if plasma rotates at \(-\omega_r/k\). 

- Maximum response of \(\tilde{\psi}(r_t)\) to static error field at \(\nu = -\omega_r/k\) with \(\nu \rightarrow -\nu\) symmetry, unlike diamagnetic drift stabilization.  
  \(\Rightarrow\) \(\omega_\ast\) does not come in a complex conjugate pair. 

- This \(\omega_r\) effect also occurs in the viscoresistive (VR) regime\(^1\).
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Quasilinear theory for locking to a static error field

**Linear theory:** The reconnecting flux is proportional to the error field $\epsilon$:

$$\tilde{\psi}(r_t) \propto \frac{\tilde{\psi}(r_w)}{\Delta' - \Delta(\gamma + ikv)} \propto \frac{\epsilon}{\Delta' - \Delta(\gamma_d)}$$

(2)

$\Delta(\gamma_d) \to \Delta(ikv)$ – Doppler shifted from the error field ($\gamma = 0$) because of the plasma rotation at $r = r_t$

Plasma rotation at $r = r_t$ is determined by the balance between:

1. The EM torque $\propto R \int r dr \langle j_z b_r \rangle_{\theta}$ by the error field
   $$N_M \propto \frac{|\tilde{\psi}(r_w)|^2 (\Delta_{\text{imag}}(ikv))}{|\Delta' - \Delta(ikv)|^2},$$
   (3)

2. Viscous torque with a momentum source $v_0$:
   $$N_V = N_0(\nu - v_0).$$
   (4)

$N_M$ is largest when mode is closest to marginal stability in the complex plane: both $(\Delta' - \Delta_{\text{real}}(ikv))^2$ and $\Delta_{\text{imag}}(ikv)^2$ small.
Fields lock to the static error field while the plasma flow locks to a finite frequency/velocity $\gtrsim \omega_r/k$.

- High-slip states where plasma at $r = r_t$ rotates fast enough to shield the error field ($|\tilde{\psi}(r_t)| \ll |\tilde{\psi}(r_w)|$).
- Locked states where the plasma rotation at $r = r_t$ is slowed down ($|\tilde{\psi}(r_t)| \sim |\tilde{\psi}(r_w)|$), but not stopped: $v(r_t) \rightarrow \omega_r/k$ as $|\psi(r_w)| \rightarrow \infty$.
- Upward/downward bifurcations between unlocked (high-slip) and locked states are possible.

Nonlinear Locking

Tibbar Technologies

NIMROD/Sherwood 2018
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For a large enough island width $W$ such that $k'_\parallel WC_s \gtrsim \omega_*$, sound wave flattens pressure across island.

$\Rightarrow$ slows diamagnetic propagation,
$\Rightarrow$ weakens the finite rotation locking effect: plasma locks back to zero velocity

**Question:** Is there a corresponding Scott effect for the Glasser effect? We expect the Scott effect to slow down $\omega_r$ in the same fashion as $\omega_*$ and cause the plasma to lock back to zero velocity.
Use a periodic cylinder to simulate a large-aspect ratio \((R/r_w = 10)\) torus

- A hollow pressure (quadratic) profile to mimic favorable average curvature in a torus \((D_m \rightarrow D_s < 0)\).
- \(1.6 \leq q(r) \leq 4.4\) with the rational surface located at \(r_t = 0.38\) for \((m, n) = (2, 1)\) tearing mode.
- \(B_z\) very slightly paramagnetic.
Visco-resistive MHD is implemented with the NIMROD multi-fluid framework.

\[ \frac{\partial n}{\partial t} + \nabla \cdot (nv) = \nabla \cdot D_n \nabla n, \quad (5) \]

\[ \rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = J \times B - \nabla p - \nabla \cdot \vec{\Pi}, \quad (6) \]

\[ \frac{\partial B}{\partial t} = -\nabla \times E + \kappa \nabla \cdot B \nabla \nabla \cdot B, \quad (7) \]

\[ n \left( \frac{\partial T}{\partial t} + v \cdot \nabla T \right) = - (\Gamma - 1) \frac{\rho}{2} \nabla \cdot v - \nabla \cdot q, \quad (8) \]

where \( E + v \times B = \eta J \), \( \rho = m_in \), \( \nabla \times B = J \), \( \rho = n(T_i + T_e) = 2nT \),

\( q = -\kappa_{||} \nabla_{||} T - \kappa_{\perp} \nabla_{\perp} T \) and \( \vec{\Pi} = \mu \nabla v \) or \( \mu \left[ \nabla v + (\nabla v)^T - \frac{2}{3} \nabla \cdot v \right] \)

- Dimensionless: \( r \rightarrow r/r_w, B \rightarrow B/B_0 \), and \( t \rightarrow t/\tau_A \). \( v_A = r_w/\tau_A \)
- Constant and uniform diffusivities but anisotropic heat conduction.
- The equilibrium is kept static in time: introduces a momentum source.
Typical simulation parameters

- Aspect ratio, $R/r_w = 10$, $r_w = 1$ with $1.6 \leq q(r) \leq 4.4$
- $\beta \leq 0.002$
- Initial $B_z(r = 0) \equiv B_0 = 1$ such that $v_A = 10^7$ m/s for a chosen $\rho$.
- Lundquist number $S = \tau_R/\tau_A = 10^5$; Prandtl number: $Pr = 0.1$
- $(m, n) = (2, 1)$ error field with $10^{-7} \leq \epsilon \leq 10^{-3}$
- Equilibrium (axial) flow: $v_0 \leq 100$ km/s ($= 0.009v_A$).
- Glasser phase speed: $\omega_r/k = 19.2$ km/s
- $D_n = 0.05 < \kappa_\perp = 0.1 < \mu = 10.9 < \eta = 109 < \kappa_B = 10^4 < \kappa_\parallel = 10^5$ with $\kappa_\parallel/\kappa_\perp = 10^6$
- Coarse poloidal resolution: $m = 0, 1, 2$. Convergence checked up to 11 modes for a few cases.
- Time asymptotic state for nonlinear simulations takes several $\tau_R$
Two major issues that have been overcome

1. **poly_divv_auto=T**: reduced polynomial representation for $\nabla \cdot \mathbf{v}$ when $\beta \neq 0$ (see C. Sovinec, JCP 319 (2016) and past NIMROD meetings) in spite of good curvature everywhere!

2. **p_computation='at nodes' otherwise**
   - **p_computation='at quads'** causes numerical instability for $\beta \gtrsim 0.001$
   
   The following parameter controls whether 1) pressure used in the velocity advance is computed from the nT product at the nodes of the FE expansion and interpolated to the quad points or 2) n and T are interpolated then combined.
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Linear simulations show real frequency and stabilization of the $(2, 1)$ tearing mode as $\beta$ increases ($v_0 = 0$)

Glasser oscillations: $\omega_r > 0$ (green) for $\beta > 10^{-4}$.

The mode becomes damped: $\gamma < 0$ (blue) for $\beta \geq 0.0018$. 
Linear simulations show maximum reconnected flux $|\tilde{\psi}(r_t)|$ when plasma rotates at the phase speed $v_0 = \pm \omega_r/k$

- $\nu \rightarrow - \nu$ symmetry.
- Values normalized to their respective maxima for each trace.
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Weakening of the Glasser effect observed in nonlinear simulations for error field $\epsilon \gtrsim 10^{-5}$

- $\epsilon$ provides the means to control the island width $W$ because $W \propto |\tilde{\psi}(r_t)|^{1/2} \propto |\tilde{\psi}(r_w)|^{1/2}$ and $\epsilon = im\tilde{\psi}(r_w)/r_w$.
- Flattening of axisymmetric pressure and an $m = 2$ island are observed.

Pressure profile ($\epsilon = 10^{-4}$)  

Helical flux $\chi$ contours ($\epsilon = 10^{-4}$)

Scott parameter: $k'_\parallel Wc_s/\omega_r = 1.6$ for $\epsilon = 10^{-4}$
Nonlinear simulations to look for bifurcation/locking to finite frequency

- Control parameters: $\tilde{\psi}(r_w)$ (or $\epsilon$) and $v_0$
- Order parameters: $\tilde{\psi}(r_t)$ and $v(r_t)$
- Simulations at very small error fields show NO locking, because
  - $N_v \gg N_M$
  - $v_0$ is still too small (à la bifurcation diagram of Slide 9)
Summary

- Periodic cylinder with $R/r_w = 10$ with
  - a hollow pressure profile to model average good curvature
  - an equilibrium unstable to $(2, 1)$ tearing mode at $r = 0.38$

- $\gamma \rightarrow \gamma \pm i\omega_r$ with $\gamma \leq 0$ at $\beta \geq 0.0018$ due to the Glasser effect.

- Resonant response to a static error field observed for $v_0 = \pm \omega_r/k$.
  - A pair of complex conjugate roots lead to $v \rightarrow -v$ symmetry.
  - No such symmetry for $\omega_*$.

- **Quasilinear theory**: Plasma locks to finite velocity under the influence of EM and viscous torques.

- **Scott effect**: when the island width $W$ is large, the pressure is flattened across the island and $\omega_*$ slowed down.
  - Plasma locks back to zero velocity
  - **Q**: Does the Scott effect also slow down $\omega_r$ and lock to 0 velocity?

- Flattening of the pressure is observed for $\epsilon \gtrsim 10^{-5}$.

- Still looking for the locked state.