Linearized drift kinetic equation in NIMROD, and Efforts for strongly coupled plasmas, HEDLP[1]

E. D. Held, J. Andrew Spencer, T. V. Taylor and D. B. Hansen

Department of Physics
UtahStateUniversity

NIMROD meeting at the Sherwood Fusion Theory Conference, April 23-25, 2018, Auburn, AL

[1]Work supported by DOE under grant nos. DE-FC02-08ER54973, DE-FG02-04ER54746, and DE-SC0018146.

Outline

Verification of consistency of drift kinetic equations.

Collision operator for strongly coupled plasmas.

Various forms for the drift kinetic equation

The drift-kinetic equation may be written using several sets of variables:

1. Hazeltine[1]

$$\bar{f}\,(U,\mu,\mathbf{x},t) \rightarrow \frac{\partial \bar{f}}{\partial t} + \mathbf{v}_{\mathrm{gc}}\cdot\nabla\bar{f} + \frac{dU}{dt}\frac{\partial \bar{f}}{\partial U} + \frac{d\mu}{dt}\frac{\partial \bar{f}}{\partial \mu} = C$$

2. Ramos[2]

$$\bar{f}\left(v_{\parallel},v_{\perp},\mathbf{x},t\right) \rightarrow \frac{\partial \bar{f}}{\partial t} + \mathbf{v}_{\mathrm{gc}}\cdot\nabla\bar{f} + \frac{dv_{\parallel}}{dt}\frac{\partial \bar{f}}{\partial v_{\parallel}} + \frac{dv_{\perp}}{dt}\frac{\partial \bar{f}}{\partial v_{\perp}} = C$$

3. NIMROD[3]

$$\bar{f}\,(s,\xi,\mathbf{x},t) \rightarrow \frac{\partial \bar{f}}{\partial t} + \mathbf{v}_{\mathrm{gc}}\cdot\nabla\bar{f} + \frac{ds}{dt}\frac{\partial \bar{f}}{\partial s} + \frac{d\xi}{dt}\frac{\partial \bar{f}}{\partial \xi} = C$$

[1] Hazeltine and Meiss, Plasma Confinement (Adisson-Wesley, RedwoodCity, 1992)
[2] Ramos, Phys Plasmas 15, 082106 (2008)
[3] Held, *et al*, Phys Plasmas 22, 032511 (2015)

Hazeltine DKE

In magnetic moment, μ, and total energy, U, coordinates, the DKE was originally written using

$$
\begin{aligned}
\mathbf{v}_{\rm gc} &= v_\parallel \mathbf{b} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{2qB}\left(2v_\parallel^2 + v_\perp^2\right)\mathbf{b}\times\nabla\ln B + \\
&\quad \frac{m}{qB^2}\left[v_\parallel^2\left(\mathbf{I}-\mathbf{bb}\right) + \frac{1}{2}v_\perp^2\mathbf{bb}\right]\cdot\nabla\times\mathbf{B} + \frac{mv_\parallel}{qB^2}\mathbf{b}\times\frac{\partial\mathbf{B}}{\partial t} \\
\dot{\mu} &= -\frac{mv_\perp^2}{2B^2}\left[\mathbf{b}\cdot\nabla\times\mathbf{b}\right]\mathbf{b}\cdot\frac{\partial\mathbf{A}}{\partial t} + \frac{m^2 v_\parallel v_\perp^2}{2qB}\mathbf{b}\cdot\nabla\left(\frac{v_\parallel\mathbf{b}\cdot\nabla\times\mathbf{b}}{B}\right) \\
&\quad -\frac{m^2 v_\parallel v_\perp^2}{2qB^2}\nabla\cdot\left(\mathbf{b}\times\frac{\partial\mathbf{b}}{\partial t}\right) \\
\dot{U} &= q\frac{\partial\phi}{\partial t} + \mu\frac{\partial B}{\partial t} - q\mathbf{v}_{\rm gc}\cdot\frac{\partial\mathbf{A}}{\partial t}
\end{aligned}
$$

Recast Hazeltine DKE

Transforming Hazeltine's coordinates to those of Ramos: $(\mathbf{x}, t, \mu, U) \rightarrow (\mathbf{x}, t, v_\parallel, v_\perp)$ yields

$$\begin{aligned}
\mathbf{v}_{\rm gc} &= v_\parallel \mathbf{b} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{2qB}\left(2v_\parallel^2 + v_\perp^2\right)\mathbf{b}\times\nabla\ln B \\
&\quad + \frac{m}{qB^2}\left[v_\parallel^2\left(\mathbf{I}-\mathbf{bb}\right) + \frac{1}{2}v_\perp^2\mathbf{bb}\right]\cdot\nabla\times\mathbf{B} + \frac{mv_\parallel}{qB^2}\mathbf{b}\times\frac{\partial\mathbf{B}}{\partial t} \\
\dot{v}_\parallel &= \left(\mathbf{b} + \frac{v_\parallel}{\Omega_c B}\left(\mathbf{I}-\mathbf{bb}\right)\cdot\nabla\times\mathbf{B} + \frac{1}{\Omega_c B}\mathbf{b}\times\frac{\partial\mathbf{B}}{\partial t}\right)\cdot\left(\frac{q\mathbf{E}}{m} - \frac{v_\perp^2}{2}\nabla\ln B\right) \\
&\quad + \frac{v_\parallel}{\Omega_c}\mathbf{b}\times\nabla\ln B\cdot\frac{q\mathbf{E}}{m} - \frac{mv_\parallel v_\perp^2}{2q}\mathbf{b}\cdot\nabla\left(\frac{\mathbf{b}\cdot\nabla\times\mathbf{B}}{B^2}\right) + \frac{mv_\perp^2}{2qB}\nabla\cdot\left(\mathbf{b}\times\frac{\partial\mathbf{b}}{\partial t}\right) \\
\dot{v}_\perp &= \frac{v_\perp}{2}\left\{\frac{\partial\ln B}{\partial t} + \left[\mathbf{bb}\cdot\nabla\times\mathbf{B}\right]\mathbf{b}\cdot\frac{\mathbf{E}}{B^2}\right. \\
&\quad + \left[\mathbf{v}_\parallel + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{qB^2}v_\parallel^2\left(\mathbf{I}-\mathbf{bb}\right)\cdot\nabla\times\mathbf{B} + \frac{mv_\parallel}{qB^2}\mathbf{b}\times\frac{\partial\mathbf{B}}{\partial t}\right]\cdot\nabla\ln B \\
&\quad \left. + \frac{mv_\parallel^2}{q}\mathbf{b}\cdot\nabla\left(\frac{\mathbf{b}\cdot\nabla\times\mathbf{B}}{B^2}\right) - \frac{mv_\parallel}{qB}\nabla\cdot\left(\mathbf{b}\times\frac{\partial\mathbf{b}}{\partial t}\right)\right\}
\end{aligned}$$

Here the green terms, which Ramos refers to as the "twist function", bring in corrections from $\dot{\mu}$.

Ramos' DKE

Ramos' DKE, written in the macroscopic flow frame, is

$$
\begin{aligned}
\dot{\mathbf{x}} =& \mathbf{u} + \mathbf{u}_F + v'_\parallel \mathbf{b} + \frac{v'^2_\perp}{2} \nabla \times \left(\frac{\mathbf{b}}{\Omega_c} \right) + \frac{\mathbf{b}}{\Omega_c} \times \left[2 v'_\parallel \left(\mathbf{b} \cdot \nabla \right) \mathbf{u} + \left(v'^2_\parallel - \frac{v'^2_\perp}{2} \right) \boldsymbol{\kappa} \right] \\
\dot{v}'_\parallel =& \frac{\mathbf{b} \cdot \mathbf{F}}{mn} - v'_\parallel \mathbf{b} \cdot \left[\left(\mathbf{b} \cdot \nabla \right) \left(\mathbf{u} + \mathbf{u}_F \right) \right] + \frac{v'^2_\perp}{2} \nabla \cdot \left\{ \frac{\mathbf{b}}{\Omega_c} \times \left[\left(\nabla \times \mathbf{u} \right) \times \mathbf{b} + v'_\parallel \boldsymbol{\kappa} \right] \right\} \\
& + \left\{ \frac{\hat{\mathbf{b}}}{\Omega_c} \times \left[\left(\nabla \times \mathbf{u} \right) \times \mathbf{b} \right] \right\} \cdot \left[\frac{\mathbf{F}}{mn} - 2 v'_\parallel \left(\mathbf{b} \cdot \nabla \right) \mathbf{u} - v'^2_\parallel \boldsymbol{\kappa} \right] - \frac{v'^2_\perp}{2} \mathbf{b} \cdot \nabla \ln B \\
& - 2 v'^2_\parallel \left(\frac{\mathbf{b}}{\Omega_c} \times \boldsymbol{\kappa} \right) \cdot \left[\left(\mathbf{b} \cdot \nabla \right) \mathbf{u} \right] - \frac{v'^2_\perp}{8 \Omega_c} \epsilon_{jkl} b_j \left(\frac{\partial b_k}{\partial x_m} + \frac{\partial b_m}{\partial x_k} \right) \left(\delta_{mn} - b_m b_n \right) \left(\frac{\partial u_l}{\partial x_n} + \frac{\partial u_n}{\partial x_l} \right) \\
\dot{v}'_\perp =& \frac{v'_\perp}{2} + v'_\parallel \mathbf{b} \cdot \nabla \ln B \left\{ \mathbf{b} \left[\left(\mathbf{b} \cdot \nabla \right) \left(\mathbf{u} + \mathbf{u}_F \right) \right] - \nabla \cdot \left(\mathbf{u} + \mathbf{u}_F \right) + v'_\parallel \mathbf{b} \cdot \nabla \ln B \right. \\
& - v'_\parallel \nabla \cdot \left[\frac{\mathbf{b}}{\Omega_c} \times \left[2 \left(\mathbf{b} \cdot \nabla \right) \mathbf{u} + v'_\parallel \boldsymbol{\kappa} \right] \right] + 2 \left[\frac{\mathbf{b}}{\Omega_c} \times \left(\boldsymbol{\omega} \times \mathbf{b} \right) \right] \cdot \left[\left(\mathbf{b} \cdot \nabla \right) \mathbf{u} + v'_\parallel \boldsymbol{\kappa} \right] \\
& \left. + 4 v'_\parallel \left(\frac{\mathbf{b}}{\Omega_c} \times \boldsymbol{\kappa} \right) \cdot \left[\left(\mathbf{b} \cdot \nabla \right) \mathbf{u} \right] \right\}
\end{aligned}
$$

Recast Ramos' DKE

In the lab frame, with $\mathbf{F} = \mathbf{E} \times \mathbf{B}/B^2$ Ramos' DKE becomes

$$
\begin{aligned}
\mathbf{v}_{\rm gc} &= v_\parallel \mathbf{b} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{2qB}\left(2v_\parallel^2 + v_\perp^2\right)\mathbf{b}\times\nabla\ln B \\
&\quad + \frac{m}{qB^2}\left[v_\parallel^2\left(\mathbf{I} - \mathbf{bb}\right) + \frac{1}{2}v_\perp^2\mathbf{bb}\right]\cdot\nabla\times\mathbf{B} \\
\dot{v}_\parallel &= \left(\mathbf{b} + \frac{v_\parallel}{\Omega_c B}\left(\mathbf{I} - \mathbf{bb}\right)\cdot\nabla\times\mathbf{B}\right)\cdot\left(\frac{q\mathbf{E}}{m} - \frac{v_\perp^2}{2}\nabla\ln B\right) \\
&\quad + \frac{v_\parallel}{\Omega_c}\mathbf{b}\times\nabla\ln B\cdot\frac{q\mathbf{E}}{m} - \frac{m v_\parallel v_\perp^2}{2q}\mathbf{b}\cdot\nabla\left(\frac{\mathbf{b}\cdot\nabla\times\mathbf{B}}{B^2}\right) \\
\dot{v}_\perp &= \frac{v_\perp}{2}\left\{\frac{\partial\ln B}{\partial t} + \left[\mathbf{bb}\cdot\nabla\times\mathbf{B}\right]\mathbf{b}\cdot\frac{\mathbf{E}}{B^2}\right. \\
&\quad + \left[\mathbf{v}_\parallel + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{m}{qB^2}v_\parallel^2\left(\mathbf{I} - \mathbf{bb}\right)\cdot\nabla\times\mathbf{B}\right]\cdot\nabla\ln B \\
&\quad \left. + \frac{m v_\parallel^2}{q}\mathbf{b}\cdot\nabla\left(\frac{\mathbf{b}\cdot\nabla\times\mathbf{B}}{B^2}\right)\right\}
\end{aligned}
$$

which agrees with Hazeltine's except for the terms in red which stem from from Hazeltine's added drift velocity piece $\frac{m v_\parallel}{qB^2}\mathbf{b}\times\frac{\partial\mathbf{B}}{\partial t}$.

NIMROD DKE

The DKE implemented in NIMROD uses the coordinates $(\mathbf{x}, t, s, \xi)$, where $s = v/v_0$ and $\xi = v_\parallel/v$

$$
\begin{aligned}
\mathbf{v}_{\mathrm{gc}} &= v_0 s\xi\mathbf{b} + \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{T_0 s^2}{qB}\left(1+\xi^2\right)\mathbf{b}\times\nabla\ln B \\
&\quad + \frac{2T_0 s^2}{qB^2}\left[\xi^2\left(\mathbf{I}-\mathbf{bb}\right) + \frac{1}{2}\left(1-\xi^2\right)\mathbf{bb}\right]\cdot\nabla\times\mathbf{B} + \frac{mv_0 s\xi}{qB^2}\mathbf{b}\times\frac{\partial\mathbf{B}}{\partial t} \\
\dot{s} &= -s\frac{d\ln v_0}{dt} + \frac{s\left(1-\xi^2\right)}{2}\frac{\partial\ln B}{\partial t} + \frac{q}{2T_0 s}\left(\mathbf{v}_\parallel + \mathbf{v}_{\mathrm{c}}\right)\cdot\mathbf{E} + \frac{s}{2}\left(1+\xi^2\right)\frac{\mathbf{E}\times\mathbf{B}}{B^2}\cdot\nabla\ln B \\
\dot{\xi} &= \frac{1-\xi^2}{2\xi}\left\{-\xi^2\frac{\partial\ln B}{\partial t} + \left(\mathbf{v}_\parallel + \mathbf{v}_{\mathrm{c}}^*\right)\cdot\left(\frac{q\mathbf{E}}{T_0 s^2} - \nabla\ln B\right) + \xi^2\frac{\mathbf{E}\times\mathbf{B}}{B^2}\cdot\nabla\ln B\right\} \\
&\quad - \xi(1-\xi^2)\left\{\frac{\mu_0}{2B^2}\mathbf{J}_\parallel\cdot\mathbf{E} + \frac{T_0 s^2}{q}\mathbf{b}\cdot\nabla\left(\frac{\mu_0 J_\parallel}{B^2}\right)\right\} + (1-\xi^2)\frac{T_0 s}{v_0 qB}\nabla\cdot\left(\mathbf{b}\times\frac{\partial\mathbf{b}}{\partial t}\right)
\end{aligned}
$$

where $\mathbf{v}_\parallel + \mathbf{v}_{\mathrm{c}} = \mathbf{v}_{\mathrm{gc}} - \dfrac{\mathbf{E}\times\mathbf{B}}{B^2} + \dfrac{T_0 s^2}{qB}\left(1+\xi^2\right)\mathbf{b}\times\nabla\ln B$ and $\mathbf{v}_{\mathrm{c}}^* = \dfrac{2T_0 s^2}{qB^2}\mathbf{J}_\perp + \dfrac{mv_0 s\xi}{qB^2}\mathbf{b}\times\dfrac{\partial\mathbf{B}}{\partial t}$. This agrees exactly with Hazeltine's DKE.

Linearization of NIMROD DKE

Assuming $\mathbf{E}_0 = 0$ and using a speed norm v_0 that is constant in time yields

$$
\begin{aligned}
\delta \mathbf{v}_{gc} =& v_0 s \xi \mathbf{b}_1 + \frac{\mathbf{E}_1 \times \mathbf{B}_0}{B_0^2} + \frac{T_0 s^2}{q B_0^3} (1+\xi^2) \Big[(\mathbf{I} - 4\mathbf{b}_0\mathbf{b}_0) \cdot \mathbf{B}_1 \times \nabla B_0 + \mathbf{b}_0 \times \nabla \big(\mathbf{B}_0 \cdot \mathbf{B}_1 \big) \Big] \\
&+ \frac{T_0 s^2}{q B_0^2} \Big\{ (1 - 3\xi^2) \delta \Big(\mathbf{bb} \Big) \cdot \nabla \times \mathbf{B}_0 + \Big[2\xi^2 (\mathbf{I} - \mathbf{b}_0\mathbf{b}_0) + (1-\xi^2) \mathbf{b}_0\mathbf{b}_0 \Big] \cdot \nabla \times \mathbf{B}_1 \Big\} \\
&- \frac{2 T_0 s^2}{q} \frac{\mathbf{b}_0 \cdot \mathbf{B}_1}{B_0^3} \Big[2\xi^2 (\mathbf{I} - \mathbf{b}_0\mathbf{b}_0) + \Big(1 - \xi^2 \Big) \mathbf{b}_0\mathbf{b}_0 \Big] \cdot \nabla \times \mathbf{B}_0 + \frac{m v_0 s \xi}{q B_0^2} \mathbf{b}_0 \times \frac{\partial \mathbf{B}_1}{\partial t} \\
=& v_0 s \xi \mathbf{b}_1 + \frac{\mathbf{E}_1 \times \mathbf{B}_0}{B_0^2} + \frac{T_0 s^2}{q B_0^3} (1+\xi^2) \Big[(\mathbf{I} - 4\mathbf{b}_0\mathbf{b}_0) \cdot \mathbf{B}_1 \times \nabla B_0 + \mathbf{b}_0 \times \nabla \big(\mathbf{B}_0 \cdot \mathbf{B}_1 \big) \Big] \\
&+ \frac{2 T_0 s^2}{q B_0^2} \Big\{ \xi^2 \Big(\mathbf{J}_{\perp 1} - \delta(\mathbf{bb}) \cdot \mathbf{J}_0 - \frac{2\mathbf{B}_0 \cdot \mathbf{B}_1}{\mathbf{B}_0^2} \mathbf{J}_{\perp 0} \Big) + \frac{1-\xi^2}{2} \Big(\mathbf{J}_{||1} + \delta(\mathbf{bb}) \cdot \mathbf{J}_0 - \frac{2\mathbf{B}_0 \cdot \mathbf{B}_1}{\mathbf{B}_0^2} \mathbf{J}_{||0} \\
&+ \frac{m v_0 s \xi}{q B_0^2} \mathbf{b}_0 \times \frac{\partial \mathbf{B}_1}{\partial t}
\end{aligned}
$$

where $\mathbf{b}_1 = (\mathbf{I} - \mathbf{b}_0\mathbf{b}_0) \cdot \mathbf{B}_1 / B_0$ and $\delta(\mathbf{bb}) = \mathbf{b}_1\mathbf{b}_0 + \mathbf{b}_0\mathbf{b}_1$.

Linearization of NIMROD DKE (cont.)

Linearizing the acceleration terms yields

$$\delta\dot{s} = -\frac{s\left(1-\xi^2\right)}{2B_0^2}\mathbf{B}_0\cdot\nabla\times\mathbf{E}_1 + \frac{q}{2sT_0}\mathbf{v}_{gc}\cdot\mathbf{E}_1 - s\delta\mathbf{v}_{gc}\cdot\nabla\ln v_0$$

$$= -\frac{s\left(1-\xi^2\right)}{2B_0^2}\mathbf{B}_0\cdot\nabla\times\mathbf{E}_1 + \frac{q}{2T_0 s}\left(\mathbf{v}_\parallel + \mathbf{v}_c\right)_0\cdot\mathbf{E}_1 + \frac{s}{2}\left(1+\xi^2\right)\frac{\mathbf{E}_1\times\mathbf{B}_0}{B_0^2}\cdot\nabla\ln B_0$$

$$-s\delta\mathbf{v}_{gc}\cdot\nabla\ln v_0$$

$$\delta\dot{\xi} = \frac{1-\xi^2}{2\xi}\left\{\frac{\xi^2\mathbf{B}_0}{B_0^2}\cdot\nabla\times\mathbf{E}_1 + \left(\mathbf{v}_\parallel + \mathbf{v}_c^*\right)\cdot\left(\frac{q\mathbf{E}_1}{T_0 s^2} - \nabla\ln B_1\right) - \delta\left(\mathbf{v}_\parallel + \mathbf{v}_c^*\right)\cdot\nabla\ln B_0\right.$$

$$+\xi^2\frac{\mathbf{E}_1\times\mathbf{B}_0}{B_0^2}\cdot\nabla\ln B_0 - \frac{\xi^2\mu_0}{B_0^2}\mathbf{J}_{\parallel 0}\cdot\mathbf{E}_1 - 2\frac{T_0 s^2\xi^2}{q}\left(\mathbf{b}_0\cdot\nabla\delta\left(\frac{\mu_0 J_\parallel}{B^2}\right) + \mathbf{b}_1\cdot\nabla\left(\frac{\mu_0 J_{\parallel 0}}{B_0^2}\right)\right)$$

$$\left.+\frac{2T_0\xi s}{v_0 q B_0}\nabla\cdot\left(\mathbf{b}_0\times\frac{\partial\mathbf{b}_1}{\partial t}\right)\right\}$$

Strongly coupled plasmas

The Landau form of the collision operator,

$$C_{ss'} = -\nabla_{\mathbf{v}} \cdot \int d\mathbf{v}' \mathbf{Q}_L\left(\mathbf{u}\right) \cdot \left(\frac{\nabla_{\mathbf{v}'}}{m_{s'}} - \frac{\nabla_{\mathbf{v}}}{m_s}\right) f_s\left(\mathbf{v}\right) f_{s'}\left(\mathbf{v}'\right) \ ,$$

can be extended to strongly coupled plasmas with coupling parameter $\Gamma = Z^2 e^2 / k_B T a_0 \lesssim 100$ where $a_0 = \left(3/4\pi n\right)^{1/3}$.

Here $\mathbf{u} = \mathbf{v} - \mathbf{v}'$ and $\mathbf{Q}_L\left(\mathbf{u}\right)$ is

$$\mathbf{Q}_L\left(\mathbf{u}\right) = \frac{1}{2}\frac{m_{ss'}^2}{m_s} u \left(u^2 \mathbf{I} - \mathbf{u}\mathbf{u}\right) \bar{\sigma}_{ss'}^{(1)}\left(u\right)$$

This is accomplished by generalizing the collisional cross section, $\bar{\sigma}_{ss'}^{(1)}\left(u\right)$.

Want $\bar{\sigma}^{(1)}_{ss'}(u)$ for Landau collision operator

The l^{th} momentum-transfer cross section is

$$\bar{\sigma}^{(l)}_{ss'}(u) = 2\pi \int_0^\infty dbb \left[1 - cos^l\left(\pi - 2\Theta(u)\right)\right].$$

with the scattering angle given as

$$\begin{aligned} \Theta &= b\int_{r_o}^{\infty} drr^{-2}\left[1 - \frac{b^2}{r^2} - \frac{2\phi_{ss'}(r)}{m_{ss'}u^2}\right]^{-1/2} \\ &= \bar{b}\int_0^{\bar{r}_0^{-1}} dz\left[1 - \left(\bar{b}z\right)^2 - \frac{2ze^{-1/z}}{\Lambda\xi^2}\frac{\phi_{ss'}}{\phi_{sc}}\right]^{-1/2} \end{aligned}$$

The screened Coulomb potential $\phi_{sc} = kq_sq_{s'}e^{-r/\lambda_D}/r$, $\Lambda = m_{ss'}v_T^2/(kq_sq_{s'}/\lambda_D)$, $\xi = u/\sqrt{v_{Ts}^2 + v_{Ts'}^2}$.

Use effective potential energies, $\phi_{ss'}$, for strongly coupled plasmas (EPT = effective potential theory).

Radial distribution functions and EPT

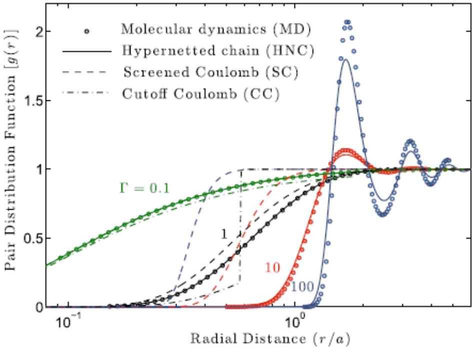


FIG. 1. Pair distribution function for a OCP obtained from MD (circles), HNC (solid lines), screened Coulomb potential (dashed lines), and the cutoff Coulomb potential (dashed-dotted lines—shown only for $\Gamma = 0.1$ and 1). [1]

The number of particles in a spherical shell surrounding a central molecule is $4\pi r^2 n g(r) dr$. Radial distribution function $g(r)$ is related to the effective potential by $g(r) = exp[-\phi_{ss'}/k_B T]$. Our plan is to get $g(r)$ from PYOZ.

[1] Baalrud and Daligault, *Phys Plasmas* **21**, 055707 (2014)

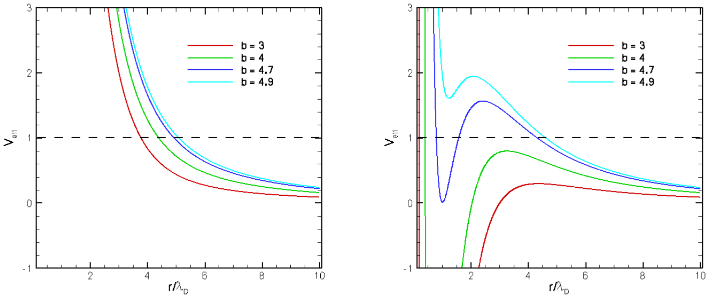
Details of $\bar{\sigma}_{ss'}^{(l)}$ calculation

Gauss-Legendre quadrature applied to momentum-transfer cross section integral:

$$\bar{\sigma}_{ss'}^{(l)}(u) = 2\pi \int_0^\infty dbb \left[1 - cos^l\left(\pi - 2\Theta(u)\right)\right].$$

Given b, λ_D, u find largest root of radical in scattering angle integral.

Plots of $V_{eff} = \frac{b^2}{r^2} + \frac{2\phi_{ss'}(r)}{m_{ss'}u^2}$ show that finding roots is difficult even for screened Coulomb potential.

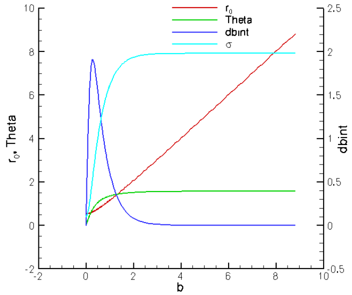


Scattering angle integral

Romberg integration applied to collision angle integral after root is found:

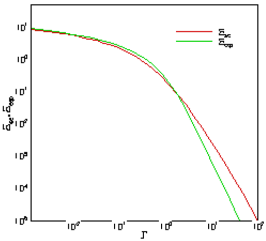
$$\Theta = \bar{b} \int_0^{\bar{r}_0^{-1}} dz \left[1 - \left(\bar{b}z\right)^2 - \frac{2ze^{-1/z}}{\Lambda \xi^2} \frac{\phi_{ss'}}{\phi_{sc}} \right]^{-1/2}$$

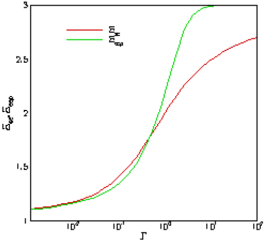
Integrand of impact parameter integral shows rapid convergence beyond a few Debye lengths.

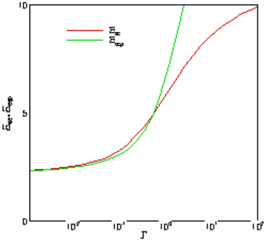


Collisional transport coefficients

Calculations of generalized Coulomb logarithms (related to collisional transport coefficients) highlights strongly coupled effects:
$\Xi_{ss'}^{(l,k)} = \frac{1}{2}\int_0^\infty d\xi \xi^{2k+3} e^{-\xi^2} \bar{\sigma}_{ss'}^{(l)}/\sigma_0$ where $\sigma_0 = (\pi q_s^2 q_{s'}^2)/(m_{ss'}^2 v_T^4)$.







Collisional cross section

Interpolate collisional cross section inside of Landau collision operator.