Stellarator Simulations With Reduced Divergence Errors
NIMROD Team Meeting

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April 21, 2018
Happy Earth Day (Eve)!
Outline

1. Project Goals and Background
2. Preliminary Results
3. Divergence Errors Coming From Boundary Conditions
4. Instability Is Uncovered
5. Ongoing and Future Work
Purpose

To study magnetic topology evolution and plasma confinement in stellarators with heating source, anisotropic conduction, and other extended MHD effects.

Goals:

- Study high beta effects in toroidal, not linear plasmas
- Analyze the effects of anisotropic thermal conduction on achievable beta
- Study the stability properties near beta limits
- Benchmark with HINT2 code
- Investigate the effects of plasma flow
Beta Effects Studied Previously With NIMROD

- Project continues the thesis work of Mark Schlutt.
- Simulations of helically symmetric (straight) stellarators show a variety of fieldline topological changes.
- Vacuum fields have analytic solutions with nested flux surfaces.
- Results show variation in achievable beta with heating rate and conduction anisotropy.

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>Heating Rate (MW/m²)</th>
<th>$\beta_{\text{max}}$ (%)</th>
<th>$\beta_{\text{onset}}$ (%)</th>
<th>$\gamma T_A$</th>
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<table>
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<tr>
<th>Symmetry</th>
<th>$\kappa_{|}/\kappa_{\perp}$</th>
<th>$\beta_{\text{max}}$ (%)</th>
<th>$\beta_{\text{onset}}$ (%)</th>
<th>$\gamma T_A$</th>
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<td>3.58</td>
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<td>$10^7$</td>
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<th>$\kappa_{|}/\kappa_{\perp}$</th>
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Increased Beta Leads to Changes in Magnetic Topology

Shafranov shift leads to break up of magnetic flux surfaces.

Pressure is supported on open fieldlines with long connection lengths.

**Q = 0.033291 MW**

**Q = 1.6646 MW**
Conduction Anisotropy Affects Temperature Distribution

Applied heating rate is fixed in these figures. Left side plot shows fixed perpendicular conduction at $\chi_\perp = 1$, right side plot shows fixed parallel conduction at $\chi_\parallel = 10^5$.
Conduction Anisotropy Affects Achievable Beta

Changes in thermal distribution result in significantly different achieved beta.
More Advanced Thermal Conduction Closures Have Been Attempted

Simulations have run with the full Braginskii magnetized conduction closure

\[ Q = 0.66583 \text{ MW} \]
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Boundary Condition Creates Magnetic Divergence Issues

- Stellarator magnetic fields are provided by fixed normal boundary conditions computed from coils.
- Finite resolution is incapable of resolving strong field variation when coils are close to the boundary leading to violation of the divergence free condition.
- The examples below (presented previously) show that divergence errors are reduced by increasing poloidal resolution.
- Attempts to improve this with additional resistivity alone had limited success.

Re $\text{div}(b)$, extrema=$(-7.511e-01, 7.511e-01)$

Re $\text{div}(b)$, extrema=$(-6.921e-02, 6.921e-02)$
Spectral Filtering Boundary Fields Reduces Divergence Errors

- Magnetic fields are computed at NIMROD quadrature positions with high toroidal resolution using the OCULUS Biot-Savart routine.
- Fields are transformed to Fourier space while directly enforcing coil toroidal periodicity.
- The largest mode number components are dropped, leading to reduced divergence errors.

\[
\text{Re } \text{div}(b), \text{ extrema}=(-1.120e+00, 1.120e+00)
\]

\[
\text{Re } \text{div}(b), \text{ extrema}=(-4.733e-03, 4.733e-03)
\]
Reducing Divergence Errors Directly Affects Current Noise

\[ \text{Re } J_Z, \text{ extrema}=(-5.327e+05, 3.497e+05) \]

\[ \text{Re } J_Z, \text{ extrema}=(-1.277e+04, 1.509e+04) \]

\[ \text{Re } J_Z, \text{ extrema}=(-8.359e+04, 9.964e+04) \]

\[ \text{Re } J_{\Phi}, \text{ extrema}=(-1.801e+04, 2.495e+04) \]
Kinetic Energy Spectrum is Vastly Different

- The magnetic energy spectrum is virtually unchanged by divergence error reduction.
- Kinetic energy is altogether different.
Removing Field Divergence Affects Beta

- Results with high $\chi_\parallel$ differ by 10-15%.
- Original simulation converged to equilibrium state, but after reducing divergence errors simulations crash with CFL drop.
- No signs of instability are present.
Simulations are restarted from equilibrium changing $z_{\text{period}} = 10$ to $z_{\text{period}} = 1$ in order to look at stability of non-stellarator symmetric modes (e.g. 1-9, 11-19, ...).
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First Signs Of Instability

First long run at low $\chi_\parallel$ with divergence reduction show instability that was not seen previously.

Log Kinetic Energy vs. $t$
Instability Has Little Effect on Pressure or Fieldlines

Results look nearly the same before and after the instability.
Most Obvious Changes are Seen in Flow and Current

Re $V_{\Phi}$, extrema=$(-1.533e+03, 1.533e+03)$

Re $V_{\Phi}$, extrema=$(-2.216e+03, 1.891e+03)$

Re $J_{\Phi}$, extrema=$(-2.923e+04, 4.342e+04)$

Re $J_{\Phi}$, extrema=$(-3.397e+04, 6.926e+04)$
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Ongoing and Future Work

- Run more simulations with reduced divergence error.
- Properly analyze stability as a function of beta.
- Consider comparison calculations with HINT2.
- Continue investigating the effects of Braginskii or other thermal conduction closures.
- More closely analyze other quantities (current evolution, effective heat transport, etc.).
- Attempt flow drive.
Extra: NIMROD Equilibria Below Beta Limit Could Be Produced Consistently

- MHD equilibria are produced by heating from vacuum with \( \text{zperiod limited} \) Fourier spectrum.
- At low beta steady state conditions are readily found.
- At high beta NIMROD fails to converge.
Converged reference has 21 modes, 24x24 grid, \( \text{poly\_degree} = 5 \).
Separate tests have been run with decreased \( dt \), increased \( n\text{modes} \), and increased \( \text{poly\_degree} \).
\( \langle \beta \rangle \) varies by at most 3% with increased resolution.
Tests with \( \text{eqn\_model} = \text{tonly} \) are consistent.
Extra: Comparison to Fixed Magnetic Fields

Magnetic fields can be held fixed in simulations as heat is applied.

By contrast, this shows that there is a complicated interplay between the thermal conduction anisotropy and the magnetic topology evolution.

**Fixed B, $\chi_{\perp,0} = 1$**
Extra: Beta Limits Have Been Studied with HINT2

- Pressure profile is fixed as $p = p_0(1 - s)(1 - s^4)$.
- At blue circle $\mathbf{J} \times \mathbf{B} = \nabla p$ can no longer be satisfied on stochastic field lines and pressure profile must be released $\rightarrow$ soft beta limit.
- At green circle hard beta limit is hit as axis is pushed into separatrix.

Extra: HINT2

- Solves for MHD equilibrium by relaxing initial condition.
- Toroidal coordinates make no assumption about magnetic geometry.
- Uses 4th order spatial finite differencing and RK4.

\[
p^{i+1} = \bar{p} = \frac{\int_{-L_{in}}^{L_{in}} \mathcal{F} p^i \frac{dl}{B}}{\int_{-L_{in}}^{L_{in}} \frac{dl}{B}}, \quad \mathcal{F} = \begin{cases} 
1 & : \text{for } L_C \geq L_{in} \\
0 & : \text{for } L_C < L_{in}
\end{cases}
\]

\[
\frac{\partial \mathbf{v}_1}{\partial t} = -\nabla p + \mathbf{j}_1 \times (\mathbf{B}_0 + \mathbf{B}_1)
\]

\[
\frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times [\mathbf{v}_1 \times (\mathbf{B}_0 + \mathbf{B}_1) - \eta (\mathbf{j}_1 - \mathbf{j}_{net})]
\]

\[
\mathbf{j}_1 = \nabla \times \mathbf{B}_1
\]