# Equilibrium Calculation with Toroidal Flow in NIMEQ

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# Motivation

- Plasma rotation has a significant impact on energy transport and macroscopic stability.
- Large flow speeds, particularly those on the order of the sound speed, imply notable alterations to equilibrium configurations with toroidal flow.
- Implementation of flow dependent equilibrium calculations establishes self-consistent for NIMEQ generated solutions (steady-state solution).

# Outline

- Theoretical Framework
- Modifications to NIMEQ
- Benchmarking NIMEQ with FLOW Code
  - Low-$\beta$ Tokamak
  - High-$\beta$ Tokamak
- Future Goals

# Assumptions and Free-Function Changes/Additions

- Isotropic thermal pressure
- Purely toroidal flow or $v = v_\phi$
- Only applicable for $\gamma = 1$, where $\gamma$ is the ratio of heat capacities
- $\Omega(\psi)$ and $D(\psi)$ models are manual inputs

$\Omega(\psi)$ : Toroidal angular velocity about the major radius

$D(\psi)$ : Density when there is no toroidal flow

$p_0(\psi)$ : Static thermal pressure

# MHD Closure and the $\gamma = 1$ Limit

As $\gamma \to 1$, the total thermal pressure goes to the analytical form

$$p(R,\psi) = p_0(\psi) e^{\frac{p_\omega(\psi)}{p_0(\psi)}\left(\frac{R^2 - R_0^2}{R_0^2}\right)}$$

where $p_\omega(\psi) = \frac{1}{2} R_0^2 D(\psi) \Omega(\psi)^2$.

Note:

- This form allows us to keep the same form of the GS equation.
- As $\Omega(\psi) \to 0$, $p(R,\psi) \to p_0(\psi)$.

The MHD closure equation can be rewritten as

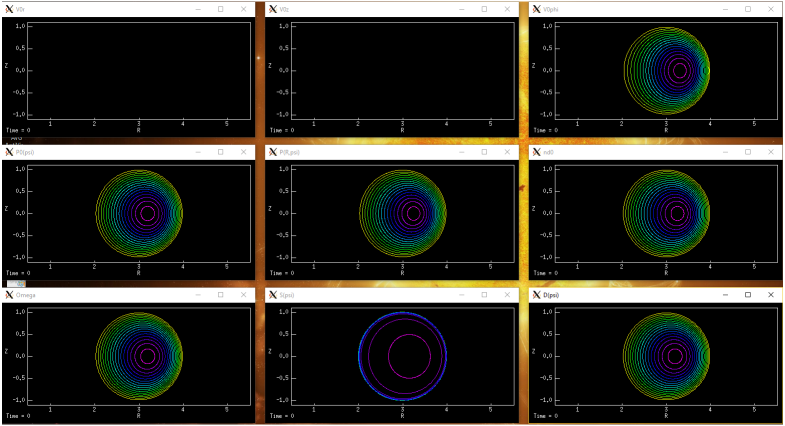
$$S(\psi) = \frac{p(R,\psi)}{\rho(R,\psi)^\gamma} = \frac{p_0(\psi)}{D(\psi)^\gamma}$$

where $S(\psi)$ is the entropy for $\gamma = \frac{5}{3}$ or the temperature for $\gamma = 1$.

# NIMEQ Changes/Adjustments

- New inputs for nimeq.in:
  - gs_flow_type="toroidal" or gs_flow_type="noflow"
  - om_model and d_model with corresponding om_func and d_func routines added
- Three new work arrays: rwork4,rwork5,rwork6
  - Only allocated for gs_flow_type="toroidal"
- Wrapper for calc_flow routines calc_gs_flow added in nimeq_mgt (similar to calc_fp but for flow parameters)
- New RHS routines for $p(R,\psi)$, $n(R,\psi)$, and $v_\phi(R,\psi)$
- New write_gradflow_bin routine in nimeq_mgt to show flow data (write_grad_bin is unaffected)

# Contour Plots for Flow Parameters



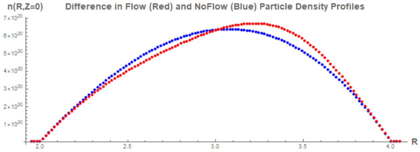

- $D(\psi)$ is converted to units of particle density at the end of gssolve
- $S(\psi)$ is in units of eV

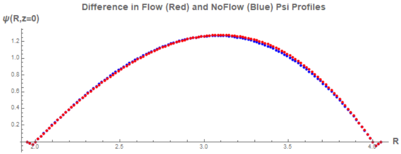
# Low-$\beta$ Equilibrium in FLOW Code

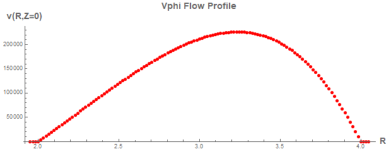
| Parameter | No Flow | Flow |
|---|---|---|
| $\psi_{max}$ | 1.272027 | 1.275802 |
| $q^*$ | 1.298187 | 1.298089 |
| $I_{tor}$ | 4.275577E+06 | 4.275901E+06 |
| $\beta_t$ | 1.222189E-02 | 1.231315E-02 |
| $\beta_p$ | 4.312505E-01 | 4.324678E-01 |
| $n_{max}$ | 6.397226E+20 | 6.512270E+20 |
| $p_{max}$ | 1.115588E+05 | 1.135243E+05 |

- Minor Radius: $a = 1\text{m}$
- Major Radius: $R_0 = 3\text{m}$
- $v_{\text{axis}} \sim 150,000$ m/s
  $M_\phi \sim 0.7$

Note: Low-beta equilibria will not see a large change in $\psi$, but the particle density is sensitive to flow.



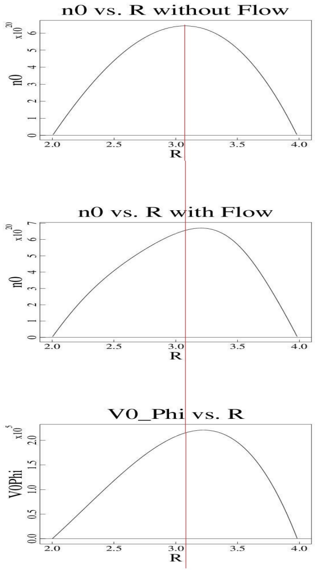



# NIMEQ Results

| Parameter | No Flow | Flow |
|---|---|---|
| $\psi_{max}$ | 1.272025 | 1.274451 |
| $q^*$ | 1.301576 | 1.294047 |
| $I_{tor}$ | 4.275314E+06 | 4.275677E+06 |
| $n_{max}$ | 6.397226E+20 | 6.539720E+20 |
| $p_{max}$ | 1.115588E+05 | 1.134690E+05 |

| Parameter | FLOW/NIMEQ Percent Difference |
|---|---|
| $\psi_{max}$ | 0.11% |
| $q^*$ | 0.31% |
| $I_{tor}$ | 0.01% |
| $n_{max}$ | 0.42% |
| $p_{max}$ | 0.05% |

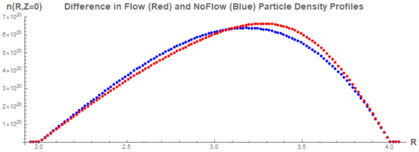
- Same shift given by FLOW
- Peak particle density increases by $\sim 2\%$

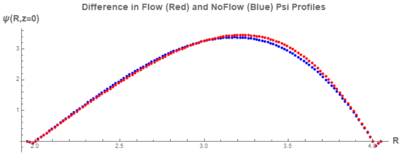


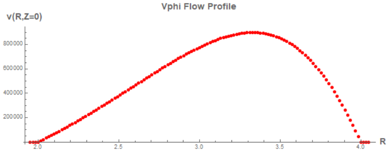
# High-$\beta$ Equilibrium in FLOW Code

| Parameter | No Flow | Flow |
|---|---|---|
| $\psi_{max}$ | 3.383617 | 3.458209 |
| $q^*$ | 5.244017E-01 | 5.249982E-01 |
| $I_{tor}$ | 1.058444E+07 | 1.057242E+07 |
| $\beta_t$ | 3.599479E-01 | 3.604553E-01 |
| $\beta_p$ | 1.861352E+00 | 1.818134E+00 |
| $n_{max}$ | 6.397226E+20 | 6.641821E+20 |
| $p_{max}$ | 3.346765E+06 | 3.473363E+06 |

- Same profiles as before
- Pressure scaled by factor of 30
- $v_{axis} \sim 800,000$ m/s<br>$M_\phi \sim$ 0.7
- Notable change in $\psi$ starting near the axis



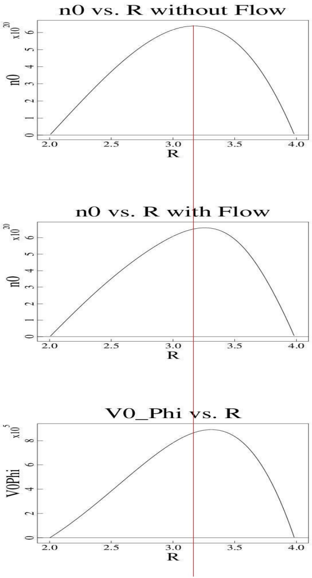



# NIMEQ Results

| Parameter | No Flow | Flow |
|---|---|---|
| $\psi_{max}$ | 3.383029 | 3.430394 |
| $I_{tor}$ | 1.057034E+07 | 1.056253E+07 |
| $n_{max}$ | 6.397230E+20 | 6.627620E+20 |
| $p_{max}$ | 3.346760E+06 | 3.466080E+06 |

| Parameter | FLOW Code/NIMEQ Percent Difference |
|---|---|
| $\psi_{max}$ | 0.81% |
| $I_{tor}$ | 0.09% |
| $n_{max}$ | 0.21% |
| $p_{max}$ | 0.21% |

- Peak particle density increases by $\sim 3.6\%$
- $\psi_{max}$ increases by $\sim 1.4\%$



# Future Work

- Extend NIMEQ's capabilities to calculate the equilibrium for purely toroidal flow for arbitrary $\gamma$ and the equilibrium for both toroidal and poloidal flow.
  - Poloidal flow velocities will dramatically effect equilibria.
  - gs_flow_type="poloidal"
- Integrate the new flow calculations into fgnimeq.

# References


1. S. Jardin, *Computational Methods in Plasma Physics*, CRC Press, Boca Raton FL, 2010
2. J. P. Freidberg, *Plasma Physics and Fusion Energy*, Cambridge University Press, Cambridge UK, 2007
3. L. Guazzotto, R. Betti, J. Manickam, S. Kaye, Physics of Plasmas 11, 604 (2004)