NIMROD simulations of Innovative Confinement Concepts

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NIMROD
Non-Ideal MHD with Rotation
Open Discussion Project

Plasma Science and Innovation
PSI-Center

Motivation

- inspired by successful SSPX simulations of coaxial flux injection and sustainment[a]
- coplanar flux injection is used in several ICC experiments (e.g. P. Bellan's experiment[b], Woodruff Scientific)
- study physics and numerics of helicity injection and flux amplification
- simulations will help improve operation and efficiency by elucidating physical processes of injection, columnation, reconnection, and flux amplification

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[a] Hooper, E. B., et al., "NIMROD resistive magnetohydrodynamic simulations of spheromak physics", PoP **15**, 2008

[b] Hsu and Bellan, "On Jets, Kinks, and spheromaks formed by a planar magnetized coaxial gun", PoP **12**, 2005

NIMROD[a] (NonIdeal MHD with Rotation - Open Discussion)

- massively parallel 3-D MHD simulation
- finite elements in poloidal plane and Fourier modes in toroidal direction → axisymmetric geometry
- utilizes Lagrange type quadrilateral structured finite elements in 2-D
- can handle extreme anisotropies, $\frac{\chi_\parallel}{\chi_\perp} \gg 1$
- flexibility to model general geometry → real experiments
- model experiment relevant parameters, $S > 10^7$
- semi-implicit advance, not restricted by magnetosonic CFL condition
- assumes a steady state background and evolves perturbed quantities $\rightarrow A(\mathbf{x}, t) = A_s(\mathbf{x}) + \delta A(\mathbf{x}, t)$
- allows linear and nonlinear simulations

NIMROD equations

- NIMROD evolves the extended MHD equations

$$
\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\
\mathbf{E} &= -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} \\
&\quad + \frac{m_e}{ne^2} \left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{JU} + \mathbf{UJ}) \right. \\
&\quad \left. + \sum_\alpha \frac{q_\alpha}{m_\alpha} \left(\nabla p_\alpha + \nabla \cdot \Pi_\alpha \right) \right] \\
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) &= \nabla \cdot D \nabla n \\
mn\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}\right) &= \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi - \nabla \cdot p_h \\
\frac{n_\alpha}{\gamma - 1}\left(\frac{\partial T_\alpha}{\partial t} + \mathbf{U}_\alpha \cdot \nabla T_\alpha\right) &= -\nabla \cdot q_\alpha + Q_\alpha \\
&\quad - p_\alpha \nabla \cdot \mathbf{U}_\alpha - \Pi_\alpha : \nabla \mathbf{U}_\alpha
\end{aligned}
$$

Inductive Flux Injection Model

- NIMROD has two available flux injection models[a]
  - direct flux injection via Faraday's law (specifying tangential **E** field at the boundary)
  - inductive flux injection via Ampere's law (specifying a tangential **B** field at the boundary)
- inductive flux injection (i.e. specifying input current) is closer to (coaxial gun source) experiment
- exploit integral Ampere's law $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$
- specify B_ϕ at the boundary corresponding to coaxial gap to induce poloidal current in simulation domain

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[a] C. Sovinec, "Ohmic Current Drive in NIMROD Simulation", NIMROD internal note, 2005

Inductive Flux Injection Model cont.

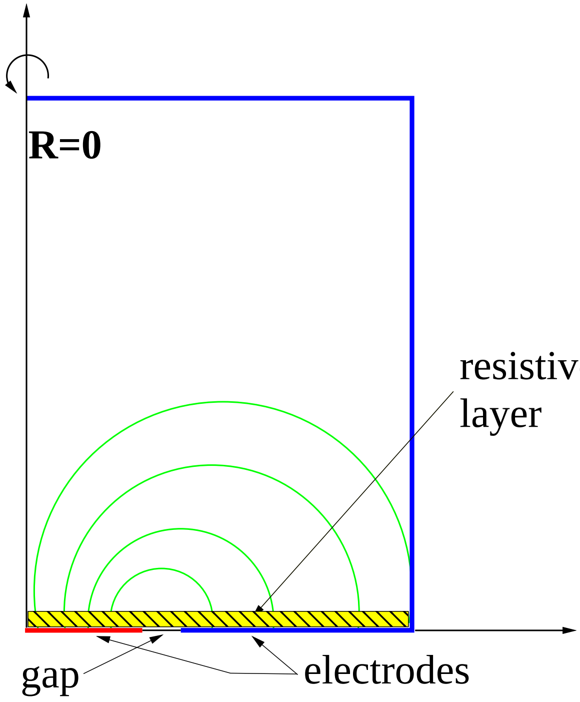
- along the boundary corresponding to flux gap specify $B_\phi R$
  - otherwise boundary condition is no-slip perfect conductor
- prescribed $B_\phi R$ induces a poloidal current in the simulation domain
- amplitude of $B_\phi R$ may vary with time (e.g. constant slope or programmed from experiment)

? thin highly resistive layer along bottom to rapidly diffuse flux across bottom

- the resulting $\mathbf{J} \times \mathbf{B}$ force pulls in the flux, drives columnation

Coplanar Flux Injection Simulation

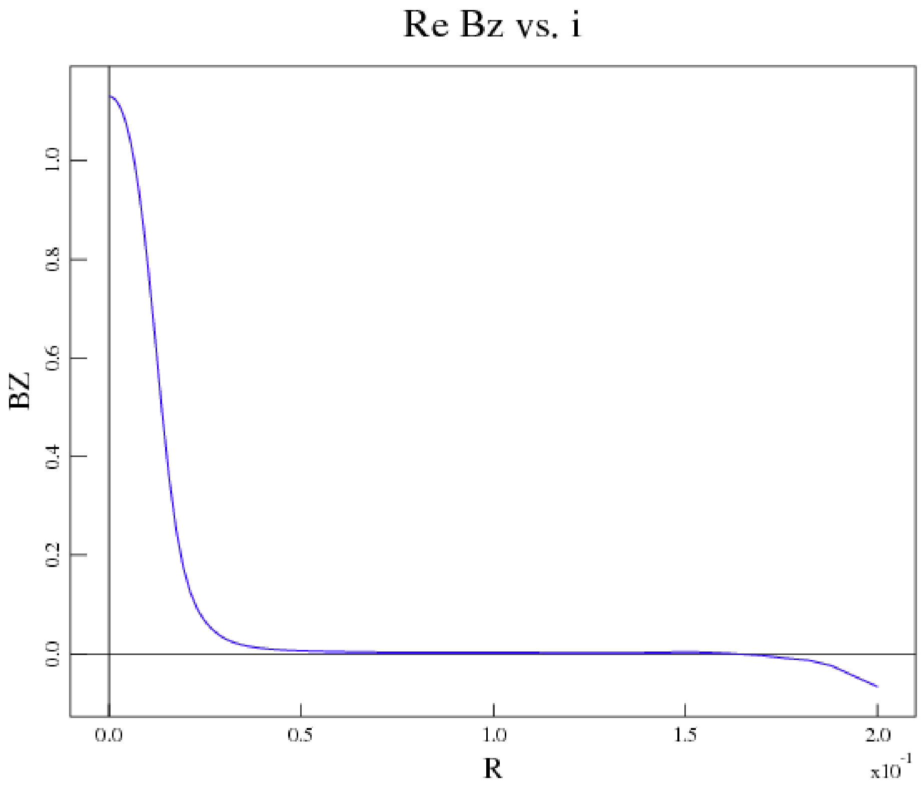
- cylindrical vessel with small flux gap of a few centimeters

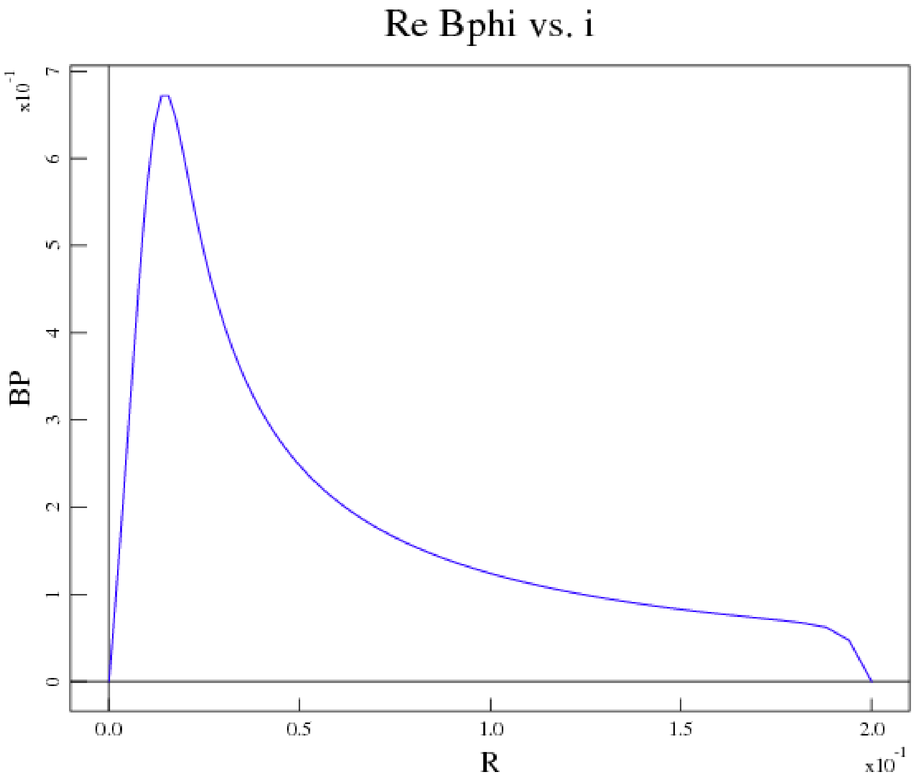


- vacuum field is dipole-like $\sim 1mWb$
  - flux gap located at top of the arc
- peak current is $40 - 100kA$ ramped over $1 - 10\mu s$
- thin highly resistive layer across bottom 10^5 larger than background resistivity

Formation of Axisymmetric Screw Pinch

- domain is cylinder radius $20cm$, height $25cm$, electrode gap at $[5.5cm, 6cm]$
- density 1×10^{19}, $v_A \simeq .5 \times 10^6$, $1mWb$
- current ramped over $5\mu s$ to $80kA$ and flat topped, $\lambda_{gun} \simeq 90$


Re Bz vs. i


Re Bphi vs. i

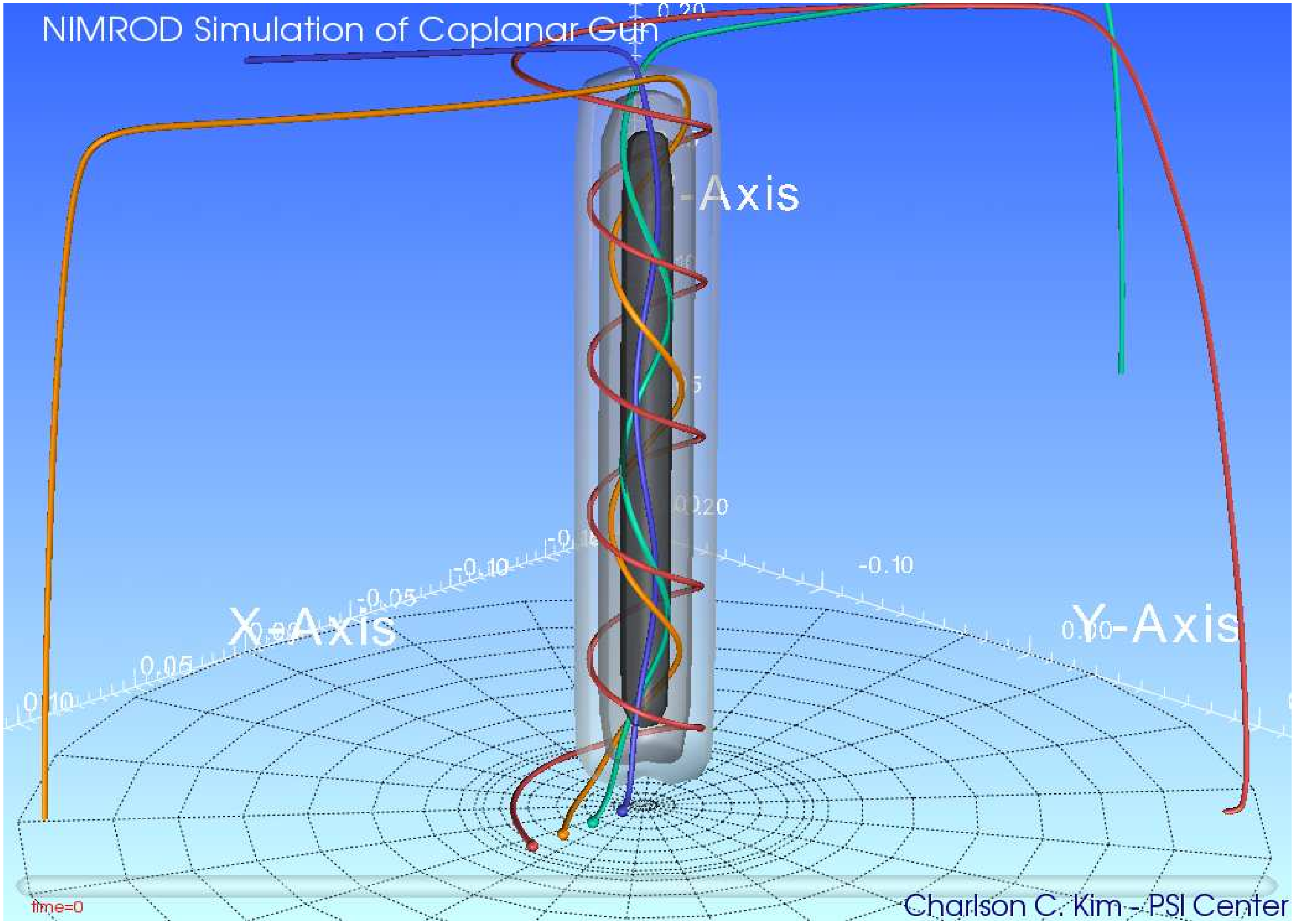
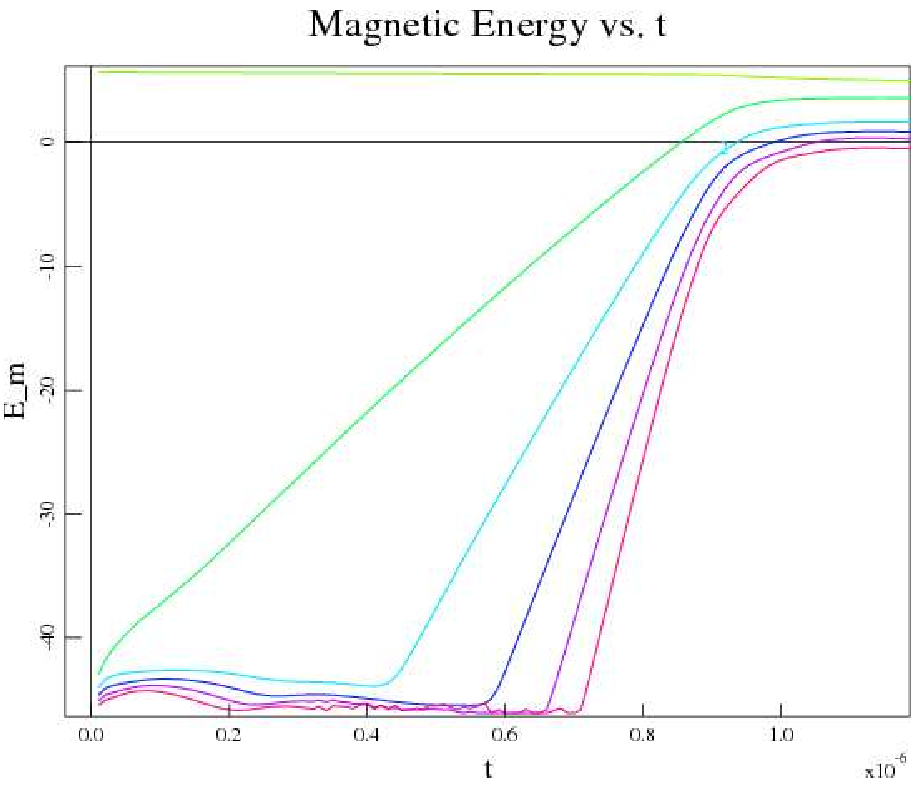


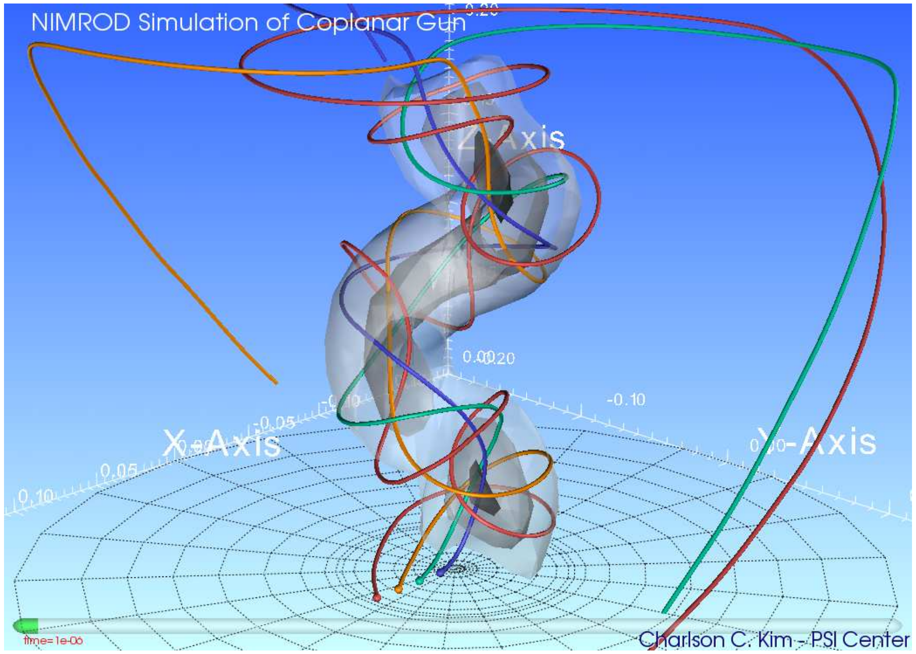
Figure 1: screw pinch - contours [.3, .6, .9]$|B|_{max}$ with field lines

- serves as initial condition for subsequent simulations

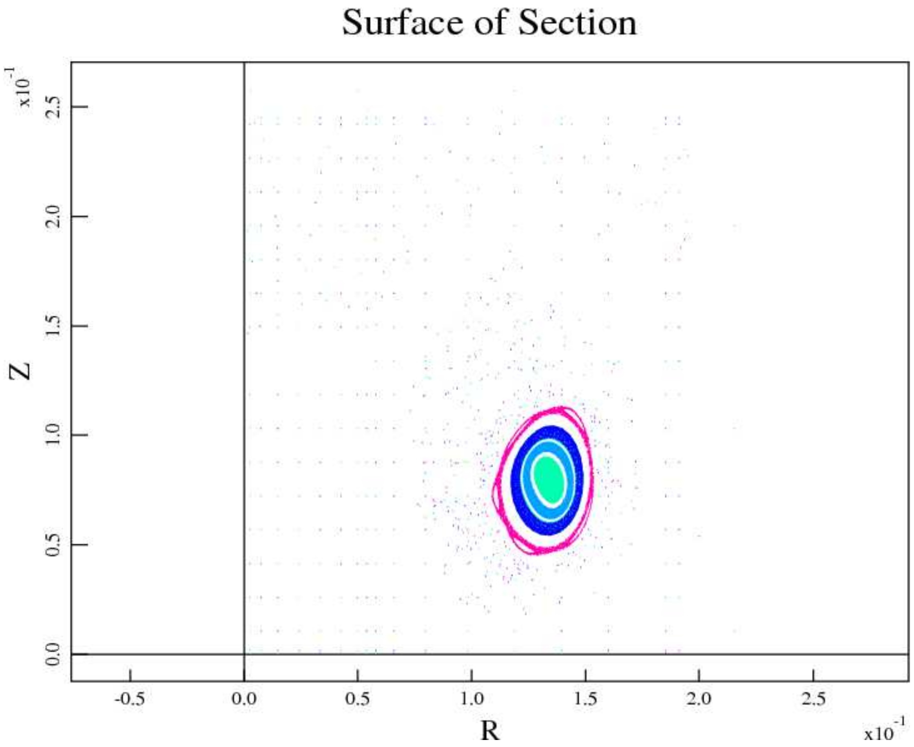
Kink and Initial Relaxed State

- current is shut off and screw pinch initialized with toroidal modes $n = [0 - 5]$
- very unstable, fast growth rate ($\sim 4.9 \times 10^7 s^{-1}$)

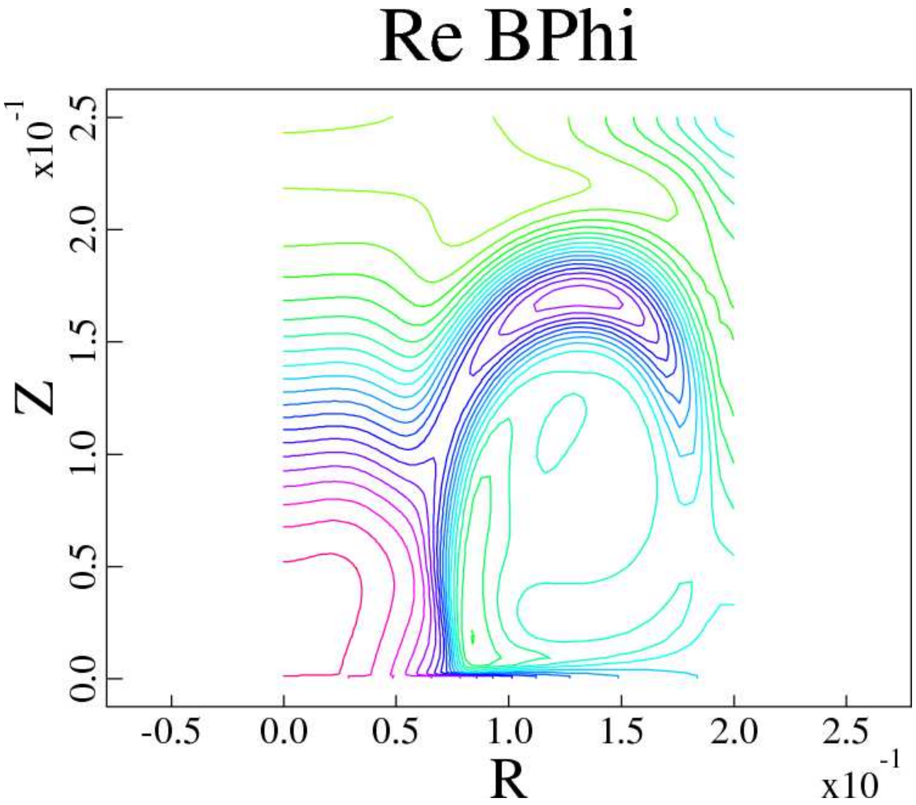




- small spheromak is formed but buried by initial dipole field



(a) surface of section $60\mu s$

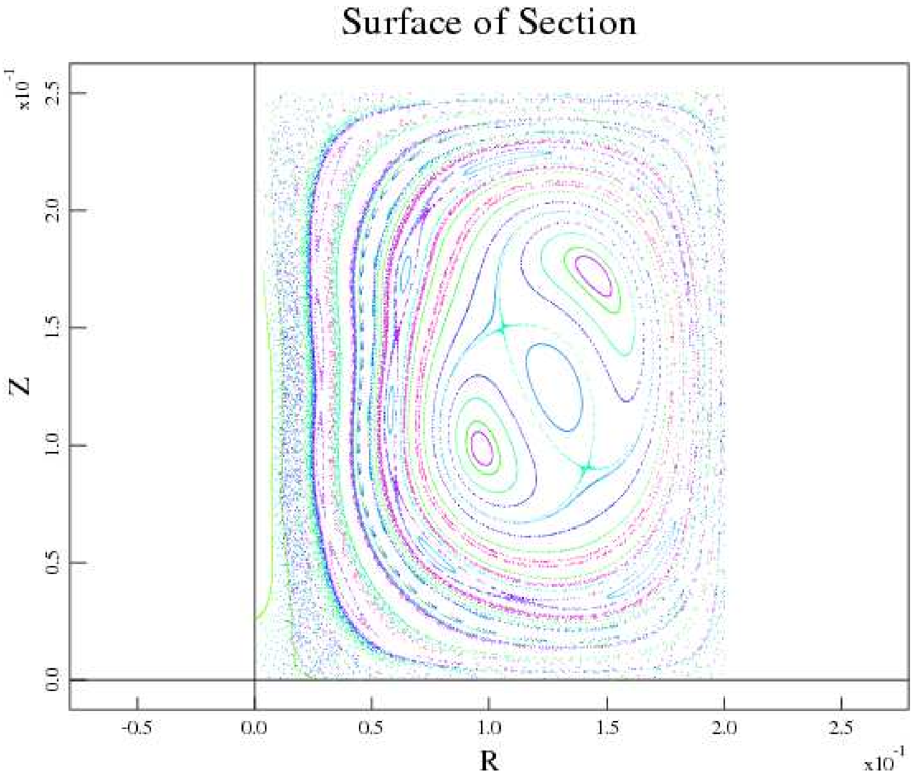


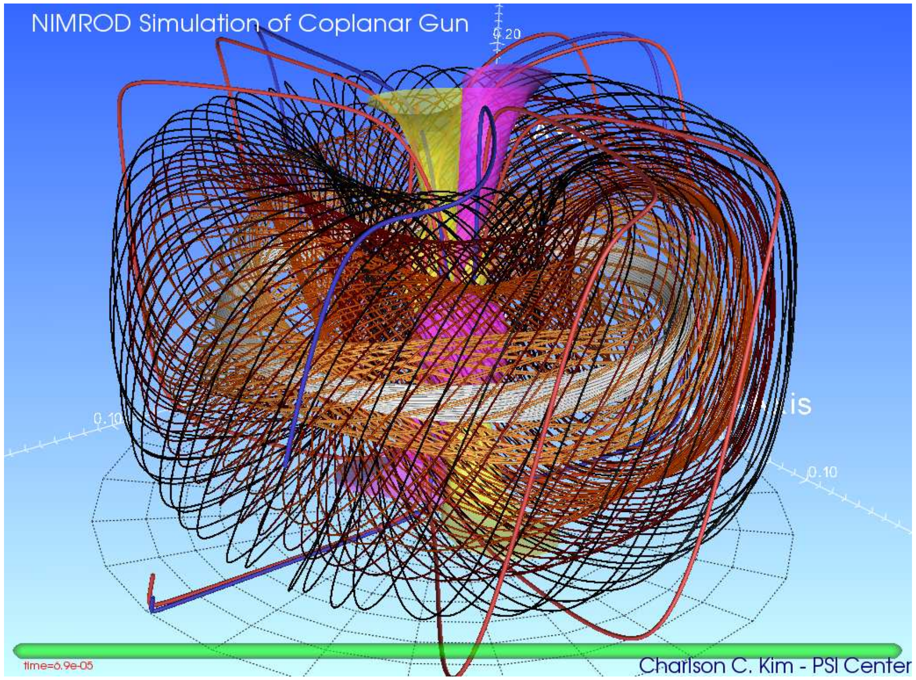
(b) $B_\phi n = 1$

- volumetrically $n = 1$ energy is 10% of axisymmetric energy, dominantly in remnant dipole field
- quiescent in region of closed flux

Numerically ‘clip’ magnetic footpoints at the electrodes

- without line-tying form a robust spheromak

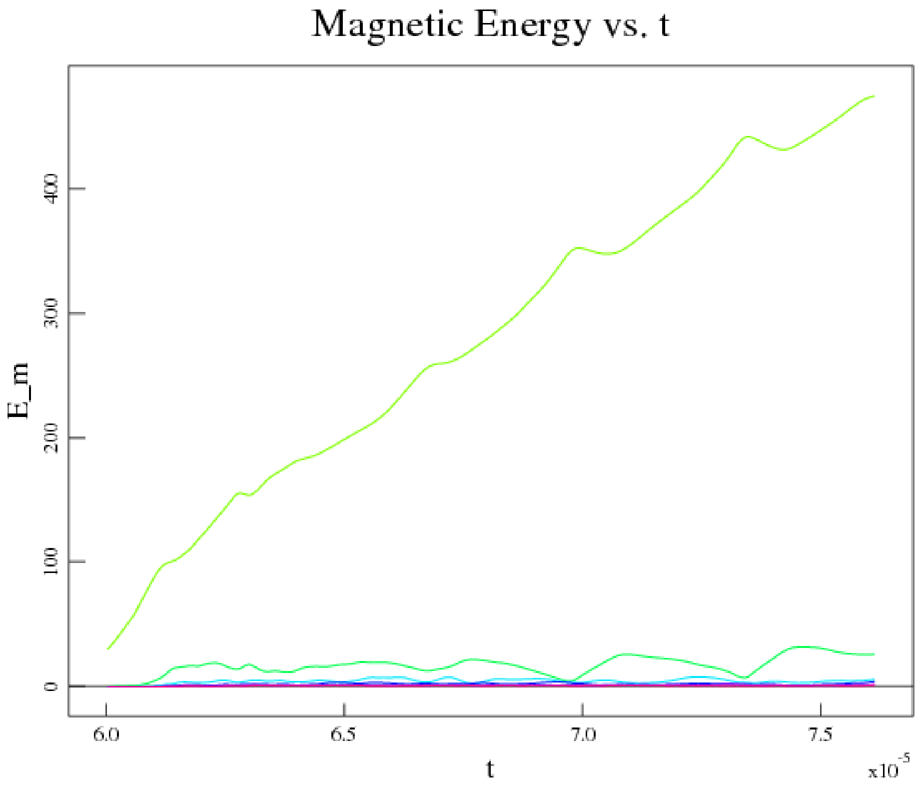




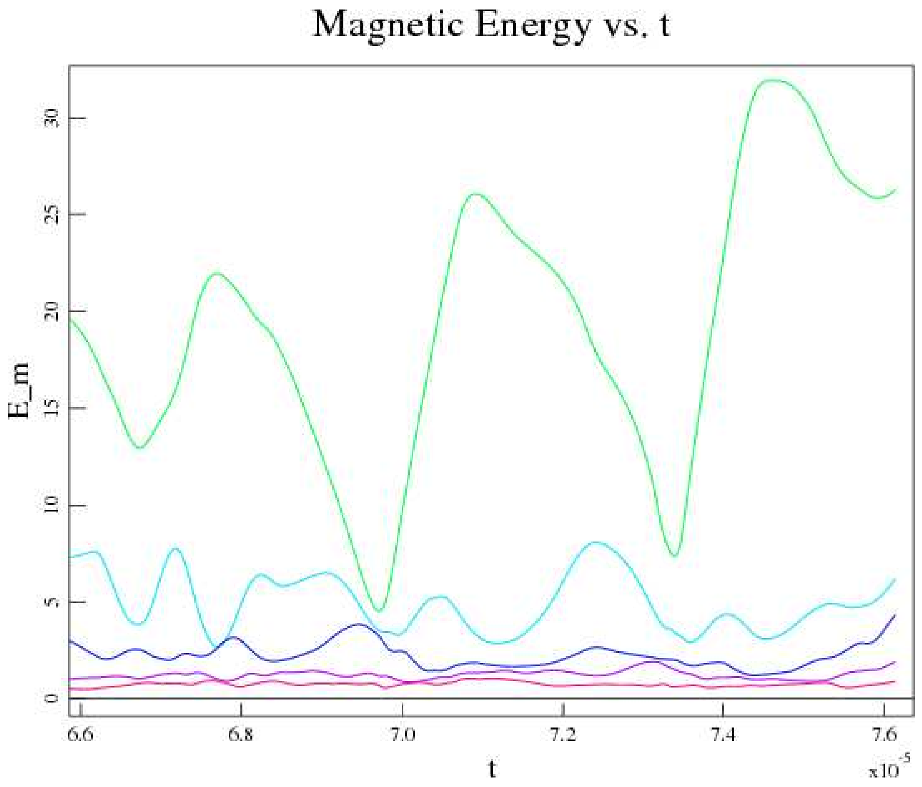
- one can infer strong impact of line tied fields

Driven Spheromak

- turn the current back on - $80kA$
- observe interesting sawtooth-like behavior in n=1 energy



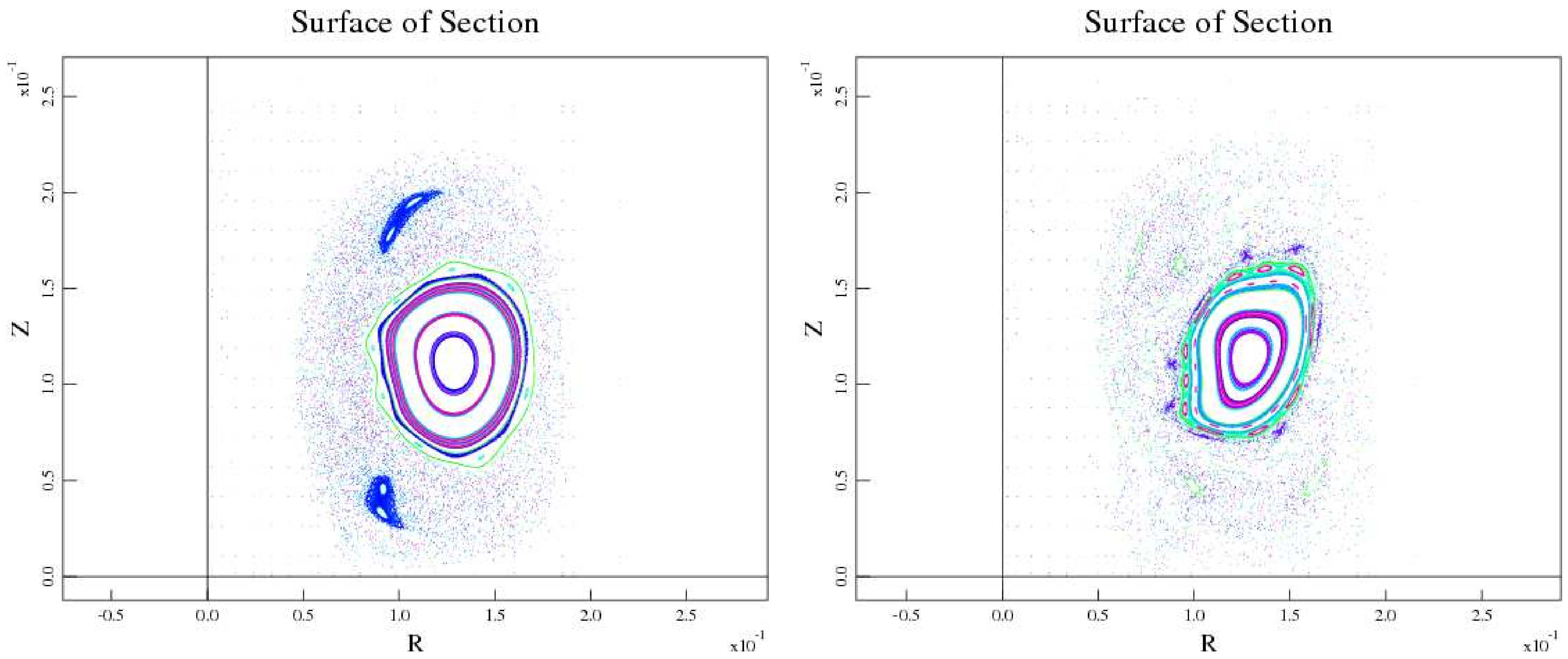
(c) Magnetic Energy



(d) Blowup of n=1-5

Second Relaxation

- two simulations with current shut off at $67.65\mu s$ and $70.9\mu s$



(e) $67.65\mu s$

(f) $70.9\mu s$

Figure 2: surface of sections $40\mu s$ after current shut off

- does it matter when the current is shut off - e.g. peak or trough

Summary

- coplanar flux injection model robust
- demonstrate formation, relaxation, drive cycle of spheromak with coplanar gun
- initial dipole field strongly impacts quality of spheromak formation
- driven spheromak exhibits sawtooth like behavior
- driven spheromaks produce larger spheromaks
- very preliminary simulations
- much more physics to turn on (compressibility,heating,2fl,etc)