Preliminary results on kinetic effects of energetic particles on nonlinear resistive MHD instability

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Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET

Experimental data from the DIII-D, Asdex, JT-60U and JET experiments show only JET breaks the model of onset of the 2/1 near ideal MHD limit.

• Model: parametric $\Delta'$ near ideal limit (Brennan 2002/3) in modified Rutherford equation for a $\rho_i^*$ dependence of onset (La Haye 2008).

Fit with pole at 1.2 to $\rho_i^* \Delta' r$

(La Haye et al. N. Fusion 2008)

Classic theory: The linear tearing stability index
Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET, the 2/1 is stable in JET

Buttery (2007, APS)
Buttery et al (IAEA, 2008)

Puzzle: Why does the JET experiment not show instability like the others?

Likely reason: energetic particles stabilize the 2/1 mode.

- JET ($\beta_{\text{frac}} > 30\%$),
- DIII-D, JT-60U ($\beta_{\text{frac}} < 20\%$


OTHER Possible Causes?

- Accurate $\Delta'$ calculation (Brennan 2002/3/6).
- Accurate equilibrium.
- Other physics, two-fluid effects … ?
Recent Results Show Energetic Particle/MHD Coupling Important and Computationally Viable

Historical focus has been on the simplified effects on the 1/1 mode.

Recent Computational Efforts Successful

• Choi, Turnbull, Chan (GA) Show highly accurate prediction of the sawtooth crash in DIII-D (2007).


Our resistive MHD analyses suggest possible energetic particle stabilization of resistive 2/1 modes at high energetic particle beta fractions.
In the Hybrid-Kinetic Approach, Initial value MHD computations are coupled to a $\delta f$ model

In the limit \( n_h \ll n_0 \) and \( \beta_h \sim \beta_0 \) quasi-neutrality, the only Modification of the MHD equations is addition of a energetic particle tensor in the momentum equation

\[
\rho \frac{d\mathbf{V}}{dt} = \rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbf{p}_b - \nabla \cdot \mathbf{p}_h
\]

where \( \mathbf{p}_h = \mathbf{p}_{h0} + \delta \mathbf{p}_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix} = \int m(\mathbf{v} - \mathbf{v}_h)^2 \delta f(x,\mathbf{v})d\tau \)

Is computed from a code advancing the change in the distribution function $\delta f$
Steady state fields satisfy a scalar pressure force balance

\[ \mathbf{J}_0 \times \mathbf{B}_0 = \nabla p_0 + \nabla p_{h0}, \]

where the assumption is that the equilibrium anisotropic energetic pressure component is 0 and the tensorial \( p_{h0} \) reduces to a scalar \( p_{h0} \).

The steady state fields satisfy a scalar pressure force balance, which limits the form of the equilibrium energetic particle distribution to isotropic distribution.
The $\delta f$ PIC model

- PIC is a Lagrangian simulation of phase space $f(x,v)$
- PIC evolves the $f(x(t),v(t))$
- $\delta f$ PIC reduces the discrete particle noise associate with conventional PIC
- Vlasov equation
  \[
  \frac{\partial f(z)}{\partial t} + \dot{z} \cdot \frac{\partial f(\dot{z})}{\partial t} = 0
  \]
- Evolution equation for $\delta f$, $\dot{\delta f} = -\delta z \cdot \frac{\partial f_0}{\partial t}$.

- the drift kinetic equations of motion are used as the particle characteristics

\[
\begin{align*}
\dot{x} &= v_\parallel \hat{b} + \frac{E \times B}{B^2} + \frac{m^2}{eB^4} (v_\parallel^2 + \frac{v^2_\perp}{2})(B \times \nabla \frac{B^2}{2}) - \frac{\mu_0 m v^2_\parallel}{eB^2} J_\perp, \\
mv_\parallel &= -\hat{b} \cdot (\mu \nabla B - eE).
\end{align*}
\]
The slowing down distribution function for energetic particles

The slowing down distribution function

\[ f = \frac{P_0 \exp\left(\frac{P_\zeta}{\psi_n}\right)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}, \quad P_\zeta \propto \psi, \quad \psi_n = C \psi_0 \]

The linearized evolution equation for \( \delta f \) becomes

\[ \delta \dot{f} = f_0 \left\{ \frac{mg}{e \psi_n B^3} \left[ (v_\parallel^2 + \frac{v_{\perp}^2}{2}) \delta \mathbf{B} \cdot \nabla B - \mu_0 v_\parallel \mathbf{J}_\perp \cdot \delta \mathbf{E} \right] \right. \]

\[ + \frac{\mathbf{v}_D \cdot (\nabla \psi_n - \rho_\parallel \nabla g)}{\psi_n} + \frac{3}{2} \frac{e \varepsilon^{1/2}}{\varepsilon^{3/2} + \varepsilon_c^{3/2}} \mathbf{v}_D \cdot \delta \mathbf{E} \}, \]

where

\[ \mathbf{v}_D = \frac{mg}{eB^3} \left( v_\parallel^2 + \frac{v_{\perp}^2}{2} \right) (\mathbf{B} \times \nabla B) + \frac{\mu_0 m v_{\perp}^2}{eB^2} \mathbf{J}_\perp, \]

\[ \delta \mathbf{v} = \frac{\delta \mathbf{E} \times \mathbf{B}}{B^2} + v_\parallel \frac{\delta \mathbf{B}}{B} \cdot \delta \mathbf{E}. \]
Equilibrium pressure and safety factor profiles as a function of $\psi$ in the D shape


$Pr$ (the ratio of the viscosity to electric diffusivity) = 100

$$f \sim \exp(\psi / C)$$

$q_{\text{min}} \approx 1.5, \quad q_{95} \approx 4.4$
Anisotropic pressure of energetic particles produces real frequency, the 2/1 mode rotates

\[ \beta_{\text{frac}} = 12.5\% , \]
\[ S = 2.7 \times 10^6 , \]
\[ \beta_N/4l_i = 0.9 , \]
\[ \gamma \tau_A = 4.0 \times 10^{-4} , \]
\[ \omega \tau_A = 0.8 \times 10^{-4} \]

ideal 1/1 mode
\[ \beta_{\text{frac}} = 12.5\% , \]
\[ \beta_N/4l_i = 0.41 , \]
\[ \gamma \tau_A = 9.5 \times 10^{-3} , \]
\[ \omega \tau_A = 1.7 \times 10^{-3} \]

2/1 modes with real frequencies observed

• Similar to ideal 1/1 mode (Kim 04,08)
The eigen function of $V_r$, the $n=1$ spatial projection of $\delta f$ in phase space, Trapped cone

Trapped particle region of phase space
Linear Growth rates (of the resistive 2/1 mode) as a function of S for MHD only cases, $\exp(-4\psi)$
Growth rates (of the resistive 2/1 mode) as a function of S (linear cases), MHD only, $\exp(-4\psi)$


$S$ (the ratio of the resistive time to Alfvén time), Pr (the ratio of the viscosity to electric diffusivity) = 100
Growth rates and real frequencies (of the resistive 2/1 mode) as a function of S (linear cases)

\[ p \propto \exp(-4\psi) \]

\[ \beta_{\text{frac}} = 0.0\% \]

\[ \beta_{\text{frac}} = 6.25\% \]

\[ \beta_{\text{frac}} = 12.5\% \]

\[ \beta_{\text{frac}} = 25.0\% \]

\[ \beta_{N}/4l_i = 0.75 \]

\[ \beta_{N}/4l_i = 0.82 \]

\[ \beta_{N}/4l_i = 0.90 \]

\[ \text{O : marginal cases} \]
Growth rates and real frequencies (of the resistive 2/1 mode) as a function of $S$ (linear cases)

$p \propto \exp(-4\psi)$

$\beta_{N/4l_i} = 0.90$

$\beta_{N/4l_i} = 1.05$
Growth rates for series of equilibria \( (\beta_N / 4l_i) \)

(stability diagram sketch)

\[ \gamma \tau_A \times 10^{-2} \]

- Ideal limit
- ideal 2/1 mode
- resistive 2/1 mode (\( S=10^6 \))

ideal stable

Nucl. Fusion
MHD only nonlinear results \((\beta_N / 4l_i = 0.83, S = 10^6)\)
Nonlinear results ($\beta_N / 4l_i = 0.83, \ S=10^6$)

With energetic particles

It does not reach to the nonlinear phase yet …

The same weight function is used for the each modes.
A real frequency of the 2/1 mode ($\beta_N / 4 \iota_i = 0.83$)

nonlinear results

At the nonlinear phase, a real frequency increases? or decreases?

Spin up?

Spin down?
A test case for the nonlinear result \((\beta_N/4l_i=0.83)\)

The case of \(\beta_{frac}=1\%\), \((mx,my)=(48,32)\)
Conclusion and Discussion

Coupled Energetic Particles and resistive MHD

• Linear resistive MHD analyses suggest energetic particle stabilization of 2/1 modes at high energetic particle fractions. -->no onset 2/1 mode, JET
  • The growth rate as a function of S are damped for higher particle fractions, accompanied by an increasing real frequency.
  • The growth rates significantly reduce with $\beta$ due to mode resonance of the trapped particles and “barely passing” particles. (Similar to Kim08.)
  • An energetic particle effect driven in the bulk of the plasma, not a direct effect in the tearing layer. Thus, strongly affects the resistive mode.
  • Near the ideal limit, still, but (weaker) damping effects (2/1 ideal mode has a “weaker” damping effects).
Conclusion and Discussion

• Nonlinear
  • Precession rate --> two fluid model.
  • nlayers, rblocks, NIM(RE)SET… etc.
• Analytic, pseudo-analytic analysis. --> Explain the physics of the stabilizing effects.
  • resistive wall mode with energetic particle … etc.
  e.g. B. Hu & R. Betti (2004,PRL).
• PEST-III development.