Benchmarking NIMROD's Simultaneous Advance of Temperature and CEL Kinetic Closures with Ion Acoustic Modes*

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Abstract

The Chapman-Enskog-like (CEL) method [1] is a mechanism to provide self-consistent kinetic closures for plasma fluid models, extending their scope of validity to the long mean free path regime. The continuum kinetic CEL formulations used in NIMROD couple the ion and electron drift kinetic equations to extended MHD providing closures for collisional friction and energy exchange and parallel heat flows and stresses. In order to handle the tight, nonlinear fluid/kinetic coupling, a Newton-Krylov method is used to advance temperatures and kinetic distortion simultaneously over time steps large relative to thermal transport processes. To test if this approach is an improvement over other CEL formulations in NIMROD, we examine the nonlinear damping of sound waves.

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[1] J. J. Ramos, Phys Plasmas 17, 082502 (2010); J. J. Ramos, Phys Plasmas 18, 102506 (2011).

Fluid Equations in NIMROD

$$n_\mathrm{e} = Z_\mathrm{eff} n_\mathrm{i}$$

$$\frac{\partial n_\mathrm{e}}{\partial t} + \nabla \cdot (n_\mathrm{e} \mathbf{u}) = 0$$

$$m_\mathrm{i} n_\mathrm{i} \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi}_\mathrm{i}$$

$$\frac{3}{2} n_\mathrm{i} \left(\frac{\partial T_\mathrm{i}}{\partial t} + \mathbf{u}_\mathrm{i} \cdot \nabla T_\mathrm{i} \right) = -n_\mathrm{i} T_\mathrm{i} \left(\nabla \cdot \mathbf{u}_\mathrm{i} \right) - \nabla \cdot \mathbf{q}_\mathrm{i}$$

$$\frac{3}{2} n_\mathrm{e} \left(\frac{\partial T_\mathrm{e}}{\partial t} + \mathbf{u}_\mathrm{e} \cdot \nabla T_\mathrm{e} \right) = -n_\mathrm{e} T_\mathrm{e} \left(\nabla \cdot \mathbf{u}_\mathrm{e} \right) - \nabla \cdot \mathbf{q}_\mathrm{e}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(-\mathbf{u} \times \mathbf{B} + \eta \mathbf{J} \right)$$

The CEL DKE* can provide self-consistent continuum kinetic closures in NIMROD

$$\frac{\partial f_{\mathrm{NM}}}{\partial t} + v_0 s\xi \mathbf{b}\cdot\nabla f_{\mathrm{NM}} - \frac{1-\xi^2}{2\xi} v_0 s\xi \mathbf{b}\cdot\nabla \ln B \frac{\partial f_{\mathrm{NM}}}{\partial \xi}$$
$$+\frac{v_0}{2}\left(\mathbf{b}\cdot\nabla\ln n\right)\left[\xi\frac{\partial f_{\mathrm{NM}}}{\partial s} + \frac{1-\xi^2}{s}\frac{\partial f_{\mathrm{NM}}}{\partial \xi}\right] - s\left[\xi\mathbf{b}\cdot\nabla + \frac{\partial}{\partial t}\right]\ln v_0 \frac{\partial f_{\mathrm{NM}}}{\partial s} = \langle C\left(f\right)\rangle$$
$$+[\left(\frac{5}{2} - s^2\right) v_0 s\xi \mathbf{b}\cdot\nabla\ln T + \frac{v_0 s\xi}{nT}\mathbf{b}\cdot\left[\frac{2}{3}\nabla\pi_{\parallel} - \pi_{\parallel}\nabla\ln B - \mathbf{R}\right]$$
$$+2s^2\left(\frac{3}{2}\xi^2 - \frac{1}{2}\right)\left[\frac{1}{3}\nabla\cdot\mathbf{u} - \mathbf{bb}\cdot\nabla\mathbf{u}\right] + \frac{2}{3nT}\left(s^2 - \frac{3}{2}\right)\left[\mathbf{b}\cdot\nabla q_{\parallel} - q_{\parallel}\mathbf{b}\cdot\nabla\ln B - Q\right]$$
$$+\frac{2}{3eB}s^2\left(\frac{3}{2}\xi^2 - \frac{1}{2}\right)\left[\left(\frac{5}{2} - s^2\right)\left(\nabla\ln B - 2\kappa\right) + \nabla\ln n\right]\cdot\nabla T\times\mathbf{b}$$
$$+\frac{4}{3eB}\left(\frac{s^4}{2} - \frac{5}{2}s^2 + \frac{15}{8}\right)\left(\nabla\ln B + \kappa\right)\cdot\nabla T\times\mathbf{b}]f_{\mathrm{M}}$$

Numerical Methods

- Semi-implicit FD (time)
- FEM (2D Poloidal plane + pitch angle)
- Fourier (Toroidal angle)
- Collocation (speed)

$$(s,\xi) \equiv \left(\left|\mathbf{v}-\mathbf{u}\right|/v_T, \left(\mathbf{v}-\mathbf{u}\right)\cdot\mathbf{b}/\left|\mathbf{v}-\mathbf{u}\right|\right)$$

*J. J. Ramos, *Phys. Plasmas* **17**, 082502 (2010).

Time-advance of DKE+Fluid System

- Question:
  How should the distribution function be advanced in NIMROD?

$$[\mathbf{u}, f_{\mathrm{NMi}}]\left(t^{k}\right) \quad n, [T_{\mathrm{e}}, T_{\mathrm{i}}, \mathbf{B}, f_{\mathrm{NMe}}]\left(t^{k+\frac{1}{2}}\right) \quad [\mathbf{u}, f_{\mathrm{NMi}}]\left(t^{k+1}\right) \quad n, [T_{\mathrm{e}}, T_{\mathrm{i}}, \mathbf{B}, f_{\mathrm{NMe}}]\left(t^{k+\frac{3}{2}}\right)$$

- Stability is a concern*

$$[\mathbf{u}, f_{\mathrm{NMe}}]\left(t^{k}\right) \quad n, [T_{\mathrm{e}}, T_{\mathrm{i}}, \mathbf{B}, f_{\mathrm{NMi}}]\left(t^{k+\frac{1}{2}}\right) \quad [\mathbf{u}, f_{\mathrm{NMe}}]\left(t^{k+1}\right) \quad n, [T_{\mathrm{e}}, T_{\mathrm{i}}, \mathbf{B}, f_{\mathrm{NMi}}]\left(t^{k+\frac{3}{2}}\right)$$

- Possible Kinetic Closures:
  $q_{s\|}$ and Q_{ei} in T eqns
  $\pi_{\mathrm{i}\|}$ in the CoM flow eqn
  $\pi_{\mathrm{e}\|}$ and $\mathbf{R}_{\mathrm{e}}$ in Ohm's law
  $\pi_{\mathrm{i}\|}$ and $q_{\mathrm{i}\perp}$ in T_{i} eqn
  $\mathbf{R}_{s}$ in T eqns

- Combination of Picard and Simultaneous Newton-Krylov

$$q_{s\|} = \pi m_s v_{Ts}^6 \int_0^\infty ds s^5 \int_{-1}^1 d\xi \xi f_{\mathrm{NM}s}$$

$$\pi_{s\|} = \pi m_s v_{Ts}^5 \int_0^\infty ds s^4 \int_{-1}^1 d\xi \left(3\xi^2 - 1\right) f_{\mathrm{NM}s}$$

$$Q_{\mathrm{ei}} = \pi m_{\mathrm{e}} v_{T\mathrm{e}}^5 \int_0^\infty ds \int_{-1}^1 d\xi s^4 \left[C_{\mathrm{ei}}\left(f_{\mathrm{Me}}, f_{\mathrm{i}}\right) + C_{\mathrm{ei}}\left(f_{\mathrm{NMe}}, f_{\mathrm{Mi}}\right)\right]$$

$$\mathbf{R}_{\mathrm{e}} = \eta_\perp \mathbf{J} + \sum_b 2\pi m_{\mathrm{e}} v_{T\mathrm{e}}^4 \int ds s^3 \int_{-1}^1 d\xi \xi \left[C\left(f_{\mathrm{NMe}}, f_{\mathrm{M}b}\right) + C\left(f_{\mathrm{M}b}, f_{\mathrm{NMe}}\right)\right]$$

*Recall Joseph Jepson's talk from yesterday

Semi-implicit Newton-Krylov method

$$\frac{\partial \boldsymbol{f}}{\partial t} = \boldsymbol{\mathcal{L}}(\boldsymbol{f}) + \boldsymbol{\mathcal{S}} \xrightarrow{\text{semi-implicit time-discretization}} \frac{\boldsymbol{f}^{k+1} - \boldsymbol{f}^{k}}{\Delta t} = \theta \boldsymbol{\mathcal{L}}\left(\boldsymbol{f}^{k+1}\right) + (1-\theta)\,\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}^{k}\right) + \boldsymbol{\mathcal{S}}$$

$$\underbrace{\boldsymbol{f}^{k+1} - \theta \Delta t \left[\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}^{k+1}\right)\right]}_{\mathbf{A}\left(\boldsymbol{f}^{k+1}\right)} = \underbrace{\underbrace{\Delta t \left[\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}^{k}\right) + \boldsymbol{\mathcal{S}}\right]}_{\mathbf{b}\left(\boldsymbol{f}^{k}\right)} + \underbrace{\left\{\boldsymbol{f}^{k} - \theta \Delta t \left[\boldsymbol{\mathcal{L}}\left(\boldsymbol{f}^{k}\right)\right]\right\}}_{\mathbf{A}\left(\boldsymbol{f}^{k}\right)}}_{\mathbf{b}^{*}\left(\boldsymbol{f}^{k}\right)}$$

Use preconditioned GMRES to solve

$$\mathbf{J}\left(\boldsymbol{f}^{k+1,n}\right) \cdot \left(\boldsymbol{f}^{k+1,n+1} - \boldsymbol{f}^{k+1,n}\right) = \left[\mathbf{b}^{*}\left(\boldsymbol{f}^{k}\right) - \mathbf{A}\left(\boldsymbol{f}^{k+1,n}\right)\right]$$

where

$$J_{ij}\left(\boldsymbol{f}^{k+1,n}\right) = \partial A_i / \partial f_j \left(\boldsymbol{f}^{k+1,n}\right)$$

Iterate until satisfied with residual

Benchmark Application: Ion Acoustic Waves (IAWs)

Frequency and Damping Rate

$$\omega = \omega_0 \sqrt{\gamma_\mathsf{e} + \gamma_\mathsf{i} \frac{T_i}{T_e}}$$

$$\omega_0 = k\sqrt{k_\mathsf{B} T_\mathsf{e} / m_\mathsf{i}}$$

Empirical formula* for approximate solution to dispersion relation

$$\frac{\gamma_{\mathsf{LD}}}{\omega} = 1.1 \times \left(\frac{T_i}{T_e}\right)^{7/4} \exp\left[-\left(\frac{T_i}{T_e}\right)^2\right]$$

*Francis F. Chen, *Introduction to Plasma Physics and Controlled Fusion,* (1974)

Hybrid Fluid+Particle Method in M3D-C1-K*

$$\rho\left[\frac{\partial \mathbf{v}_\perp}{\partial t}+\left(\mathbf{v}_\perp+v_\parallel \mathbf{b}\right)\cdot\nabla\mathbf{v}_\perp\right]=\mathbf{J}\times\mathbf{B}-\nabla_\perp p_\mathrm{e}-\nabla_\perp\cdot\left[P_{\mathrm{i}\parallel}\mathbf{bb}+P_{\mathrm{i}\perp}\left(\mathbf{I}-\mathbf{bb}\right)\right]$$
$$-\nabla_\perp\cdot\left[P_{f\parallel}\mathbf{bb}+P_{f\perp}\left(\mathbf{I}-\mathbf{bb}\right)\right]+\nu\nabla^2\mathbf{v}_\perp$$

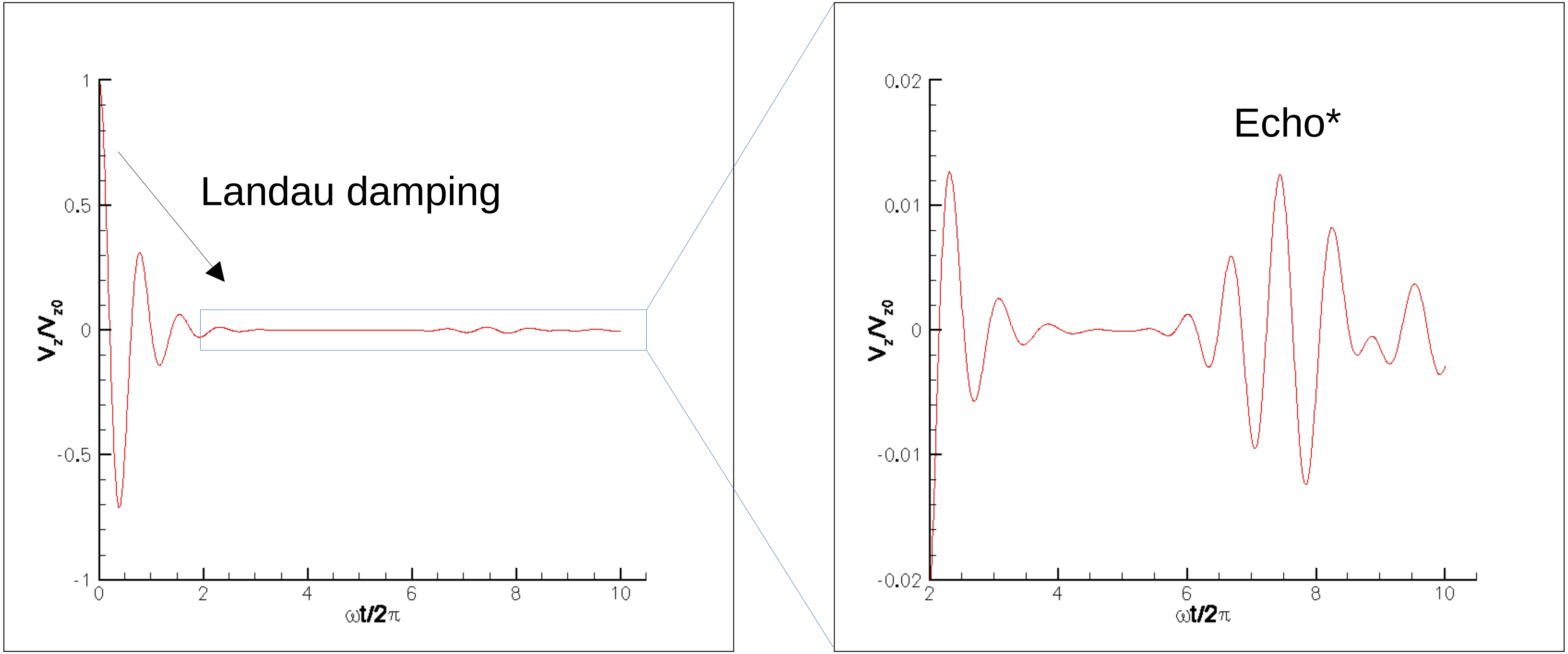
$$\mathbf{J}=\frac{1}{\mu_0}\nabla\times\mathbf{B}\qquad\qquad \frac{\partial\mathbf{B}}{\partial t}=-\nabla\times\mathbf{E}\qquad\qquad \mathbf{E}=-\mathbf{v}_\perp\times\mathbf{B}+\eta\mathbf{J}$$

$$\frac{\partial T_\mathrm{e}}{\partial t}+\left(\mathbf{v}_\perp+v_\parallel\mathbf{b}\right)\cdot\nabla T_\mathrm{e}=-\left(\gamma_\mathrm{e}-1\right)T_\mathrm{e}\nabla\cdot\left(\mathbf{v}_\perp+v_\parallel\mathbf{b}\right)+\nabla\cdot\left(\kappa_\perp\mathbf{I}+\kappa_\parallel\mathbf{bb}\right)\cdot\nabla T_\mathrm{e}$$

Closures from particles: $n_{i,f}\rightarrow(\rho,n_e)$, $v_\parallel$, $P_{\parallel i,f}$, and $P_{\perp i,f}$

*Chang Liu, et al, *J. Plasma Phys*. **88**, 905880610 (2022)

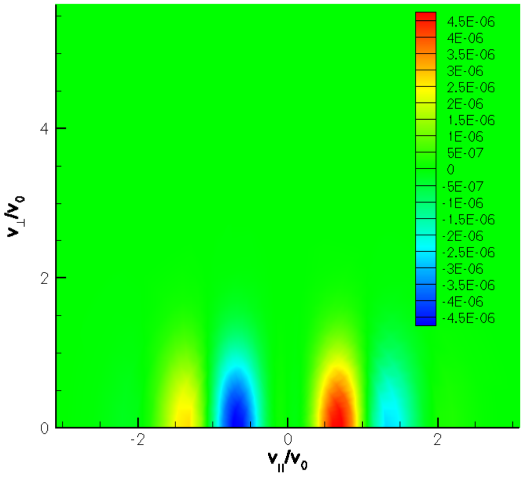
Ion acoustic wave time evolution



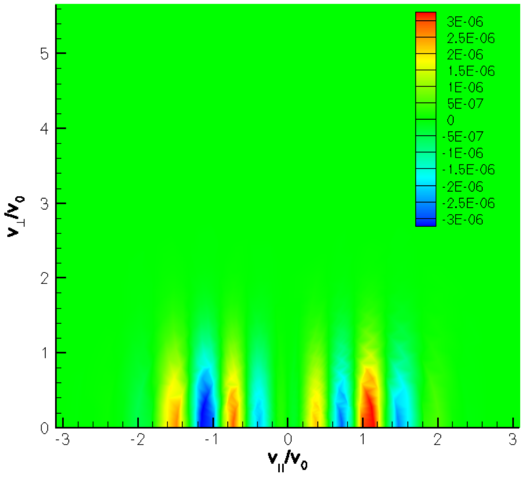
*If damping due to other diffusive processes is much slower than Landau damping, then an echo can occur due to the distribution retaining a ‘memory’ of previous oscillations

Time evolution of the non-Maxwellian distribution

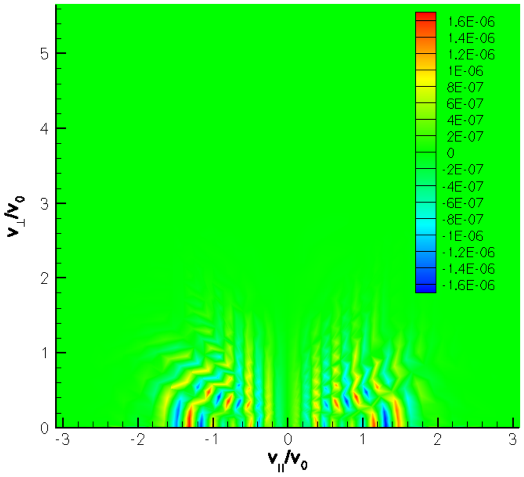
Landau damping (phase mixing) → Echo



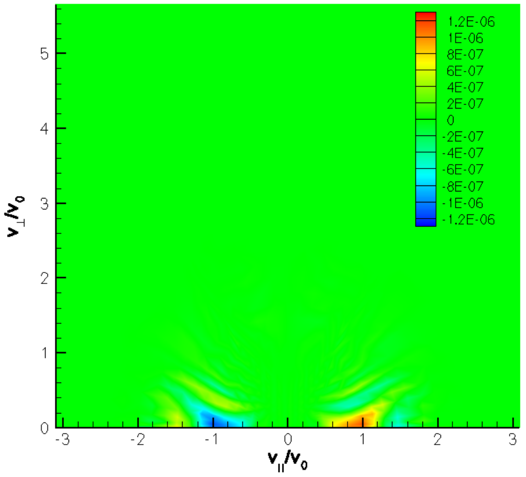
$$\omega t/2\pi \approx 0.75$$



$$\omega t/2\pi \approx 1.5$$

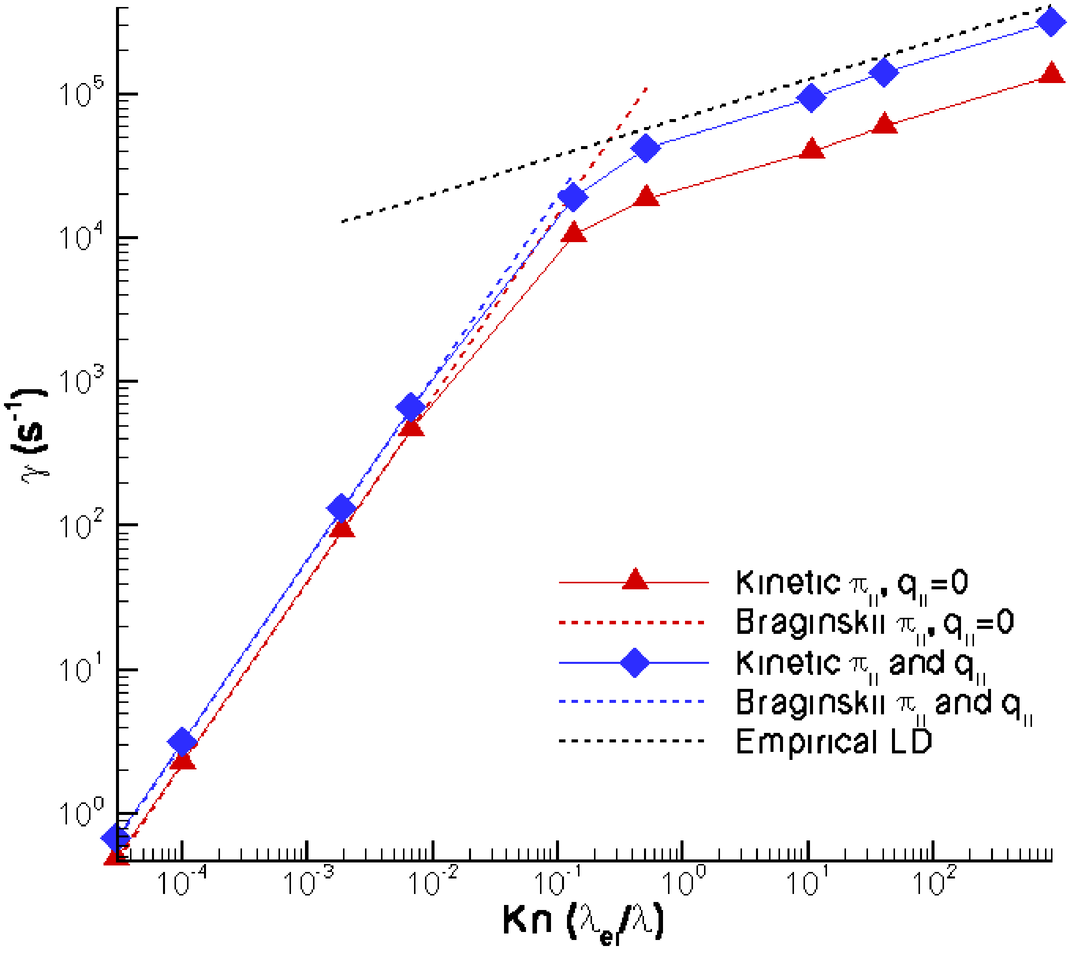


$$\omega t/2\pi \approx 4.5$$

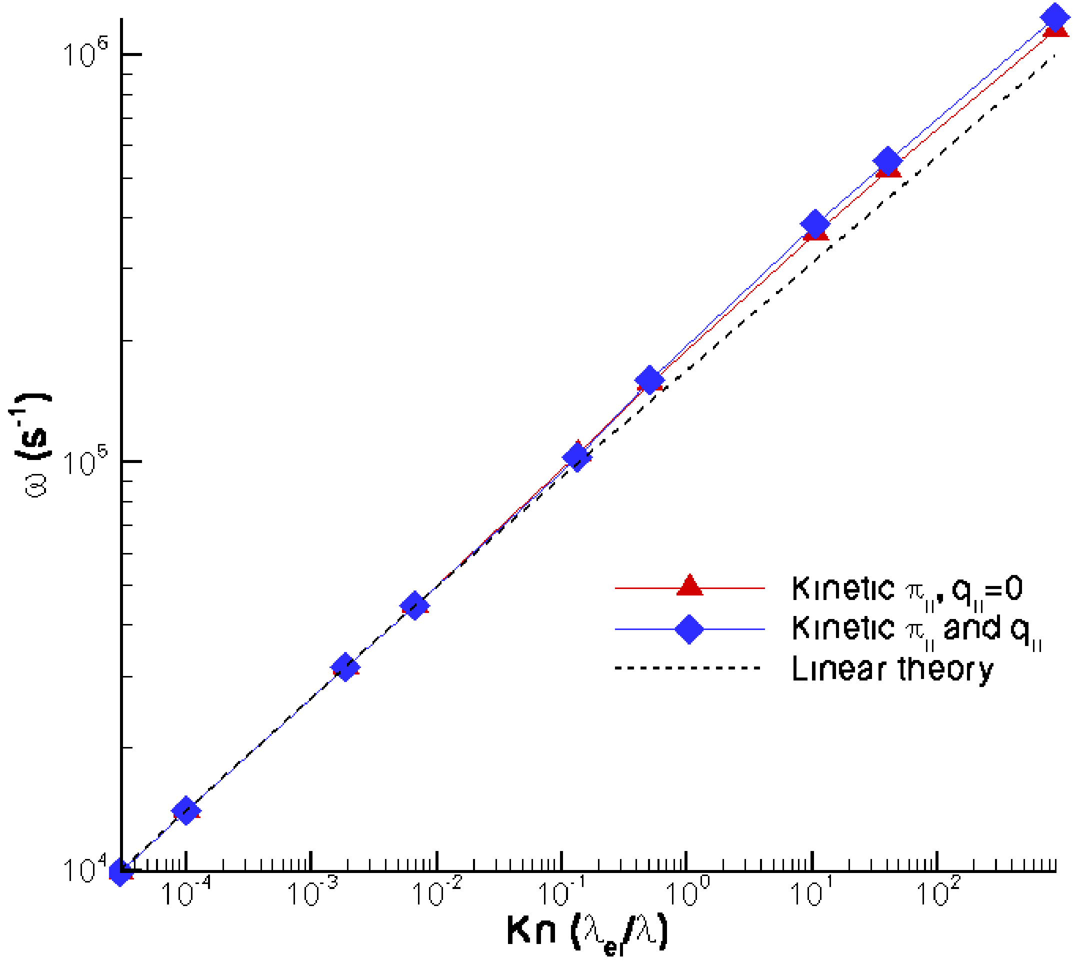


$$\omega t/2\pi \approx 9$$

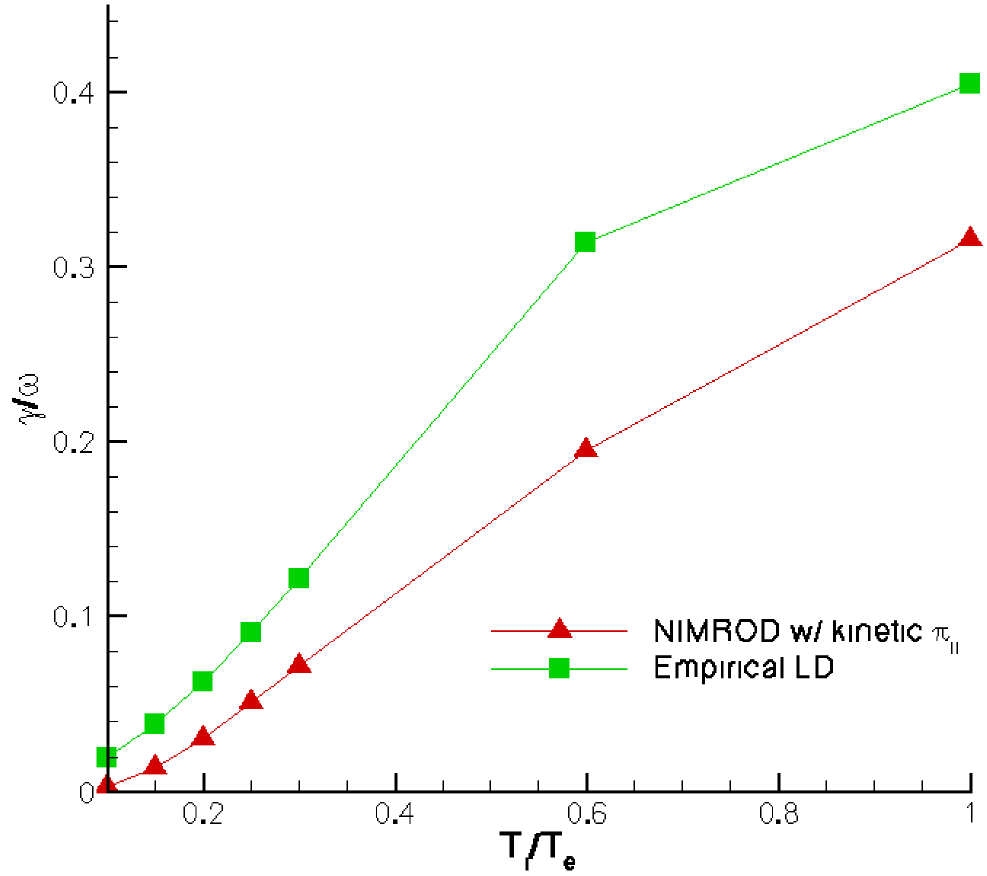
IAW damping rate vs Knudsen number



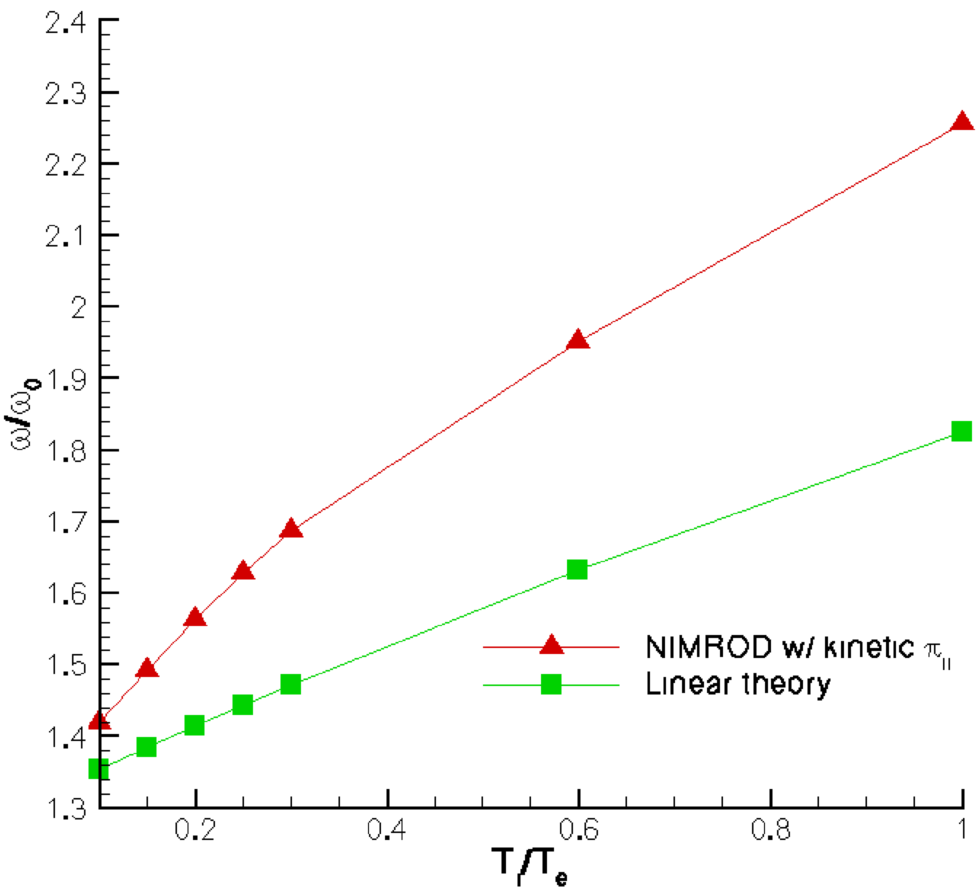
IAW frequency vs Knudsen number



IAW damping rate vs T_i/T_e



IAW frequency vs T_i/T_e



Future Work

- Direct comparison with M3D-C1-K
  (ensure both codes solve the same set of equations)
- Linearize the equations and compare nonlinear-linear solutions
- Bring in electron parallel heat flux
- Additional problems
  - Try CEL model to compute bootstrap current
  - Consider i-e collision operator
- Bring in B field
  - Picard iteration with B?
  - Fully implicit NK?