Verification of NIMROD with Fluid “ITG-like” Modes

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\textsuperscript{1}TriAlpha Energy
“ITG mode” = Ion Temperature Gradient Mode
Verification and Validation

• Verification
  – Are the equations being solved correctly?
  – Comparison with known solutions, or benchmarking with independent codes

• Validation
  – Are the right equations being solved?
  – Direct comparison with experiment

• Here we will deal with Verification
Verification of NIMROD in MHD

- NIMROD has been successfully verified in most realms of ideal and resistive MHD
  - Ideal MHD waves and instabilities
  - Resistive instabilities (linear and non-linear) in slab, cylindrical, and toroidal geometry
  - Anisotropic thermal conduction (comparison with theory)
  - Peeling and ballooning edge modes (comparison with ELITE)
  - Saweeth (comparison with M3D)
  - High-$\beta$ disruption (comparison with theory)
Verification of NIMROD in Extended MHD

• Energetic minority ion species
  – Kink stabilization/TAE destabilization (comparison with NOVA-K and M3D)

• Two-Fluid/FLR
  – Stabilization of g-mode in slab geometry (comparison with theory)
  – Drift-tearing modes (King)
  – De-stabilization of parallel sound wave by FLR effects (ITG-like mode)
    • Comparison with theory
    • Hope for comparison with kinetic code
FLR Effects on Fluid Modes
Two-fluid/FLR Equations

• Low order moments for ions and electrons

\[
\frac{\partial n}{\partial t} = -\nabla \cdot n \mathbf{V}_i = -\nabla \cdot n \mathbf{V}_e \quad \text{(quasi-neutrality)}
\]

\[
Mn \left( \frac{\partial \mathbf{V}_i}{\partial t} + \mathbf{V}_i \cdot \nabla \mathbf{V}_i \right) = ne \left( \mathbf{E} + \mathbf{V}_i \times \mathbf{B} \right) - \nabla p_i - \nabla \cdot \mathbf{\Pi}_i \quad \text{(ion viscous stress)}
\]

\[
0 = -ne \left( \mathbf{E} + \mathbf{V}_e \times \mathbf{B} \right) - \nabla p_e \quad \text{\(m_e = 0\), Ohm's law)}
\]

\[
\frac{\partial p_i}{\partial t} + \mathbf{V}_i \cdot \nabla p_i = -\frac{5}{3} \rho_i \nabla \mathbf{V}_i - \frac{2}{3} \nabla \cdot \mathbf{q}_i + Q \quad \text{\(\Gamma_i = \frac{5}{3}\)}\]

\[
\frac{\partial p_e}{\partial t} + \mathbf{V}_e \cdot \nabla p_e = -p_e \nabla \mathbf{V} \quad \text{\(\Gamma_e = 1\), isothermal)}
\]

\[
\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}
\]

\[
\nabla \cdot \mathbf{A} = 0, \quad \mathbf{J} = ne \left( \mathbf{V}_i - \mathbf{V}_e \right), \quad \nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}
\]
Extended MHD

• 2-fluid equations can be combined into “single fluid form” (extended MHD)

\[
\frac{\partial n}{\partial t} = -\nabla \cdot n \mathbf{V}
\]

\[
Mn \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla \left( p_i + p_e \right) + \mathbf{J} \times \mathbf{B} - \nabla \cdot \Pi_{\perp}
\]

\[
\frac{\partial p_i}{\partial t} + \mathbf{V} \cdot \nabla p_i = -\frac{5}{3} p_i \nabla \mathbf{V} - \frac{2}{3} \nabla \cdot q_{\perp}, \quad \frac{\partial p_e}{\partial t} + \mathbf{V} \cdot \nabla p_e = -p_e \nabla \mathbf{V}_e
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad \mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{1}{ne} \left( \mathbf{J} \times \mathbf{B} - \nabla p_e \right) + \eta \mathbf{J}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad \mathbf{V}_e = \mathbf{V} - \frac{\mathbf{J}}{ne}
\]
Expressions for Stress Tensor in Magnetized Plasma

- Can be decomposed as \( \Pi = \Pi_\parallel + \Pi_\perp + \Pi_\lambda \) orthogonal components

\[ \Pi_\parallel = \hat{b}\hat{b} \cdot \Pi = \frac{3p}{2v_c} \left( \hat{b} \cdot \mathbf{W} \cdot \hat{b} \right) \left( \hat{b}\hat{b} - \frac{1}{3} \mathbf{I} \right) \sim \frac{1}{v_c} \quad \text{unphysical as } v_c \to 0 \]

\[ \Pi_\perp = \left( \hat{b} \times \mathbf{l} \right) \cdot \Pi = \frac{p}{4\Omega} \left[ \left( \hat{b} \times \mathbf{W} \right) \cdot \left( \mathbf{I} + 3\hat{b}\hat{b} \right) + \left( \mathbf{I} + 3\hat{b}\hat{b} \right) \cdot \left( \mathbf{W} \times \hat{b} \right) \right] \quad \text{independent of } v_c \]

\[ \Pi_\lambda = \hat{b} \times \left( \hat{b} \times \mathbf{l} \right) \cdot \Pi = \frac{pv_c}{\Omega^2} \left\{ \left( \mathbf{I} - \hat{b}\hat{b} \right) \cdot \mathbf{W} \cdot \left( \mathbf{I} - \hat{b}\hat{b} \right) - \frac{1}{2} \left( \mathbf{I} - \hat{b}\hat{b} \right) \cdot \left( \mathbf{I} - \hat{b}\hat{b} \right) : \mathbf{W} \right\} + 4 \left[ \left( \mathbf{I} - \hat{b}\hat{b} \right) \cdot \mathbf{W} \cdot \hat{b}\hat{b} + \hat{b}\hat{b} \cdot \mathbf{W} \cdot \left( \mathbf{I} - \hat{b}\hat{b} \right) \right] \sim v_c, \to 0 \text{ as } v_c \to 0 \]

- Only \( \Pi_\lambda \) is independent of collision frequency
- Called the \emph{gyro-viscosity}
- Captures lowest order (in \( k_\perp Q_i \ll 1 \)) effect of finite ion Larmor radius
Properties of Gyro-viscosity

• Independent of collisions
  – Remains in collisionless limit

• Causes no heating or dissipation
  \[ Q_i \equiv \Pi : \nabla v = 0 \]

• Completely reversible transport of momentum due to spatial distribution of ion Larmor orbits
  – FLR effect
  – No increase in entropy
Closures: Heat Flux

- Can be decomposed as \( q_i = q_{||} + q_{\wedge} + q_{\perp} \)
  
  \[
  q_{||} = \hat{b}\hat{b} \cdot q_i = -\kappa_{||} \hat{b}\hat{b} \cdot \nabla T_i
  \]
  
  \[
  q_{\wedge} = \hat{b} \times q_i = +\kappa_{\wedge} \hat{b} \times \nabla T_i
  \]
  
  \[
  q_{\perp} = \hat{b} \times (\hat{b} \times q_i) = -\kappa_{\perp} (1 - \hat{b}\hat{b}) \cdot \nabla T_i
  \]

- Dependence on collision frequency
  
  \[
  \kappa_{||} \sim 1 / v_c , \quad \kappa_{\wedge} \text{ independent of } v_c , \quad \kappa_{\perp} \sim v_c
  \]

- \( \kappa_{\wedge} \) survives for collisionless model

- Ion diamagnetic heat flux

- Reversible flux of heat due to spatial distribution of ion Larmor orbits
  - No increase in entropy
Diamagnetic Flows and Fluxes

• Solve ion momentum equation for velocity

\[
\mathbf{V}_{i\perp} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\mathbf{B} \times \nabla p_i}{neB^2} + \frac{M}{eB^2} \mathbf{B} \times \frac{d\mathbf{V}_i}{dt} + \frac{\mathbf{B} \times \nabla \cdot \Pi}{neB^2}
\]

= \mathbf{V}_E + \mathbf{V}_{i*} + \ldots

- MHD velocity
- Diamagnetic drift velocity

• These can be consider ordered in \( \rho_i/L \ll 1 \)

• These flows can cause ``transport” by fluxes, i.e. \( \sim nV_\ast \)

• This is the origin of the FLR closures
Diamagnetic Current

Reversible flux of electric charge due to distribution of Larmor orbits

\[ J = T \frac{B \times \nabla n}{B^2} \]
Diamagnetic Heat Flux

Reversible flux of heat (energy) due to spatial distribution of Larmor orbits

\[ q_A = \frac{5}{2} \frac{p_i}{eB} \hat{b} \times \nabla T_i \]
Gyro-viscosity

\[ \mathbf{E} = -\mathbf{u} \times \mathbf{B} \]
\[ E_2 = u_{1,2} B x_2 \]

\( \frac{\partial u_1}{\partial x_2} \equiv u_{1,2} \ll \Omega \)
Gyro-viscosity

\[ E = -\mathbf{u} \times \mathbf{B} \]
\[ E_2 = u_{1,2} B x_2 \]
Gyro-viscosity

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Gyro-viscosity

- Drift (not shown)
- Gyro-circle ➔ Ellipse
- Modification of Ω

\[
E = -u \times B \\
E_2 = u_{1,2} B x_2
\]
Gyro-viscosity

\[ \frac{\partial u_1}{\partial x_2} \equiv u_{1,2} \ll \Omega \]

Original Orbit

Perturbed Orbit

\[ E = -u \times B \]
\[ E_2 = u_{1,2} B x_2 \]

Drift (not shown)
Gyro-circle \( \Rightarrow \) Ellipse
Modification of \( \Omega \)

Gyro-viscosity

Stress tensor components:

$$\Pi_{i,j} \sim mn \langle v_i v_j \rangle = mn \langle \dot{x}_i \dot{x}_j \rangle$$

Perturbed orbits:

$$x_1 = x_1^{(0)} + u_{1,2} x_2^{(0)} t + \rho \sqrt{\frac{\Omega}{\Omega - u_{1,2}}} \cos \left[ \sqrt{\Omega (\Omega - u_{1,2})} t + \alpha \right]$$

$$x_2 = x_2^{(0)} + \rho \sin \left[ \sqrt{\Omega (\Omega - u_{1,2})} t + \alpha \right]$$

Particles passing through origin at $x_1 = x_2 = t = 0$:

$$\dot{x}_1 = \rho (\Omega - u_{1,2}) \sin \alpha, \quad \dot{x}_2 = \rho \sqrt{\Omega (\Omega - u_{1,2})} \cos \alpha$$

To first order in $u_{1,2}$:

$$\langle \dot{x}_1^2 \rangle = \frac{1}{2} \langle \rho^2 \rangle (\Omega^2 - 2\Omega u_{1,2}), \quad \langle \dot{x}_2^2 \rangle = \frac{1}{2} \langle \rho^2 \rangle (\Omega^2 - \Omega u_{1,2})$$
Gyro-viscosity

Stress tensor component:

$$\Pi_{1,1} - \Pi_{2,2} = mn\langle \dot{x}_1^2 - \dot{x}_2^2 \rangle = -\frac{1}{2} mn \langle \rho^2 \rangle \Omega u_{1,2} = -\frac{1}{2} mn \langle v_\perp^2 \rangle u_{1,2} / \Omega$$

$$= -(p_\perp / \Omega) u_{1,2}$$

Do same calculation for $u_{2,1} \neq 0$:

$$\Pi_{1,1} - \Pi_{2,2} = -(p_\perp / \Omega) u_{2,1}$$

Combine:

$$\Pi_{1,1} - \Pi_{2,2} = -\frac{2 p_\perp}{\Omega} U_{1,2} \equiv -\frac{p_\perp}{\Omega} \left( \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right)$$

Off-diagonal components found by considering $(x_1, x_3)$ orbits
Gyro-viscous Cancellation

• In Drift MHD (small deviations from equilibrium), acceleration ~ stress

\[ n \frac{dV_i}{dt} \equiv n \left[ \frac{\partial V_i}{\partial t} + (V_E + V_\ast + V_\parallel) \cdot \nabla V_i \right] \sim -\nabla \cdot \Pi_\Lambda \]

• Since \( \Pi_\Lambda \) arises from drifts, there is a partial cancellation between the gyro-viscous force and advection by \( V_\ast \):

\[ nV_\ast \cdot \nabla V_i + \nabla \cdot \Pi_\Lambda \approx 0 \]

• This is the gyro-viscous cancellation

• It is often assumed to be complete:

\[ nV_\ast \cdot \nabla V_i + \nabla \cdot \Pi_\Lambda = 0 \]
~ GV Cancellation can be seen from Form of GV Stress Tensor

• For unsheared slab equilibrium with $p_i = p_i(x)$:

$$V_{*i} = \frac{B \times \nabla p}{neB^2} = \frac{1}{neB} \frac{dp}{dx} \hat{e}_y, \quad \Pi_{xx} = -\frac{p}{2\Omega} \left( \frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right)$$

$$Mn(V_{*i} \cdot \nabla V_i)_y + \nabla \cdot \Pi_\lambda \approx \frac{1}{\Omega} \frac{dp}{dx} \frac{\partial V_x}{\partial y} - \frac{1}{2} \frac{1}{\Omega} \frac{dp}{dx} \frac{\partial V_x}{\partial y} + \ldots$$

• Gyro-viscous cancellation is incomplete, but ...

• Assuming it is exact is often a good approximation, and simplifies the algebra
Diamagnetic Heat Flux Cancellation

- Diamagnetic heat flux in ion energy cancels advection by diamagnetic drift in continuity equation

\[ \nabla \cdot q_A = \frac{5}{2} \frac{1}{B} \hat{b} \cdot \nabla T_i \times \nabla n \]

- Electrostatic: \[ \nabla \cdot V_i = -n \nabla \cdot V_i - (V_i + V_{*i}) \cdot \nabla n , \quad (\nabla \cdot V_{*i} = 0) \]

\[
\frac{\partial n}{\partial t} = -n \nabla \cdot V_i - (V_i + V_{*i}) \cdot \nabla n
\]

\[
\frac{\partial p_i}{\partial t} + (V_i + V_{*i}) \cdot \nabla p_i = -\frac{5}{3} \nabla \cdot V_i - \frac{2}{3} \nabla \cdot q_A
\]

\[= \frac{5}{3} \frac{p_i}{n} \frac{\partial n}{\partial t} + \frac{5}{3} \frac{p_i}{n} V_i \cdot \nabla n + \frac{5}{3} \frac{p_i}{n} V_{*i} \cdot \nabla n - \frac{2}{3} \nabla \cdot q_A \]

- Then

\[ V_{*i} \cdot \nabla n = \frac{1}{neB} \hat{b} \times \nabla p_i \cdot \nabla n = \frac{1}{eB} \hat{b} \cdot \nabla T_i \times \nabla n \]

\[\frac{5}{3} \frac{p_i}{n} V_{*i} \cdot \nabla n - \frac{2}{3} \nabla \cdot q_A = \frac{5}{3} \frac{T_i}{eB} \hat{b} \cdot \nabla T_i \times \nabla n - \left( \frac{2}{3} \right) \left( \frac{5}{2} \right) \frac{T_i}{eB} \hat{b} \cdot \nabla T_i \times \nabla n = 0 \]

- Diamagnetic heat flux cancellation is complete in electrostatics
FLR Effects on Modes

- Interchange type modes
  - $g$-mode: $\omega(\omega-\omega_*)+\gamma^2_{\text{MHD}} = 0$, \hspace{1cm} $\gamma^2_{\text{MHD}} = g/L_{n0}$
    - *Unstable* in MHD ($\omega_* = 0$)
    - *Stable* if $\omega_* > 2 \gamma_{\text{MHD}}$
  - MRI, driven by plasma rotation (Ferraro)
    - Gyro-viscosity completely stabilizing if $\beta >> 1$

- Parallel sound waves
  - *Stable* in MHD and Hall MHD
  - *Destabilized* by FLR (GV and IDHF)
  - ITG-like fluid modes
ITG-like Fluid Mode

• Consider modes driven by ion temperature gradient in slab geometry
  – No density gradient, \( n_0(x) = n_0 \)
  – Constant electron temperature, \( T_{e0}(x) = T_{e0} \)
  – No magnetic shear, \( B_0 = B_{z0}(x) e_z \)
  – Ion temperature gradient, \( T_{i0}(x) = T_{i0} e^{x/L} \)
  – Parallel sound wave driven unstable by FLR effects (compare with \( g \)-mode)

• Originally derived from kinetic theory

• Very important mode in tokamak transport (toroidal effects, magnetic shear, etc.)…one of the most studied modes in plasma physics

• Stable in ideal, resistive, and Hall MHD!

• Requires FLR effects for instability
  – Ion gyro-viscous stress
  – Ion diamagnetic heat flux

• Good mode for verification of extended fluid model
Fluid Dispersion Relation

\[
\omega^3 - \frac{1}{2} k_z^2 \left( \beta_e + \frac{5}{3} \beta_i \right) \omega - \frac{1}{4} k_y k_z \beta_i \left( \beta_e + \frac{5}{3} \beta_i \right) \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{Ti0}} = 0
\]
Fluid Dispersion Relation

\[ \omega^3 - \frac{1}{2} k_z^2 \left( \beta_e + \frac{5}{3} \beta_i \right) \omega - \frac{1}{4} k_y k_z^2 \beta_i \left( \beta_e + \frac{5}{3} \beta_i \right) \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{Ti0}} = 0 \]

\[ \omega^2 = \frac{1}{2} C_s^2 k_z^2 \]

Sound Wave
Fluid Dispersion Relation

\[ \omega^3 - \frac{1}{2} k_z^2 \left( \beta_e + \frac{5}{3} \beta_i \right) \omega - \frac{1}{4} k_y k_z^2 \beta_i \left( \beta_e + \frac{5}{3} \beta_i \right) \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{\text{Ti0}}} = 0 \]

\[ \omega^2 = \frac{1}{2} C_s^2 k_z^2 \]

Sound Wave

\[ \omega = -2 k_y \beta_i \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{\text{Ti0}}} \]

Low freq. “drift” mode
Fluid Dispersion Relation

\[ \omega^3 - \frac{1}{2} k_z^2 \left( \beta_e + \frac{5}{3} \beta_i \right) \omega = -\frac{1}{4} k_y k_z \beta_i \left( \beta_e + \frac{5}{3} \beta_i \right) \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{T_i0}} = 0 \]

\[ \omega^2 = \frac{1}{2} C_s^2 k_z^2 \]

Sound Wave

\[ \omega^3 = \frac{1}{4} k_y k_z \beta_i \left( \beta_e + \frac{5}{3} \beta_i \right) \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{T_i0}} \]

“ITG”

\[ \omega = -2 k_y \beta_i \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{T_i0}} \]

Low freq. “drift” mode
Fluid Dispersion Relation

\[ \omega^3 - \frac{1}{2} k_z^2 \left( \beta_e + \frac{5}{3} \beta_i \right) \omega - \frac{1}{4} k_y k_z^2 \beta_i \left( \beta_e + \frac{5}{3} \beta_i \right) \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{Ti0}} = 0 \]

\[ \omega^2 = \frac{1}{2} C_s^2 k_z^2 \]

Sound Wave

\[ \omega^3 = \frac{1}{4} k_y k_z^2 \beta_i \left( \beta_e + \frac{5}{3} \beta_i \right) \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{Ti0}} \]

“ITG”

\[ \omega = -2 k_y \beta_i \left( 1 + \frac{1}{6} \beta_i \right) \frac{d_i}{L_{Ti0}} \]

Low freq. “drift” mode

- Electro-static
- Ballooning ordering: \( k_z \sim 1, k_y \sim 1/\varepsilon^2, d_i/L \sim \varepsilon^2 \)
- Local approximation: \( f \sim e^{i(k_y y + k_z z)} \)
- No gyro-viscous or diamagnetic cancellations assumed
Physical Picture(?)

Parallel sound wave, no pressure gradient:

$V_x$  $V_z$

Pressure
Physical Picture (?)

Parallel sound wave, no pressure gradient:

\[ V_x \uparrow \quad V_z \downarrow \]

\[ + \quad - \quad + \quad - \quad + \]

\[ \text{Pressure} \]

Parallel sound wave, pressure gradient:

\[ \frac{dP_i}{dx} \]

\[ V_x \downarrow \quad V_z \uparrow \]

\[ + \quad - \quad + \quad - \quad + \]

\[ \text{Pressure} \]
Parallel sound wave, no pressure gradient:

\[ V_x \uparrow \quad V_z \downarrow \]

\[ + \quad - \quad + \quad - \quad + \]

\[ \text{Pressure} \]

Parallel sound wave, pressure gradient:

\[ V_x \uparrow \quad V_z \downarrow \]

\[ \pi \text{ phase shift in } V_{xi} : \quad \delta p = V_{xi} \frac{dP_{i0}}{dx} \text{ reinforces pressure perturbation; } \]

FLR cancels diamagnetic advective contributions
Electrostatic Marginal Stability

- **Cubic of form:**  
  \[ w^3 - 3Aw + B = 0 \]
  
  \[ w = \sqrt[3]{A \left( z + \frac{1}{z} \right)} \rightarrow \xi^2 + \frac{B}{A^{3/2}} \xi + 1 = 0 , \quad \xi = z^3 \]

  \[ z^3 = -\frac{B}{2A^{3/2}} + \sqrt{\frac{B^2}{4A^3} - 1} \rightarrow z = \left( -\frac{B}{2A^{3/2}} + \sqrt{\frac{B^2}{4A^3} - 1} \right)^{1/3} e^{2\pi il/3} , \quad l = 0,1,2 \]

- **Unstable if**  \( \frac{B^2}{4A^3} > 1 \)

- **Approximate instability condition:**  
  \[ \eta_i \rho_i = \frac{\rho_i}{L_i} > \eta_i^{\text{crit}} \rho_i \approx \frac{k_{\parallel}}{k_{\perp}} \]
Behavior of Roots, \( f_3(w; \eta) = 0 \)

Threshold in \( \eta_i \)

Threshold in \( k_{\text{perp}} Q_i \)

- \( \eta_i = \alpha / L_{Ti} \)
- \( k_{\text{perp}} Q_i \)

- \( \alpha_z = 0.1 \)
- \( \eta_i = 1 \)
- \( \beta_i = 0.04 \)
- \( \beta_e = 0.01 \)
- \( \delta_i = 0.0161 \)
ITG is Electrostatic

\[ \frac{\gamma}{\omega} \]

\[ \eta_i \]

- \( \alpha_y = 125.6 \)
- \( \alpha_z = 0.1 \)
- \( \beta_i = 0.01 \)
- \( \beta_\epsilon = 0.04 \)
- \( d_i / a = 0.0161 \)
- \( a = 1 \text{ m} \)
Growth Rate Scaling Depends on How $k_z/k_y$ Varies
NIMROD Results
Computational Geometry

Computational problem is 2D in \((x,z)\) plane, 1-D in \(x\) and \(k_z\)
Equilibrium Pressure Profile

\[ p_{i0}(x) = p_{i0}(0) \left[ 1 + 0.9 \tanh \frac{x}{L_{Ti0}} \right] \]
Instability Drive in Tanh Model

\[ \frac{d \ln T_i}{dx} \]

Biased towards \( x < 0 \)
Theory/Computation Comparison:

$\gamma_{\text{LOCAL}}, \gamma_{\text{AVG}}, \gamma_{\text{NIMROD}} \text{ vs. } \eta_i$
Theory/Computation Comparison:

\( \gamma_{\text{LOCAL}}, \gamma_{\text{NIMROD}} \) vs. \( k_{\text{perp}} Q_i \)
Computational Eigenfunction Structure

- Perturbed ion temperature
- More structure as $1/L$ increases
- Resolved
- Local theory gives no eigenfunction structure
- Slightly biased toward $x < 0$. 
Computation/Theory Comparison:

\[ \gamma_{\text{LOCAL}}, \gamma_{\text{NIMROD}} \text{ vs. } f_e = \frac{\beta_e}{\beta} \]
Comparison with Kinetic Theory

• Preliminary!
• Comparison between local analytic fluid and kinetic models (Cheng, Parker)
• No computational comparisons yet…..
• Still a lot of work to be done!
Growth Rate, $k_z/k_y = 0.01$
Real Frequency, $k_z/k_y = 0.01$
Discussion

• Fluid theory requires both:
  – Ion gyro-viscous stress
  – Ion diamagnetic heat flux
  – Only details depend on form of “gyro-viscous cancellation”
• Instability threshold in both $1/L_{Ti0}$ and $k_{perp} Q_i$ (at fixed $k_z$)
• Reasonable agreement between NIMROD and local theory on growth rate behavior
• Comparison not possible on eigenmode structure
• *NIMROD is verified where theory and computation can be compared reasonably*
• Fluid theory has no natural stabilizing mechanism at high $k$
  – Implications for non-linear extended MHD computations
• Preliminary comparison of local fluid and kinetic analytic models
• Await direct comparison between NIMROD and kinetic codes
The End
Physical Behavior(?)

Parallel sound wave: Pressure and $V_z$ perturbation parallel to $\mathbf{B}$
Physical Behavior(?)

Parallel sound wave: Pressure and $V_z$ perturbation parallel to $\mathbf{B}$

$V_x$ when $dP_{i0}/dx = 0$
Physical Behavior(?)

Parallel sound wave: Pressure and $V_z$ perturbation parallel to $\mathbf{B}$

$V_x$ when $dP_{i0}/dx = 0$

$V_x$ when $dP_{i0}/dx \neq 0$

- $dP_{i0}/dx$ induces phase shift in $V_x$
- $\delta p = V_x dP_{i0}/dx$ re-inforces pressure perturbation
- Instability