Evaluation of Mercier Criterion on Non-Aligned Meshes

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A utility has been created within NIMEQ that performs poloidal field line traces to calculate the Mercier stability criterion.

- Both the ideal and resistive stability criteria are calculated from the axisymmetric fields.
- Calculations take around 5 minutes on my laptop using 250 field line integrals on a mesh of 1920 bi-cubic elements.
- Calculated values agree with fluxgrid, but are not limited to a flux aligned mesh!
- The machinery can easily be modified to calculate other 2D flux surface integrals.

Outline of the talk

- Review of Mercier criterion
- Overview of Code
- Benchmarking with fluxgrid
- Application to spheromak calculations

The Mercier Criterion is a necessary a condition for stability against local pressure driven interchange in toroidal geometry.

- Mercier represents a competition between pressure drive, average magnetic curvature, and magnetic shear.
- $D_m > 0$ is ideal interchange unstable.
- $D_r > 0$ is resistive interchange unstable.
- The Suydam criterion is the analogous condition for instability in a cylinder.
- $D_s = -\frac{2\mu_0 p'}{rB_z^2}\frac{q^2}{q'^2}$
    - $D_s > \frac{1}{4}$ is ideal unstable.
    - $D_s > 0$ is resistive unstable.

Greene's formulation of the Mercier criterion avoids derivatives in flux surface integrals.

- $D_m = -\frac{1}{4} + \frac{\mu_0 p' V'}{q'^2} W_m$
- $D_r = D_m + (\frac{1}{2} - H)^2$
- $W_m = \frac{-V''}{4\pi^2} < \frac{B^2}{(RB_p)^2} > + \frac{Fq'}{2\pi} < \frac{1}{(RB_p)^2} >$
  $+ \frac{\mu_0 p' V'}{4\pi^2} < \frac{1}{B^2} >< \frac{B^2}{(RB_p)^2} >$
  $+ \frac{\mu_0 p' V' F^2}{4\pi^2} (< \frac{1}{B^2 (RB_p)^2} >< \frac{B^2}{(RB_p)^2} > - < \frac{1}{(RB_p)^2} >^2)$
- $H = \frac{\mu_0 p' V' F}{2\pi q'} (< \frac{1}{(RB_p)^2} > - < \frac{B^2}{(RB_p)^2} > / < B^2 >)$
- $V' = \frac{2\pi q}{F < 1/R^2 >}$
- $F(\psi) = RB_T$
- $q(\psi) = \frac{1}{2\pi} \oint \frac{B_\phi}{RB_p} dl_p$
- $< Q > = \frac{\oint Q dl_p / B_p}{\oint dl_p / B_p}$
- Comments on Modern Physics Part E, 17 (1997)

Calculation of Mercier requires the evaluation numerous flux surface integrals and derivatives.

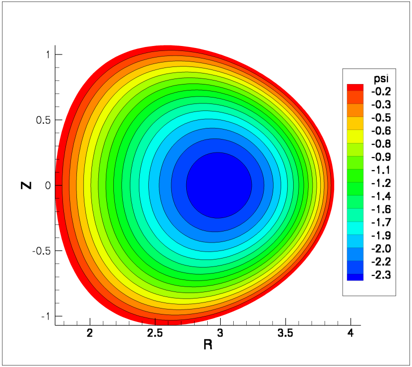
- LSODE is used to perform the flux surface integrals.
    - A modified version int_segment traces the field lines in the poloidal plane.
    - LSODE will not converge if the integrand is discontinuous across elements!
        - An earlier version of the utility that used a different formulation of D_m projected the derivatives of $\vec{B}$ onto an auxiliary field.
- NIMEQ is used to find ψ from J_ϕ.
    - Finite differencing is used to find q', p', and V''.
    - Smoothing is needed near the magnetic axis where the derivatives are noisy.
    - Spline interpolation has trouble calculating q' when a separatrix is present.

A new subroutine has been added to NIMEQ to calculate the stability parameters

- Calculate Ψ from J_ϕ
- Locate the magnetic axis
- Calculate the starting points for the field line traces
  - Start near the magnetic axis and move radially outwards
- Evaluate P, F, and Ψ for each flux surface
- Perform the field line integrals
  - Check if the field line leaves domain
  - Refine last step to minimize overshoot
- Calculate V', P', q', and V''
- Assemble D_m and D_r

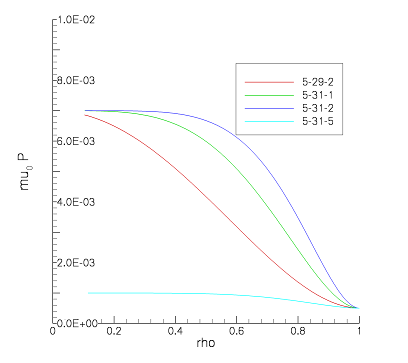
The calculated stability criteria agree with fluxgrid to within 1% for a Solovev equilibrium.

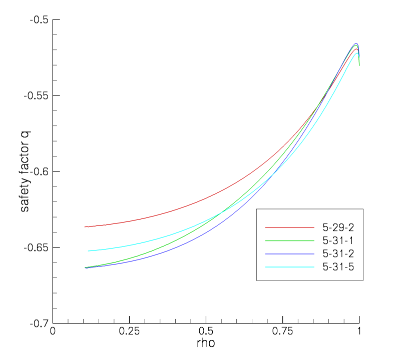
- $\Delta^*\psi = -FF' - \mu_0 R^2 P'$
  - Uniform $F = 3Tm$
  - Uniform $\mu_0 P' = 0.177 T^2/Wb$
- Fluxgrid calculates:
  - $D_m = -.8058$ at $\rho = .95$
  - $D_m = -6.699$ at $\rho = 0.2$
- The Mercier utility calculates:
  - $D_m = -.8027$ at $\rho = .95$
  - $D_m = -6.652$ at $\rho = 0.2$



- Similar agreement is observed when comparing a fluxgrid equilibrium with a nimeq equilibrium on a rectangular mesh.

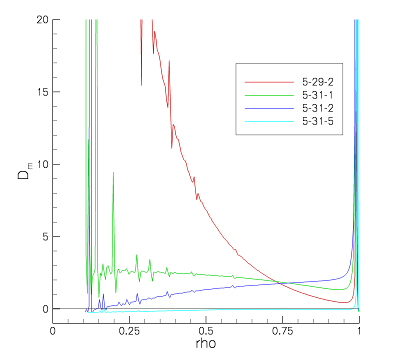
A family of similar tuna-can spheromak equilibria are used to study the Mercier stability of different pressure profile.

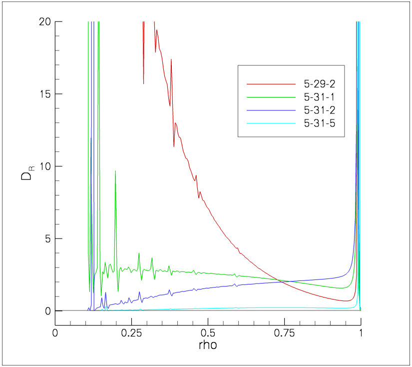




- Cases 5292, 5301, and 5302 are roughly based off of SSPX shot 14590 and have peak $T_e \sim 350eV$.
- The temperature in case 5305 is scaled to be ideal Mercier stable.

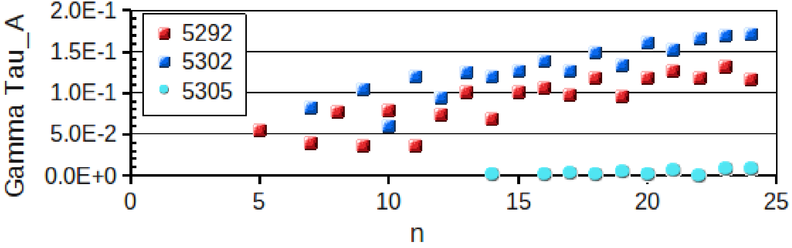
Moving the pressure gradient outwards is stabilizing near the magnetic axis but destabilizing near the separatrix.





- Case 5292 uses a quadratic pressure profile and D_m diverges near $\rho = \sqrt{\psi} = 0$
- The noise in D_m and D_r is primarily due to noise in calculating q'.

Preliminary calculations show large MHD growth when $D_m > 1$.



- The low n resonant core modes $m = 5, n = 3$ and $m = 5, n = 8$ are not yet observed for case 5302.
  - These modes may still be unstable but with slower growth rates.
- The growth rates for the ideal Mercier stable case, 5305, are greatly reduced. ($\gamma\tau_A \sim 1.0 - 9.0x10^{-3}$)

A new diagnostic that calculates Mercier Stability has been added to NIMEQ

- The diagnostic is quick and not limited to flux aligned meshes.
- The calculated stability criteria agree with fluxgrid for Solovev equilibria.
- Linear calculations for spheromak equilibria predict large growth rates when ideal Mercier is violated.
- I either need to find a better way to calculate the derivatives or add some smoothing.

Final thoughts

- What would other people find useful?
  - Fluxgrid can calculate D_i,D_r, and D_{nc}.
  - Does this belong in NIMEQ or NIMFL?
  - Field line integrals are quick in 2-D. The code runs in 5min.
- The machinery needed for an equilibrium q-solver is there.
  - $q = \frac{F}{2\pi} \oint \frac{dl_p}{R^2 B_p}$
  - $\Delta^* \psi = -\mu_0 R^2 p' - \frac{1}{2} \frac{d}{d\psi} (\frac{2\pi q}{\oint \frac{dl_p}{R^2 B_p}})^2$