Resistive Wall Mode Study in NIMROD

Andi Montgomery
with C. C. Hegna, A. J. Cole, (U of Wisconsin, Madison)
and S. E. Kruger (Tech-X Corp)

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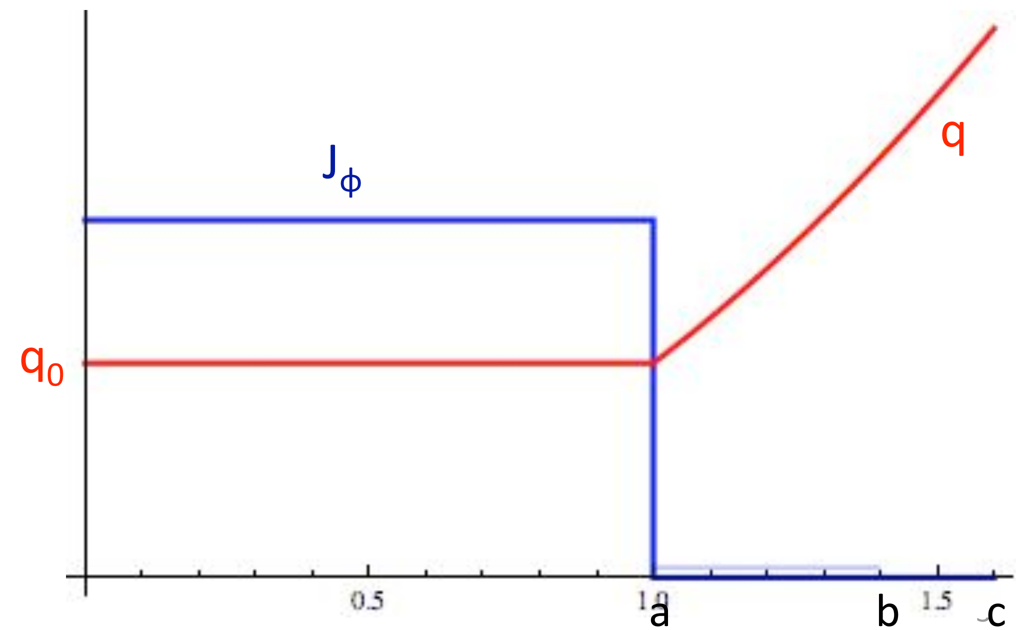
Outline

- Equilibrium and how it is represented in NIMROD
- Analytic solution for periodic cylinder with resistive wall
- Implementation in NIMROD
- Preliminary Results
- What's next?

Equilibrium

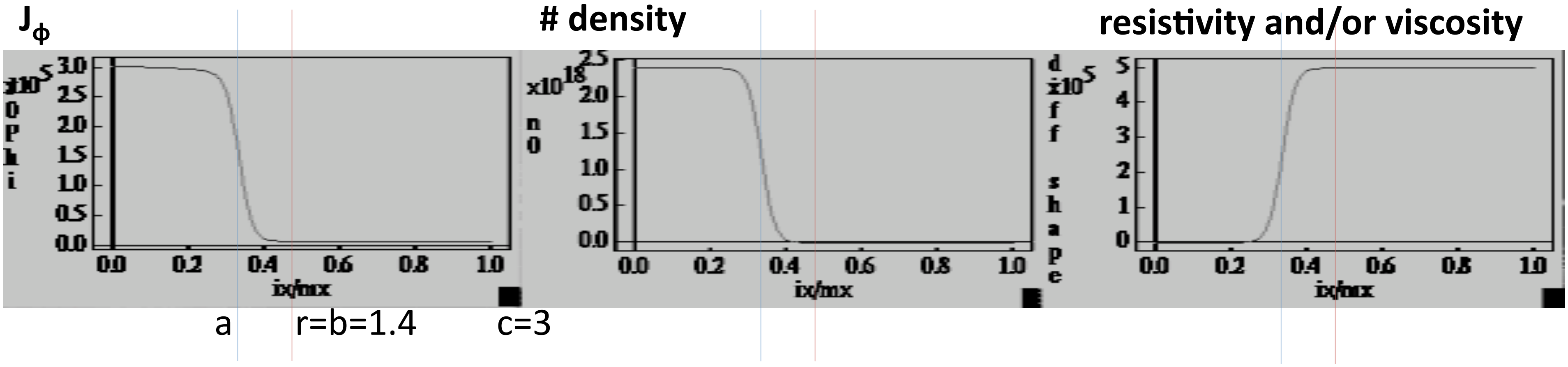
- Goal is to reproduce analytic results of J. M. Finn [Phys. Plasmas **2**, 198 (1995)] for RWM in a periodic cylinder.
  - Ideal plasma inside r=a, vacuum outside r=a
  - Step-function jump in axial current at r=a
  - q=2 rational surface at r=b
  - wall located at r=c

B_0= 1. T
q_0= 1.05
β = 0
m= 2
n = 1



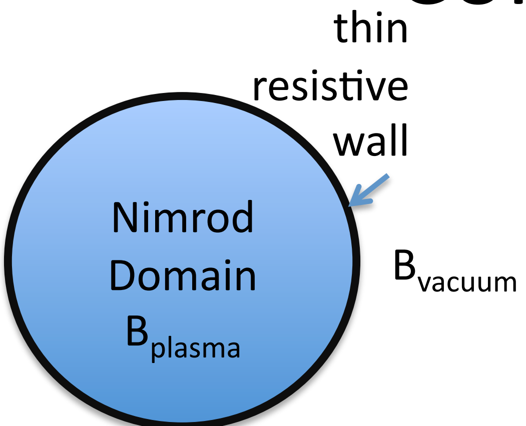
Equilibrium in NIMROD

- Some combination of jumps in current, # density, resistivity, and viscosity - implemented using a tanh function



- Instead of a vacuum, use high resistivity – still have velocity in "vacuum"
- For the RWM, which is slow growing, we don't want to see a tearing mode at the q=2 resonant surface, so we move the wall to r=1.4. For r=1.4 there is no growth for a perfectly conducting wall.

Analytic Solution for Boundary Conditions in Cylinder



$$B_{vac} = -\nabla\chi$$

$$\nabla^2\chi = 0$$

$$\chi = p_1 K_m(kr)e^{i(\omega t + m\theta - kz)}$$

$$B_{vac} = -\nabla\chi = p_1\left[K_m'(kr)\hat{e}_r + \frac{im}{r}K_m(kr)\hat{e}_\theta + ikK_m(kr)\hat{e}_z\right]e^{i(\omega t + m\theta - kz)}$$

Matching Conditions on normal and tangential components of B:

$$\hat{r}\cdot\left[B_{vac} - B_{plasma}\right] = 0$$

<- lets us solve for constant p_1

$$\hat{r}\times\left[B_{vac} - B_{plasma}\right] = \mu_0 K = \frac{\mu_0\delta_{wall}}{\eta_{wall}}E_T$$

<- gives an expression for E_T in terms of B_{plasma}

$$\mathbf{E}_T = B_{pm}\frac{i\eta_w}{\mu_0\delta_w}\frac{K_m(kb)}{K_m'(kb)}\left[-k\hat{e}_\theta - \frac{m}{b}\hat{e}_z\right] - \frac{\eta_w}{\mu_0\delta_w}\left[(B_{eqz} - B_{pz})\hat{e}_\theta - (B_{eq\theta} - B_{p\theta})\hat{e}_z\right]$$

Implementation in NIMROD

- E_T is specified at the boundary (surface_e) in terms of of the B-field at the edge of the domain. This is used in the induction equation to update B.
- Input parameter specifying wall characteristics is vwall.
$$vwall = \frac{\tau_{wall}}{r_{wall}} = \frac{\mu_0 \delta_{wall}}{\eta_{wall}}$$
- B_r is left floating, and we neglect the time shift between B and v solves (assume that $B_r^{n-1/2}$ is sufficient to specify E_t^n)
  - This should be the case for $\frac{\Delta t}{\tau_{wall}} << 1$
- All components of velocity set to 0 at the wall.

Analytic Growth Rate and Eigenfunctions (for ideal plasma and vacuum)

For $\gamma^2 << \gamma_\infty^2$ (no wall), we have a simple expression for γ.

$$\gamma = \frac{2m}{\tau_w\left(1 - a^{2m}/c^{2m}\right)}\left(-\frac{\gamma_\infty^2}{\gamma_c^2}\right)$$

γ_c^2 is for a perfectly conducting wall

Using vwall=6.e-3, $\gamma = 108\ s^{-1}$

For B_r the eigenfunction should be a combination of r^m and r^{-m}

Equilibrium Trials for RWM

- Began with no jump in resistivity, viscosity, or density.
  - Large fluctuations in velocity near the wall in the "vacuum" region – no true vacuum, and v is driven by proximity to non-zero E in the wall.
- Put large, sharp jump in resistivity and viscosity close to the wall, and grid-pack around this jump.
  - See a fast growing (gamma =1.e+6) numerical (?) mode. Profiles, however, similar to eigenfunctions we would expect for RWM.
- Most recent case has a jump in resistivity and viscosity at the plasma-vacuum interface. There is no grid-packing.

RWM Case – Input Parameters

mx=40

my=16

poly_degree=4

xvac=1

xmax=1.4 (wall location)

xo=5

pert_m=2

pert_n=1

ndens=2.4e+18

be0=1.

q0=1.05 (using lam0 parameter)

dtm=1.e-5

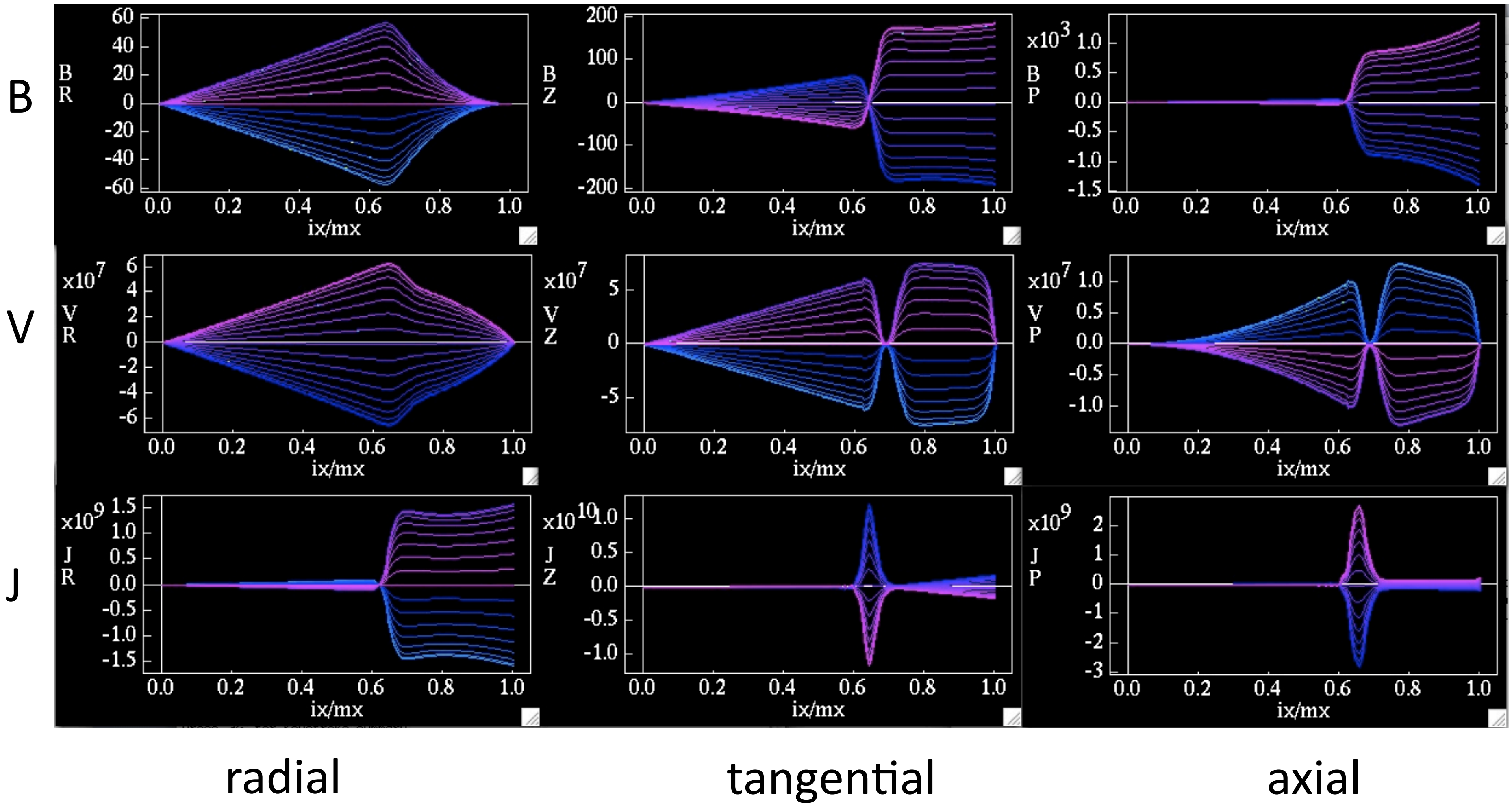
divbd=1.e5

vwall=6.e-3

- Linear run
- Jumps in resistivity and viscoity (at plasma-vac interface) with tanh width = .03
- Density constant throughout domain.
- No packing

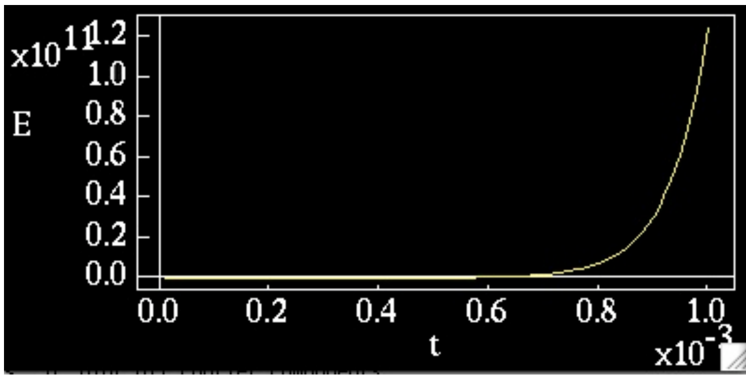
	Plasma Value	Vacuum Value
elecd	1.e-2	100
S	1.7e+9	1.7e+5
kin_visc	1.e-2	100
P	1.	1.

RWM Case – Profiles

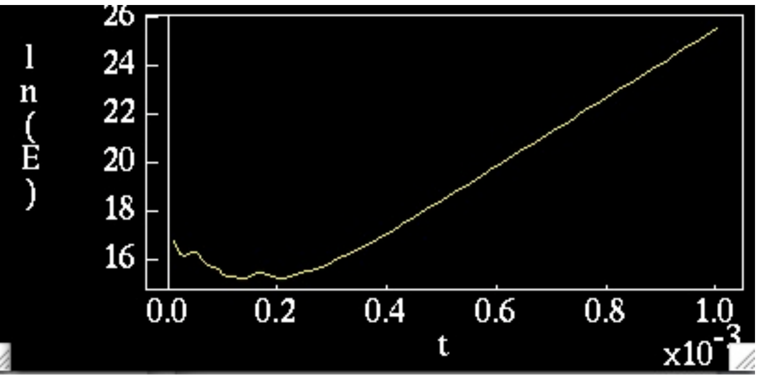


RWM Case – Discharge and Growth Rate

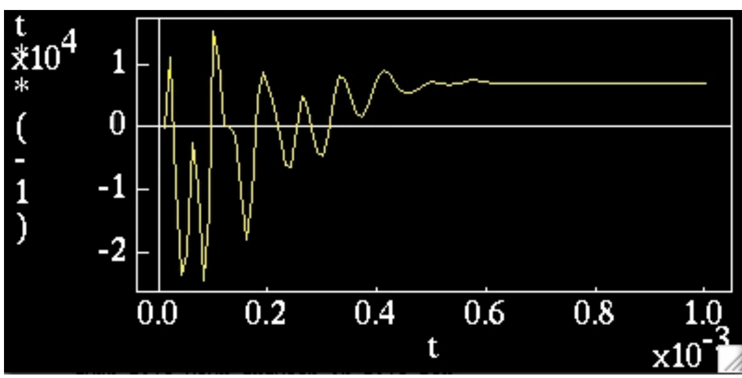
Total Energy



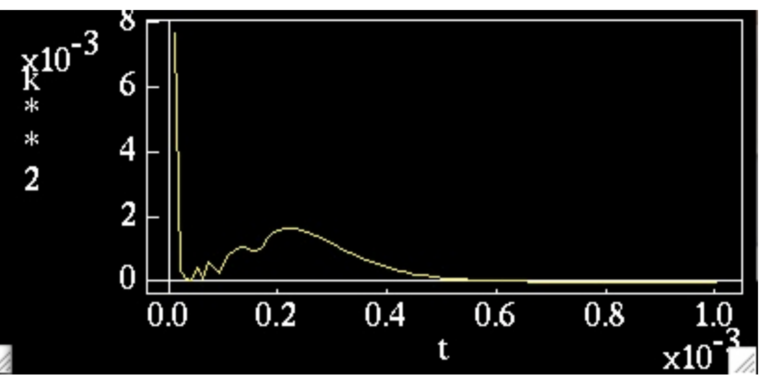
ln(Energy)



Growth Rate



k_divb**2



gamma=7040 s^{-1} (gamma for RWM=108 s^{-1})

- halving τ_w (vwall=3.e-3) doubles gamma: gamma=14108 s^{-1} (gamma for RWM=216 s^{-1})

What's next?

- Put grid packing around plasma-vacuum interface to allow for a larger jump in resistivity (and a smaller S in the "vacuum" region").
- Diagnose the fast-growing mode when the jump is near the wall.
- Once RWM in cylinder is correct:
  - RW BC for toroidal equilibrium using Green's function solver instead of analytic solution.
  - Include flow in the cylindrical case