Helicity-injected current drive and open flux instabilities in spherical tokamaks

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The toroidal current driven by relaxation processes in a cylindrical spherical tokamak (ST) geometry with coaxial injected flux is estimated by use of the linear ideal stability boundary of equilibria with a high current on the open driven flux and a lower current on the closed flux. Instabilities with toroidal mode number \( \text{n} = 1 \) have been shown to play a vital role in the helicity injection current drive, being closely associated with the relaxation process which distributes current from a directly driven open flux to a closed flux. Previous results for spheromaks (1D and 2D equilibria) and STs (1D equilibria) have predicted stabilization, for a given open flux current, if the closed flux plasma current is sufficiently large, suggesting that the current drive mechanism is self-limiting. New results presented here for 2D ST equilibria are consistent with the 1D results, but new features appear in the stability maps as the axial length and toroidal field (TF) strength are varied in the equilibria. These include changes in the shape of stability boundaries and the estimated driven current, changes in the mode structure due to equilibrium changes and resonance effects which extend stability boundaries into the stable region. As the minimum and maximum of the safety factor \( q \) profile cross integer rational values, the resonant mode is destabilized, causing regions of enhanced instability in the current profile parameter space. The results show the effects on the stability of varying the geometric length ratio \( \text{R/L} \) and provide driven current estimates with varying imposed TF strengths; these results have implications both for existing STs and for the design of future devices. Preliminary nonlinear computations with NIMROD are presented where the a spheromak configuration evolves near the stability boundary for the mode.
OUTLINE

• Previous stability results are reviewed from 1D and spheromak and spherical tokamak solutions, and 2D spheromak

• The equilibrium model and results are presented for the 2D spherical tokamak.

• The linear ideal stability analysis method is reviewed
  • Includes inverse coordinate system with open flux current

• Stability results are presented for 2D spherical tokamaks with varying R/L and current profiles

• The driven current is estimated by the stability boundary.

• The equilibria are used as initial conditions in NIMROD, showing first signs of μ trajectories around stability boundary.

• Future plans include more physically realistic modeling of the plasma.
In the Laboratory: Helicity injection occurs by reconnection, and is used to drive toroidal confinement experiments

- Helicity injection using DC electrodes is widely used and proposed for current drive in spherical tokamaks (NSTX, HIT-II, proto-SPHERA, HIST,...) and spheromaks

\[
K = \int_V A \cdot B \, dV \quad B = \nabla \times A \quad \frac{dK}{dt} = 2V_{\text{gun}} \psi_{\text{gun}} - \frac{K}{\tau_{\text{diss}}}
\]

- Poloidal current directly driven on open flux (linking gun electrodes) - require mechanism to transfer this to toroidal current on the closed flux - a “relaxation process” e.g. Taylor (1974)
Spheromak

- Ultimate low-aspect ratio devices - no conductors link plasma
- Must be created by indirect means e.g. helicity injection
- All formation schemes involve magnetic reconnection
- Almost all magnetic field generated by plasma currents
Relaxation in helicity-injected systems

• Coaxial helicity-injection directly generates toroidal flux from gun (drives poloidal current on open flux)

• Need “current drive”/relaxation mechanism, involving reconnection, to convert into new poloidal closed flux (drive toroidal current on closed flux)

• Fluctuations (dynamo) play a crucial role

Fluctuations - drive current
Intermittency in $\mu$ profile observed in SPHEX

The equilibrium state evidently crosses a stability boundary in $\mu$, higher on open flux, lower on closed flux.

The equilibrium state:

$$\nabla \times B = \mu B$$
Ideal kink instability of model 1D equilibrium captures essential physics

- Conducting walls - line-tied at $z = 0, L$
- Increasing column current ($\mu_c$) destabilizes $n = 1$ mode
- Increasing closed flux current ($\mu_a$) → annulus expands and column compressed → kink instability stabilized

This is closely related to a model for a solar flare.
Mode structure peaks along high current region

- Eigenfunction of unstable mode is concentrated in open flux region (column)
- Wavelength in z determined by fieldline pitch

Spherical tokamak $I_{rod} = 50 \text{ kA}$
Stability boundaries computed for 1D spheromak and spherical tokamak at varying $B_\phi$

Upper boundary at lower $mc$, $ma$ with higher $B_f$.

Rod boundary condition initially stabilizing.

$q$ undefined in this configuration.

2D expected to be more unstable than line tied 1D.
Results Compare Favorably with experiment

- Expect operation near upper stability threshold - current drive “switches off” at sufficiently large plasma current ($\mu_a$)
- In Ti gettered operation in SPHEX, $n = 1$ mode was intermittent - current drive “switched off” when mode absent
- Inferred stability threshold according to fitted $\mu$ profile at onset of mode
Geometrically Dependant Toroidal Current Limit

- Current drive requires operation near upper boundary of stable region maximum $\mu_a$ (hence maximum $I_{\text{tor}}$) for each $\mu_c$
- Can predict variation of plasma toroidal current with electrode current ($I_{\text{gun}}$) and TF current ($I_{\text{rod}}$) for various geometries ($L, R$)
The 2D equilibrium state is chosen as a right circular cylinder with HI on one face.

High $\mu$

surrounding enclosed torus with low $\mu$

$\mu$ continuous between column & annulus

$$\mu = \mu_0 + \mu_1(tanh(\delta(1 - \psi)) - 1)$$

$$M = M_0 \frac{\text{T} \cdot \vec{B}}{B^2}$$
Simple 2D - ST equilibria adapted, with HI boundary condition

Solving the equilibrium involves the imposition of a consistent boundary condition along the gun face. This is solved as a separate computational problem.

\[ G \equiv R^2 FF' \]

\[ G_{bnd} \equiv (1 - C \psi)^m R^2 FF' \]

\[ \Delta \psi = -G \]

q profiles vary from reversed to monotonic

Generally interested in flux amplified states

\[ A_F \equiv \frac{\psi_{MA}}{\psi_{bnd}} > 1 \]

\[ s(\psi) \equiv (1 - \psi / \psi_{ma})^{1/2} \]
2D inverse stability code SCOTS employs non-orthogonal angle-like poloidal coordinate

Common forms of poloidal coordinate for inverse systems, based on integrals along field lines of constant flux, become infinite at separatrix. We employ a coordinate which can be calculated smoothly across the separatrix, with continuous first and second derivatives, and retains the straight line limit at the gun face.

\[ f = z_1 - z_2 (1 + (\tan^{-2}((\chi + \pi)/2) - 1)(r'/r' + 2a z_2 \tan^{-2}((\chi + \pi)/2))) \]

\[ r_1 = r - r_{mg} \]
\[ r_2 = r_{ma} - r_{mg} \]
\[ z_1 = z - z_{mg} \]
\[ z_2 = z_{ma} - z_{mg} \]
\[ r' = r_1 - (r_2/z_2)(z_1 - z_2) - r_2 \]

Ugly math (real space projections enormous), but nice results.
Flux amplification factors increase with $\mu$, $B_\phi$, R/L.

Flux amplification $\psi_A$ increases with $\mu$, toroidal field and R/L.

Define

$\mu^* = \frac{\mu}{\mu_e}$

Typical experiments have

$\mu_c^* = \frac{\mu_c}{\mu_e} < 3$

$\mu_a^* = \frac{\mu_a}{\mu_e} < 0.75$

$\psi_A \sim 3-4 < 6$
$q_{\text{min}}$ decreases with $R/L$, $\mu_a$, increases with $B_\phi$, $\mu_c$

The minimum can occur near edge or axis depending on reversal.

Conventional interior profiles near experimental regime, at high $\mu_a$. Low $\mu_a$ causes reversed shear.

Strongly reversed
\( q_{\text{max}} \) decreases with \( \mu_a \), generally increases with \( \mu_c, \, B_\phi \) but complicated by reversed shear at \( R/L=0.5 \)

Strongly reversed cases have \( q_{\text{max}} \) on axis

Maxima in region of interest have maxima near the separatrix

The \( n=1 \) stability properties will tend to attract the system these states
Eigenfunctions for high $\mu_c, \mu_a$ case at high $B_\phi$ show unstable column

\[ f_1 = \frac{\partial \psi}{\partial s} J v_1 \]  
Surface normal velocity

\[ f_2 = i(v \times B_0)_1 \]  
Surface normal $v \times B$ force

For this low $\mu_a$, high $B_\phi$ case:  
$(\mu_c^*, \mu_a^*) \approx (2.5, 0.4)$  
$R/L = 0.5/0.5$  
$I_{tor} = 100kA$

F2 indicates column dominant force.

F1, or displacement is more broad.

$J$ is Jacobian
Real space projections of eigenfunctions indicate mode peaks outboard at high $B_\phi$, as observed in HIT.

$(\mu_c^*, \mu_a^*) \approx (3.4, 0.7)$
$R/L = 0.5/1.0$
$I_{rod} = 0$

$(\mu_c^*, \mu_a^*) \approx (2.5, 0.4)$
$R/L = 0.5/0.5$
$I_{tor} = 100kA$
Column stability and resonances affect the shapes of stability boundaries vs R/L and Bφ

Kruskal Shafranov limit indicates column stability.

\[ P = \frac{2\pi r B_z}{B_{\phi}} \]
\[ \frac{P}{L} > 1 \rightarrow \text{Column unstable} \]

Higher \( B_{\phi} \) causes lower boundary in \( \mu_a \) and \( \mu_c \).

\( q \) resonances extend boundary
When the column mode is stable and the resonant mode persists, eigenfunctions dominant in closed flux

\((\mu_c^*, \mu_a^*) \approx (1.5, 0.4)\)

Internal resonance evident on the upper leg of the region extending to low \(\mu_c\) in the R/L=0.5/0.5, \(I_{rod}=50\text{kA}\)

Mode is resonant at \(q=2\)

Other legs are due to other \(n=1\) resonances (\(m=3\) at low \(\mu_a\)).

Relation between boundaries and \(q_{max}\) and \(q_{min}\) maps.

Note: this does not consider the large solution, and real space projection is major R in \(V_R\) not minor r.
Taking the upper boundary of the unstable region, the driven toroidal current can be estimated.

Driven closed flux toroidal current calculated for all equilibria

Experimental trajectories are generally along direction of constant $I_{\text{tor}}$ and $\psi_A$, conserving $U_M$

\[
I_{\text{tor}} = \int_{\psi_{\text{ma}}}^{2\pi\psi_{\text{tor}}} \int_0 \left( P' + \frac{FF'}{r^2} \right) Jd\psi d\chi
\]
A comparison of all estimates in R/L and B\(_\phi\) indicates ballpark agreement with experiment.

Predicted values compare well with relevant devices.

Largest current at highest \(I_{\text{tor}}\) and \(\mu_c^*\) in each case.

Note, elongation affects normalization.

\[ I_{\text{gun}} = \mu_c \psi_{\text{gun}} / \mu_0 = \mu_c \mu_c^* \psi_{\text{gun}} / \mu_0 \]
As an example, published SPHEX data is compared

Predicted values are in reasonable agreement with experiment.

![Graph](image1)

\[ \mu_c^*, \mu_a^* \approx 1.5, 0.8 \quad I_{rod} = 0 \]
\[ \mu_c^*, \mu_a^* \approx 1.4, 0.4 \quad I_{rod} = 83kA \]
\[ R/L \approx 0.34 / 0.7 \]
\[ I_{pred}(I_{rod} = 0kA) \approx 50kA \]
\[ I_{pred}(I_{rod} = 83kA) \approx 120kA \]


Figure 8. Toroidal current inferred from equilibrium modelling of internal field measurements. The full symbols indicate the total toroidal current, whereas the open symbols are for the current on closed flux surfaces only.

Figure 11. Poloidal flux amplification ratio as a function of TF current; for each measurement \( \psi_{min} \) was increased in proportion to \( I_{TF} \) to follow the position of peak poloidal field defined by the dashed line in figure 10.
Low $\mu_a$ equilibria in NIMROD show a wide column and constant $RB\phi$ in closed flux

$\mu_c = 8.0$, $\mu_a = 1.0$
High $\mu_a$ equilibria in NIMROD show a narrow column and flux amplified state in closed flux.

$\mu_c = 14.0$, $\mu_a = 4.0$
Linear modes from NIMROD are in broad agreement with SCOTS

Low $\mu_c$: wide column, core mode displacement, and low helical pitch

$\mu_c = 9$, $\mu_a = 3.5$

High $\mu_c$: narrow, dominant column and increased helical pitch

$\mu_c = 15$, $\mu_a = 3.5$
Series of linear runs with n=1 show similar features to eigenvalue-based results

Increasing column current ($\mu_c \uparrow$):
→ More energy available for mode growth
→ Increasingly unstable

Increasing annulus current ($\mu_a \uparrow$):
→ Column compresses
→ Can reach a stable region
Trajectory in $\mu$-Space is being explored

The nonlinear response to DC drive from startup and from each initial state are being investigated as a function of geometry and toroidal field.
Bellan Box-Like Configuration Set Up

- Grid packed near the lower plate
- Electrodes gap at 0.25
- Seed field (not optimal)
- $\beta=0$

```plaintext
mx=40  my=40
init_type='vboxnon0'
bamp=1.e-12
ndens=1.e19,
advect='V only'
kin_visc=100.  elecd=100.
ohms='mhd’  continuity=’n=0 only'
nd_bc='dirichlet’  gun_max=3.e4
nd_diff=2.  nd_hypd=2.e-2
p_model='aniso1'
k_pll_min=1.e3  k_perpi=1.e3  k_plle=1.e6
ohm_heat=.true.  visc_heat=.true.  insulate=.true.
```
Results suggest seed field is obstruction

- Hollow current profile seems to flow around seed field.
- Kink instability threshold found
• Preliminary mhd only 3 mode runs are encouraging
• n=1 kink develops in column
• With incremental improvements of the included physics we can begin to examine a trajectory in µ-space
Trajectory in $\mu$-Space formed with integral average

Define a rule for the spatial integral average over “high” and “low” regions with the overall average defining the boundary.

$$\mu_a = \int d\mu \quad \mu < (\mu_{a0} + \mu_{c0})/2$$

$$\mu_c = \int d\mu \quad \mu > (\mu_{a0} + \mu_{c0})/2$$
Summary

- The driven current from Helicity Injection can be estimated as a function of experimental geometry using nothing more than the ideal MHD model.

- The system will tend to oscillate in a direction crossing the stability boundary due to the conservation of flux.

- Estimated driven currents are reasonably close to experimental values.

- Nonlinear initial value simulations are being investigated to determine if the trajectories in equilibrium parameter space agree with this picture.