Continuum solution of kinetic equations in NIMROD

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Outline.

- toroidal preconditioning with mixed finite-element temperature advance
- continuum solution of drift kinetic equation
- finite-element velocity space representation for solving Fokker-Planck equation.

Toroidal preconditioning for mixed finite-element method (MFEM) can reduce solver iterations.

- MFEM solves expanded system for T and auxiliary scalar, $q_{||}$:

$$\frac{3}{2} n \Delta T + \kappa_0 \sqrt{\theta \Delta t}\, \vec{B} \cdot \vec{\nabla} q_{||} = \ldots,$$

$$\frac{\kappa_0^2 B^2}{\kappa_{||} - \kappa_{\perp}} q_{||} + \kappa_0 \sqrt{\theta \Delta t}\, \vec{B} \cdot \vec{\nabla} \Delta T = -\kappa_0 \sqrt{\frac{\Delta t}{\theta}}\, \vec{B} \cdot \vec{\nabla} T^n$$

- Toroidal preconditioning terms arise from:

$$\vec{B} \cdot \vec{\nabla} q_{||} = [\vec{B}_0 + \Sigma_{m>0} (\vec{B}_m e^{im\phi} + \vec{B}_m^t e^{-im\phi})] \cdot \vec{\nabla}$$

$$[q_{||0} + \Sigma_{n>0} (q_{||n} e^{in\phi} + q_{||n}^t e^{-in\phi})]$$

2/1 island in cylinder with heat source and dt = 10 (10 perpendicular conduction times)

- n=(0,1) and polynomial degree = 2:
  iterations without toroidal preconditioning = 295, 78, 52, 1, 1,...
  iterations with toroidal preconditioning = 149, 31, 20, 1, 1,....

- n=(0,1) and polynomial degree = 3:
  iterations without toroidal preconditioning = 921, 300, 291, 1, 1,...
  iterations with toroidal preconditioning = 452, 80, 77, 1, 1,....

- n=(0,1,2), dt = 0.1 and polynomial degree = 3:

  w/o toroidal preconditioning=414,162,221,244,219,165,121, 50,52,53.
  w/ toroidal preconditioning=388,245,184,151,139, 76, 50, 51, 1, 1.

Other preconditioning issues

- Apply MFEM to Mark's stellarator problem.

  - 2-D helical equilibrium, no dominant n=0 **B**.

  - Simple transport problem with heat source does not converge when lphi > 3 unless time step is reduced significantly.

- Toroidal preconditioning may also reduce iterations for continuum solution of drift kinetic equation at low collisionality.

Continuum solution of drift kinetic equation

- Compute $q_{||}$ and $\pi_{||}$ closure from solution of lowest order drift kinetic equation:

$$\frac{\partial F}{\partial t} + v_{||} \hat{b} \cdot \vec{\nabla} F - C(F + f_M) = thermodynamic\ drives.$$

- Expanding F = Σ F_i (**x**,v,t) P_i ($v_{||}$/ v) yields:

$$\frac{\partial \vec{F}}{\partial t} + \boldsymbol{A} v \hat{b} \cdot \vec{\nabla} \vec{F} - \boldsymbol{B} v (\hat{b} \cdot \vec{\nabla} \ln B) \vec{F} - \boldsymbol{C} \vec{F} = drives\ ,$$

- *F* couples to *T* and ***V*** advances via closures:

$$q_{||} = -T \int d\vec{v}\, v_{||} L_1^{3/2} F\ ,\ \pi_{||} = m \int d\vec{v}\, (v_{||} - \frac{v_{\perp}^2}{2}) F$$

Compare MFEM and continuum closure.

- MFEM solves expanded system for T and auxiliary scalar, $q_{||}$:

$$\frac{3}{2} n \Delta T + \kappa_0 \sqrt{\theta \Delta t} \vec{B} \cdot \vec{\nabla} q_{||} = \ldots,$$

$$\frac{\kappa_0^2 B^2}{\kappa_{||} - \kappa_{\perp}} q_{||} + \kappa_0 \sqrt{\theta \Delta t} \vec{B} \cdot \vec{\nabla} \Delta T = -\kappa_0 \sqrt{\frac{\Delta t}{\theta}} \vec{B} \cdot \vec{\nabla} T^n$$

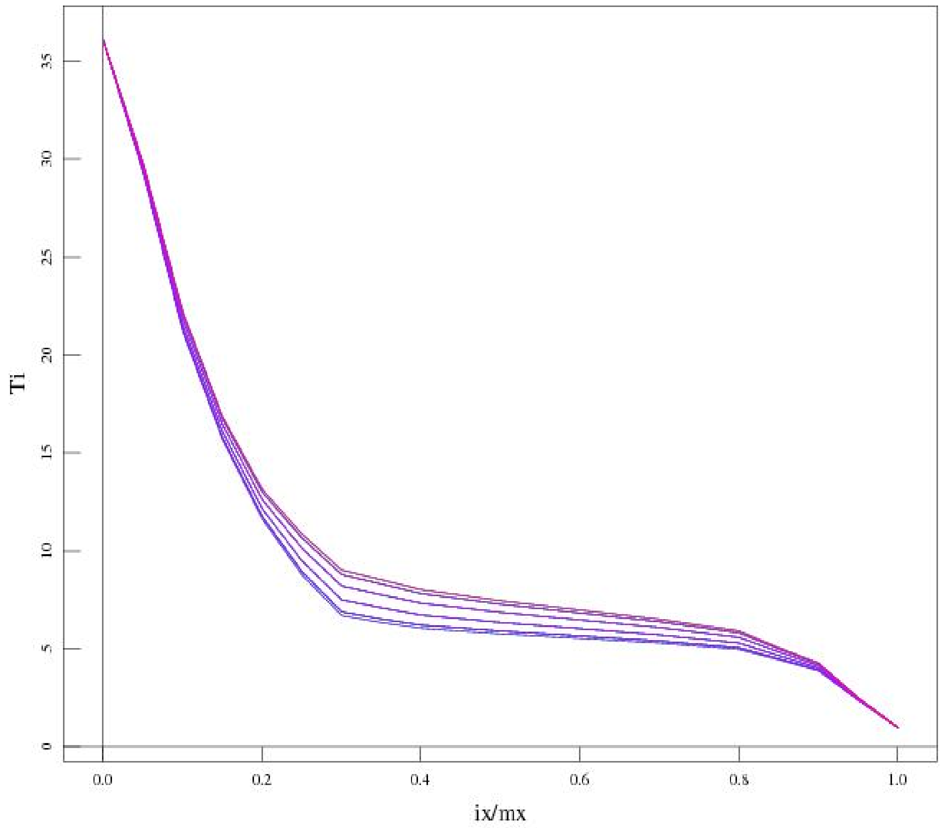
- Continuum solution solves coupled T, F system:

$$\frac{3}{2} n \frac{\partial T}{\partial t} + \vec{B} \cdot \vec{\nabla} q_{||} = \ldots,$$

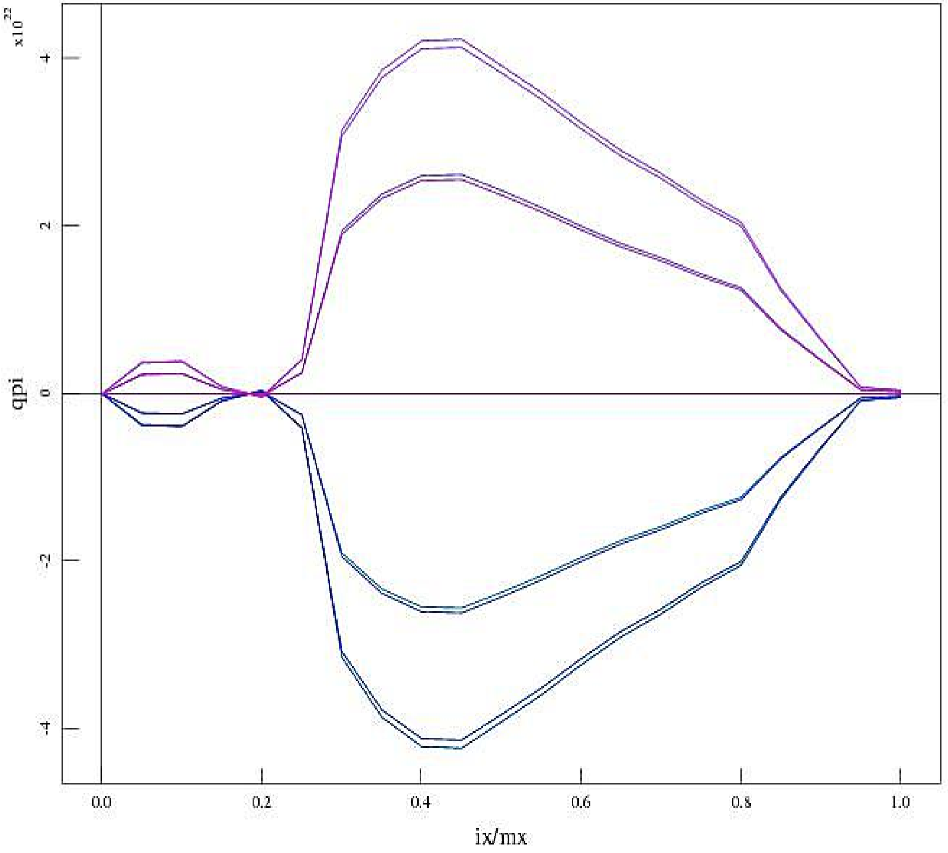
$$\frac{\partial \vec{F}}{\partial t} + \boldsymbol{A} v \hat{b} \cdot \vec{\nabla} \vec{F} - \boldsymbol{B} v (\hat{b} \cdot \vec{\nabla} \ln B) \vec{F} - \boldsymbol{C} \vec{F} = drives,$$

Do Holzl problem, 2/1 island in cylinder with a heat source.

T profiles show flattening across island.



Parallel heat flow moment

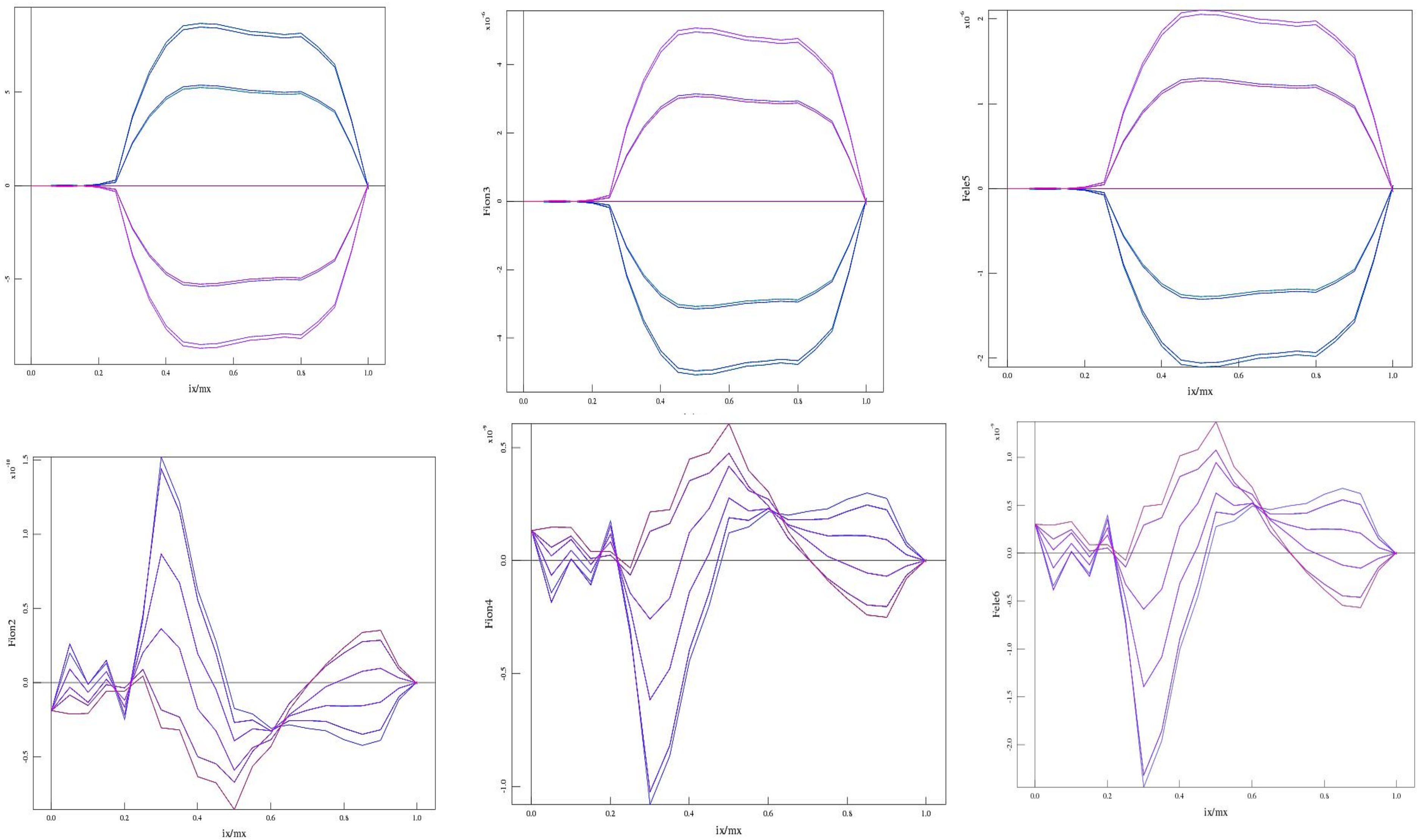


2 Legendre polynomials and 3 speed points sufficient for convergence.

s= 0.967 | s=1.818 | s=2.785

l=1

l=2



Summary of results for continuum closures.

- Done:
  - low-T cases agree with collisional Braginskii.
  - fully implicit fluid and *F* advance with preconditioned solves.
  -staggered *F* from fluid advance with parallelization over speed points.

- To do:
  - test convergence for high-T cases, other work suggests 5 speed points and 8 to 16 Legendre polynomials should be sufficient.
  - parallelize over Legendre coefficients.
  - toroidal preconditioning.
  - implement collision terms beyond Lorentz operator and test particle trapping effects in toroidal geometry
  - test on other problems like wave damping due to parallel stress.

Apply NIMROD's finite-element/Fourier representation to velocity space.

- Solve Fokker-Planck equation:

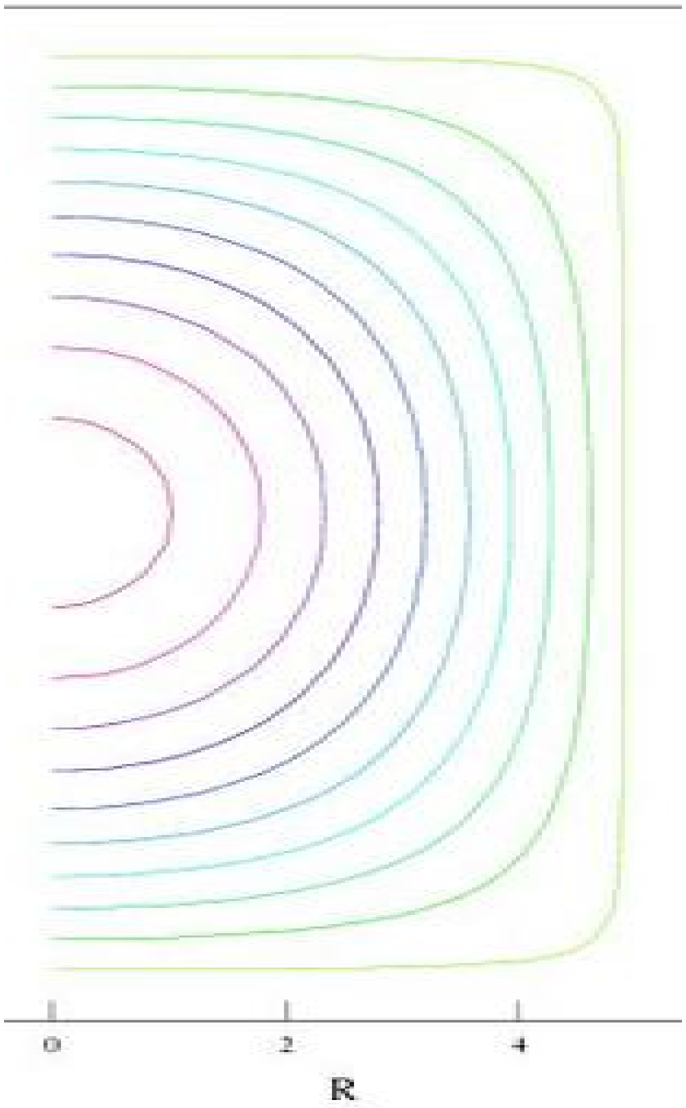
$$\frac{\partial f}{\partial t} = C(f)$$

- Expand f = $\Sigma\, f_{in}(t)\, \alpha_i(v_{||}, v_{\perp})\, e^{in\phi}$ + c.c($f_{in}(t)\, \alpha_i(v_{||}, v_{\perp})$)$e^{-in\phi}$ and use Lorentz operator initially:

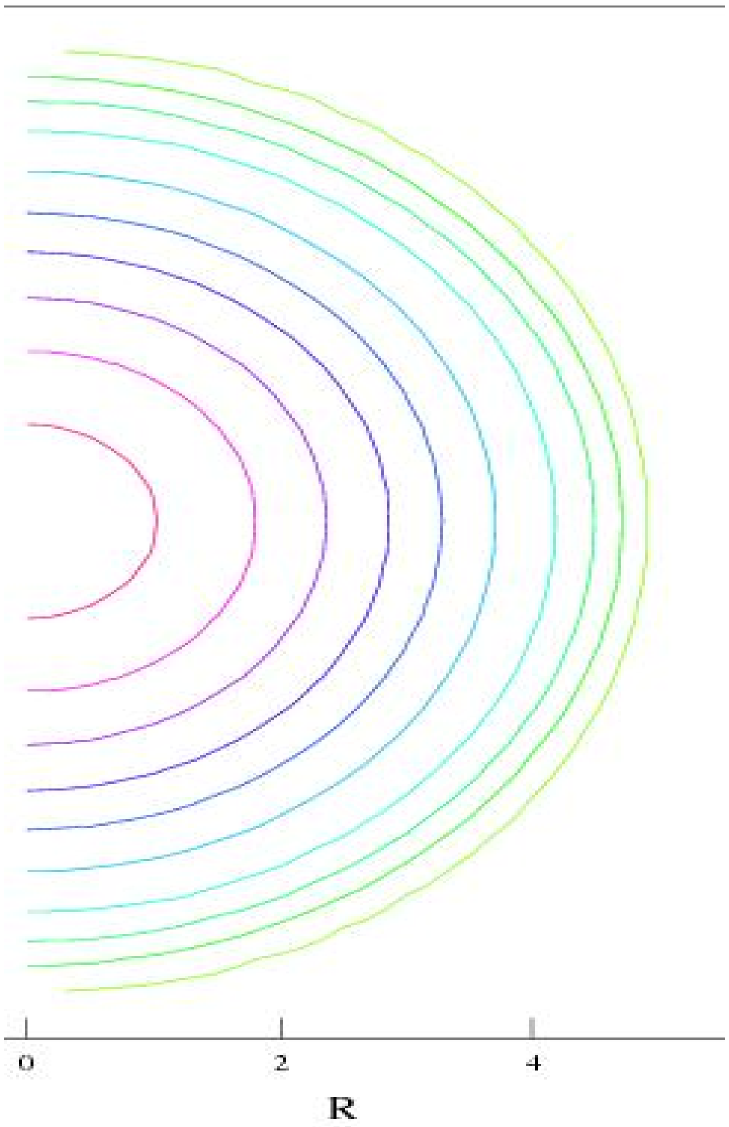
$$C_L(f) = \frac{\nu_L}{2} \vec{\nabla} \cdot (v^2 \boldsymbol{I} - \vec{v}\vec{v}) \cdot \vec{\nabla} f$$

F evolves to a Maxwellian.

t = 0



t = 5 collision times



Future work on finite-elements for velocity space.

- Implement linear terms (linearized about a Maxwellian) -requires double integration over velocity space since collision operator is integro-differential.
- Implement spherical coordinates for velocity space with proper regularity conditions.
- Implement nonlinear terms and compare with JY's moment method.
- Extend to 1D, 2D and 3D in spatial dimension.