On ion diamagnetic stabilization of ideal ballooning modes in the presence of spatially varying ω_i^*

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NIMROD Meeting

July 31, 2009

Ref: Hastie, Catto and Ramos, PoP 2000

Conventional ballooning theory relies on a WKB-like formulation with n >> 1 (Dewar et al)

- Based on a large $k_{||}/k_{perp}$ ~ 1/n << 1 expansion

$$\xi = \xi(\vec{x})e^{iS} \quad \vec{B}\cdot\nabla S = 0$$

$$---> \nabla S = n\nabla(\zeta - q\theta) + nq'\theta_k\nabla\psi$$

- To leading in 1/n, ODEs for each field line and radial wavenumber (θ_k)

$$\frac{d}{dl}[1+h^2(l)]\frac{d\xi}{dl} + \alpha g(l)\xi = \lambda\frac{[1+h^2(l)]}{\omega_A^2}\xi$$

  - λ is the local eigenvalue

$$\lambda = \lambda(\psi, \theta_k)$$

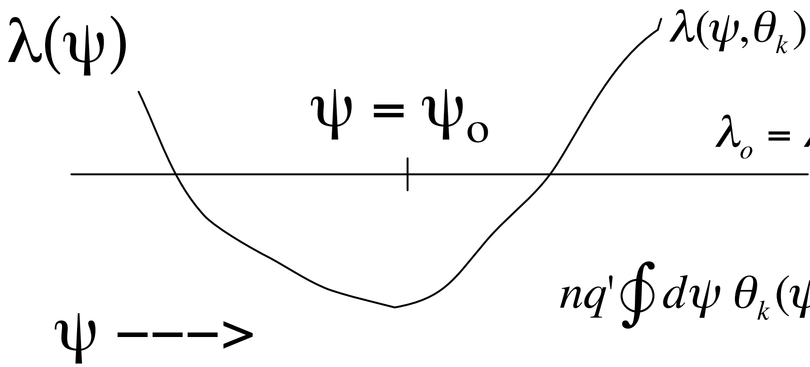
  - To calculate global eigenvalue and eigenfunction, one has to satisfy the WKB quantization condition

$$nq'\oint d\psi\ \theta_k(\psi,\lambda) = \pi(1+2N)$$

  - N = 0 most unstable

For ideal MHD, eigenvalue condition can be calculated analytically for radially localized modes

- In the absence of two-fluid/FLR, the eigenvalue condition is given by $\omega^2 = \lambda(\psi,\theta_k)$
  - Taylor expand λ near the most unstable flux surface and radial wavenumber



$$\lambda(\psi,\theta_k) = \lambda_o + \frac{1}{2}\frac{\partial^2\lambda}{\partial\psi^2}(\psi-\psi_o)^2 + \frac{1}{2}\frac{\partial^2\lambda}{\partial\theta_k{}^2}(\theta_k-\theta_{ko})^2 + \ldots$$

$$\lambda_o = \lambda(\psi_o,\theta_{ko})$$

$$nq'\oint d\psi\ \theta_k(\psi,\lambda) = \pi \quad ---->$$

$$\oint d\psi[\omega^2 - \lambda_o - \frac{1}{2}\lambda''(\psi-\psi_o)^2]^{1/2} = \frac{\pi}{nq'}\sqrt{\frac{\lambda_{\theta_k\theta_k}}{2}}$$

$$\omega^2 = \lambda_o + O(\frac{1}{n}) = -\gamma_{MHD}^2$$

In the presence of FLR, the eigenvalue condition changes --- Roberts and Taylor, '62

- With two-fluid

$$\lambda(\psi,\theta_k) = \omega[\omega - \omega_i^*(\psi)]$$

$$\omega_i^* = n\frac{1}{e_i n_i}\frac{dp_i}{d\psi}$$

  - If λ varies more rapidly in space than ω^*, we can treat ω^* as a constant. Redoing the WKB quantization condition, we obtain

$$\omega(\omega - \omega_i^*) = \lambda_o + O(\frac{1}{n}) = -\gamma_{MHD}^2$$

$$\omega = \frac{\omega_i^*}{2} \pm i\sqrt{\gamma_{MHD}^2 - \frac{\omega_i^{*2}}{4}}$$

    - Marginal stability condition

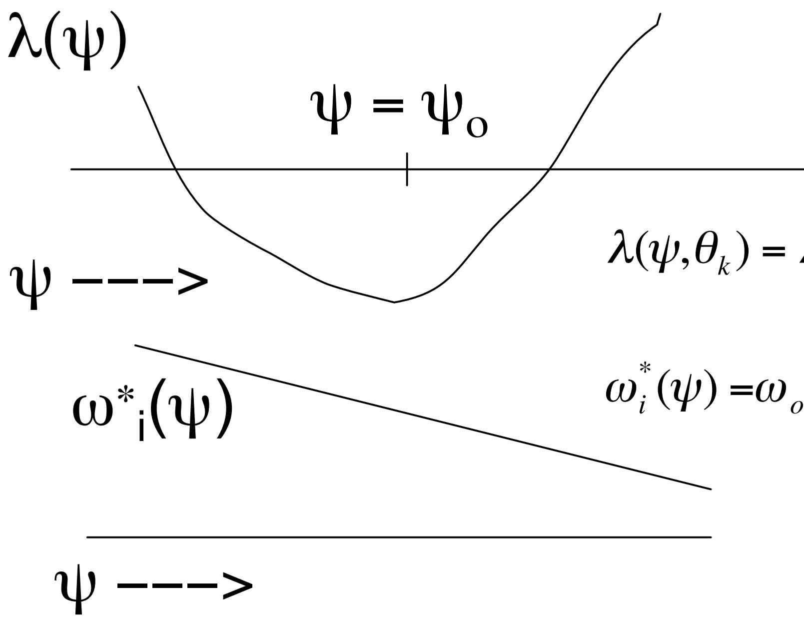
$$\gamma_{MHD}^2 = \frac{\omega_i^{*2}}{4}$$

---> $\omega*$ scales with n

--- Stabilization at high enough n

An example analytic problem is set up to account for spatially varying $\omega*$

- With two-fluid $\omega(\omega-\omega*)=\lambda$
  - Taylor expand λ near the most unstable flux surface and radial wavenumber



$$\lambda(\psi,\theta_k) = \lambda_o + \omega \frac{1}{2}\frac{\partial^2\lambda}{\partial\psi^2}(\psi-\psi_o)^2 + \frac{1}{2}\frac{\partial^2\lambda}{\partial\theta_k^{\ 2}}(\theta_k-\theta_{ko})^2 + \ldots$$

$$\omega_i^*(\psi) = \omega_o + \frac{d\omega}{d\psi}(\psi-\psi_o) + \ldots$$

Radial variation of ω* can be accounted for in the WKB quantization condition

- WKB quantization condition

$$nq'\oint d\psi\ \theta_k(\psi,\lambda) = \pi \quad --->$$

$$\oint d\psi\{\omega[\omega-\omega_i^*(\psi)]-\lambda_o-\frac{1}{2}\lambda''(\psi-\psi_o)^2\}^{1/2} = \frac{\pi}{nq'}\sqrt{\frac{\lambda_{\theta_k\theta_k}}{2}}$$

$$\oint d\psi\{\omega^2-\omega\omega_o-\omega\omega_o'(\psi-\psi_o)-\lambda_o-\frac{1}{2}\lambda''(\psi-\psi_o)^2\}^{1/2} = \frac{\pi}{nq'}\sqrt{\frac{\lambda_{\theta_k\theta_k}}{2}}$$

Radial variation in ω* shifts the radial location of the eigenfunction

- WKB quantization condition

$$nq'\oint d\psi\, \theta_k(\psi,\lambda) = \pi \quad ---->$$

$$\oint d\psi\{\omega[\omega-\omega_i^*(\psi)]-\lambda_o-\frac{1}{2}\lambda''(\psi-\psi_o)^2\}^{1/2} = \frac{\pi}{nq'}\sqrt{\frac{\lambda_{\theta_k\theta_k}}{2}}$$

$$\oint d\psi\{\omega^2-\omega\omega_o-\omega\omega_o'(\psi-\psi_o)-\lambda_o-\frac{1}{2}\lambda''(\psi-\psi_o)^2\}^{1/2} = \frac{\pi}{nq'}\sqrt{\frac{\lambda_{\theta_k\theta_k}}{2}}$$

  - Peak of eigenfunction shifts

$$\psi-\psi_o \cong -\frac{\omega_o\omega_o'}{2\lambda''+\omega_o'^2}$$

Radial variation in ω* reduces two-fluid stabilization effect

- WKB quantization condition

$$nq'\oint d\psi\ \theta_k(\psi,\lambda) = \pi \quad ---->$$

$$\oint d\psi\{\omega[\omega-\omega_i^*(\psi)]-\lambda_o-\frac{1}{2}\lambda''(\psi-\psi_o)^2\}^{1/2} = \frac{\pi}{nq'}\sqrt{\frac{\lambda_{\theta_k\theta_k}}{2}}$$

$$\oint d\psi\{\omega^2-\omega\omega_o-\omega\omega_o'(\psi-\psi_o)-\lambda_o-\frac{1}{2}\lambda''(\psi-\psi_o)^2\}^{1/2} = \frac{\pi}{nq'}\sqrt{\frac{\lambda_{\theta_k\theta_k}}{2}}$$

  - Peak of eigenfunction shifts

$$\psi-\psi_o \cong -\frac{\omega_o\omega_o'}{2\lambda''+\omega_o'^2}$$

  - Marginal Stability Condition

$$\gamma_{MHD}^2 = \frac{\omega_o^2}{4(1+\frac{\omega_o'^2}{2\lambda''})}$$

Limits:

dω*/dψ = 0 ---> Old Answer

dω*/dψ = Big ---> No ion diamagnetic stabilization! ---> Same marginal condition as ideal MHD