Investigation of Equilibrium Plasma β Limits in 3D Magnetic Topologies

Mark Schlutt and Chris C. Hegna
University of Wisconsin

Eric D. Held
Utah State University

Scott E. Kruger
Tech-X Corporation

Wednesday, 29 July 2009

Motivation.

- **Recent work indicates that in stellarators, stability may not limit β. Instead β may be limited by equilibrium physics.**[1] [2] [3] [4]
  - What maximum β is possible?
  - What factors and physics limit achievable β?
  - What equilibrium characteristics limit β?
- **The magnetic topology appears to change to limit β; flux surfaces deteriorate, enhancing transport.**
  - Pressure-induced currents may degrade magnetic surface integrity.
  - "Weakly stochastic" edge magnetic fields are produced, possibly as a result of self-consistent transport physics.

[1] M. Hirsch, et al., Plasma Phys. Control. Fusion, **50**, 1(2008).
[2] M. Sato, et al., 2008 IAEA Proceedings.
[3] A. Reiman, et al., Nucl. Fusion, **47**,572(2007)
[4] M.C. Zarnstorff, et al., 2004 IAEA Fusion Energy Conference.

Motivation and Thesis.

- **Finite transport along field lines is important to changes in the equilibrium magnetic topology. The ratio $\kappa_{\|}/\kappa_{\perp}$ may be related to maximum achievable β.**
  - Magnetic island physics is influenced by finite transport along field lines.[5] [6]
  - Stochastic regions have been observed to support a pressure gradient. [4]
- **GOAL: Study β-limiting phenomena, including the effect of self-consistent transport, using a MHD model:**
  - **Analytically: Fully 3-D MHD equilibrium island width calculations, accounting for the effect of finite parallel heat conductivity.**
  - **Numerically: Use the NIMROD code to study finite-β plasma behavior in a straight stellarator.**

---

[5] R. Fitzpatrick, Phys. Plasmas, **2**, 825(1995).

[6] Gorenlenkov, et al., Phys. Plasmas, **3**, 3379(1996).

Status at last NIMROD team meeting

Numerical work:

A 3-D magnetic field structure in a straight stellarator configuration was modeled using NIMROD.

- In vacuum, good flux surfaces are formed by analytically prescribing the magnetic field structure.
  - These vacuum solutions persist in time, even when perturbed.
  - In vacuum, magnetic islands are formed by judicious addition of small harmonics.
- When the helically symmetric system is heated, good flux surfaces remain intact, but deform.

Analytical work:

Island width equations were generated by taking into account the effect of finite parallel heat transport. It was found that the inclusion of finite parallel heat transport attenuated neoclassical effects and interchange effects, but the destabilizing Pfirsch-Schlüter current effects were unchanged. However, as a first attempt, this analysis was completed using only toroidally averaged quantities.

Work since last time

1. Answer a question from last time: Are my ”communication timescales” acceptable? That is, do my temperature profiles form over timescales that are longer than MHD instability timescales?

2. Perturbing my equilibria using reset - I'll talk more about this when I discuss future work.

3. Verifying that I have sufficient toroidal resolution. It appears that adequate resolution is achieved with lphi=6, or 22 modes.

4. Sanity check: When I change $\kappa_\perp$, $\kappa_\parallel$, or $\frac{\kappa_\parallel}{\kappa_\perp}$ do I see results that I would expect?

5. Analytical confirmation: Do back of the napkin Pfirsch-Schlüter current calculations agree with the current I see in my simulations?

6. Progress on generating an island width equation which includes toroidally non-resonant quantities.

”Communication timescales” seem acceptable for the current simulation.

Early in the simulation, the background temperature is only 1eV. So, the ion sound speed is:

$$C_s = 1.264 \cdot 10^4 \text{ m/s} \quad (1)$$

The Alfvén speed is:

$$V_A = 6.195 \cdot 10^5 \text{ m/s} \quad (2)$$

So, fast magnetosonic waves propagating perpendicular to $\vec{B}$ (compressional Alfvén waves) move with speed :

$$\left(C_s^2 + V_A^2\right)^{0.5} = 6.196 \cdot 10^5 \text{ m/s} \quad (3)$$

The plasma has a minor radius of about 0.4m, thus the ”communication timescale” is:

$$\tau = \frac{a}{\left(C_s^2 + V_A^2\right)^{0.5}} = 6.456 \cdot 10^{-7} \text{ s} \quad (4)$$

Since the temperature profiles evolve during times on the order of $1 \cdot 10^{-5}$ s, it can be concluded that the heat input rate is not too large.

Changing $\kappa_\perp$, $\kappa_\parallel$, or $\frac{\kappa_\parallel}{\kappa_\perp}$ yields results that are not necessarily expected.

Case 1: Set $\kappa_\perp = 1.0$, $\kappa_\parallel = 10^7$, and heat the helically symmetric system, non-perturbed system. On-axis equilibrium temperature $= 480eV$.
Case 2: Set $\kappa_\perp = 1.0$, $\kappa_\parallel = 10^8$, and heat the helically symmetric system, non-perturbed system. On-axis equilibrium temperature $= 111eV$.
Case 3: Set $\kappa_\perp = 1.0$, $\kappa_\parallel = 10^9$, and heat the helically symmetric system, non-perturbed system. On-axis equilibrium temperature $= 15eV$.

In principle, if $\kappa_\parallel$ is high enough, changing it shouldn't result in different core temperatures. Possible cause: low poloidal resolution is smearing $\kappa_\parallel$ and contributing to perpendicular transport.

The above cases were run with poly degree = 6, with only mx=my=8 on my mac. So, I ran the same cases with poly degree = 7.

Changing $\kappa_\perp$, $\kappa_\parallel$, or $\frac{\kappa_\parallel}{\kappa_\perp}$ yields results that are not necessarily expected.

poly degree	$\kappa_\perp (m/s^2)$	$\kappa_\parallel (m/s^2)$	T_{core} (eV)
6	1.0	10^7	480
6	1.0	10^8	111
6	1.0	10^9	15
7	1.0	10^7	415
7	1.0	10^8	114
7	1.0	10^9	16

So, this doesn't appear to be an issue with resolution. **Any suggestions?**

Analytic estimates of Pfirsch-Schlüter currents roughly match the results produced by NIMROD.

The Pfirsch-Schlüter current can be estimated from the quasi-neutrality equation:

$$(\mathbf{B} \cdot \nabla)Q = -\nabla \cdot \frac{\mathbf{B} \times \nabla p}{B^2} \tag{5}$$

where Q is the magnitude of the parallel current. Expressing Q and the Jaocbian $\mathcal{J}$ using Fourier expansions and solving yields:

$$\mathbf{J}_{\|} = Q_{mn}\mathbf{B} = \frac{-p' \mathcal{J}_{mn}}{\iota - \frac{n}{m}}\mathbf{B} \tag{6}$$

Substituting in typical near-axis values from the present simulation (after the temperature/pressure profile has evolved) gives:

$$Q = 0.47 \text{ MA} \tag{7}$$

Using nimplot, it is found that near the geometric axis, $J_\phi \approx 1.0$ MA. Thus, the analytical estimate of the Pfirsch-Schlüter current is of similar magnitude to the numerical calculation.

Future Work

1. Can we create a helically symmetric system which is above some MHD stability boundary, where adding a 3-D perturbation would trigger the onset of instability?

2. Does island behavior match analytical predictions when $\frac{\kappa_{\|}}{\kappa_{\perp}}$ are changed?

How will a 3D system react if it is above some MHD stability boundary?

Concept: As noted above, work at LHD and Wendelstein indicates that the magnetic topology changes in response to increasing β. To further study this, we take away the ability of the magnetic topology to change (keep the system in a 2D state), and heat the system. Then we introduce 3D perturbations to the system. Specifically,

- Create a helically symmetric vacuum case.
- Heat this configuration, but set bamp=0. In other words, minimize the perturbations to the system. Since the system is in principle helically symmetric, it is 2D. No flux surface destruction should be observed. At this point, we will perhaps have a state which is above some MHD stability boundary.
- Now we restart the calculation and introduce a 3D perturbation.
  - Will this perturbation trigger some MHD instability?
  - How will the system respond?
  - Will the final state of the system match that of a system that allowed 3D topology from the start?

Analytical theory predicts that changing $\frac{\kappa_\parallel}{\kappa_\perp}$ will influence island size and growth.

The previous analytic island theory was extended: the effect of finite parallel thermal conductivity has been included in a 3D configuration. This calculation was carried out for toroidally resonant quantities only. For $W < W_c$, matched asymptotic analysis yields:

$$\underbrace{\Delta' W + \frac{C}{W}}_{\text{unaffected by finite } \kappa_\parallel} \underbrace{+ 1.25 D_{NC}\left(\frac{W^2}{W_c^2}\right) + 5.80 D_R\left(\frac{W}{W_c}\right)}_{\text{island region effects} \Rightarrow \text{ finite } \kappa_\parallel \text{ important}} = 0 \tag{8}$$

where:

- Resonant Pfirsch-Schlüter effects:

  $C \sim \frac{R_0^2 \beta}{a r_s m^2 \iota'^2} \frac{\mathcal{J}_{mn}}{\mathcal{J}_{00}}$

  $\mathcal{J} = \sum_{m,n} \mathcal{J}_{mn} e^{im\theta - in\zeta} \sim \frac{1}{B^2}$

- Resistive interchange effects

  $D_R = E + F + H^2 \sim \frac{\mathcal{J}'_{00} p'_0}{\iota'^2}$

- Neoclassical effects for quasi-axisymmetry:

  $D_{NC} = -4.6(\epsilon)^{0.5} \frac{2\mu_0 p'_s R^2}{\psi_s'^2} \frac{q_s}{q'_s}$

Other future work.

- Temperature evolution studies will continue for the two cases, helically symmetric (2-D) field and the broken symmetry (3-D) case.
- Adjust the strength of the symmetry-breaking terms to obtain differing degrees of stochasticity.
- The ratio $\frac{\kappa_{\parallel}}{\kappa_{\perp}}$ will be modified and the effect on achievable β will be observed. Is the behavior related to standard stability metrics, e.g. Mercier criterion?

Changing $\kappa_\perp$, $\kappa_\|$, or $\frac{\kappa_\|}{\kappa_\perp}$ yields results that are not necessarily expected.

Case 1: Set $\kappa_\perp = 1.0$, $\kappa_\| = 10^7$, and heat the helically symmetric system, non-perturbed system. On-axis equilibrium temperature $= 480eV$.
Case 2: Set $\kappa_\perp = 1.0$, $\kappa_\| = 10^8$, and heat the helically symmetric system, non-perturbed system. On-axis equilibrium temperature $= 111eV$.
Case 3: Set $\kappa_\perp = 1.0$, $\kappa_\| = 10^9$, and heat the helically symmetric system, non-perturbed system. On-axis equilibrium temperature $= 15eV$.

In principle, if $\kappa_\|$ is high enough, changing it shouldn't result in different core temperatures. Possible cause: low poloidal resolution is smearing $\kappa_\|$ and contributing to perpendicular transport.

In a system with $\mathbf{v} = 0$,

$$\frac{3}{2}\frac{\partial T}{\partial t} + \nabla \cdot \left[\left(-\chi_\| \hat{\mathbf{b}}\hat{\mathbf{b}} - \chi_\perp(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})\right) \cdot \nabla T\right] = source \qquad (9)$$

The above cases were run with poly degree = 6, with only mx=my=8 on my mac. So, I ran the same cases with poly degree = 7.