Logan, UT

August 4, 2011

0.1 First-order DKE in the (ξ,s) velocity variables

Hazeltine's form for the drift kinetic equation (ϵ, μ) :

$$\partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f + \left(\mu \frac{\partial B}{\partial t} + e(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right) \partial_\epsilon f = C + Q.$$

Using $\xi = v_{||}/v$ and $s = v/v_0$ yields

$$\begin{aligned} \partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f - +(\mathbf{v}_{||} + v_D) \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s\right] f + \\ \left(\frac{e}{2e_0 s^2}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right) (s\partial_s f + 2g(\xi)\partial_\xi f) \;=\; C \end{aligned}$$

with general form for drift

$$v_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{e_0 s^2}{eB}\left[\mathbf{b}\times\left(\left(1-\xi^2\right)\nabla \ln B + 2\xi^2\kappa - \frac{v_0 s \xi}{e_0 B}\nabla \times \mathbf{E}\right) + (1-\xi^2)\frac{\mu_0 \mathbf{J}_{||}}{B}\right].$$

0.2 1D FE basis in ξ

- Use 1D FE grid in pitch angle. In each element expand f as :

$$f(\mathbf{r}, t, \xi, s) \;=\; \sum_i f_i(\mathbf{r}, t, s)\phi_i(x)$$

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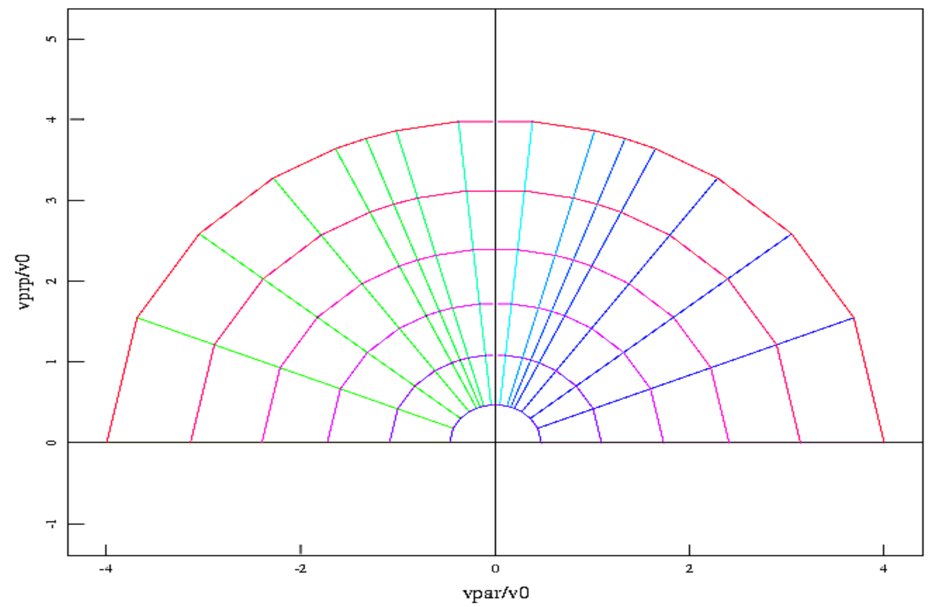
- Modal (built from Legendre polynomials) and nodal (Lagrange and Gauss-Lobatto-Legendre) bases have been implemented.

- Pitch-angle coefficients, f_i, computed on speed grid determined by quadrature on semi-infinite ($s\epsilon[0, \infty)$) or finite ($s\epsilon[0, s_{max}]$) domains.

0.3 Sample velocity grids

- 3 cells in ξ, 5th-order polynomials, 6 speed grid points = 96 unknowns.



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0.4 Use full linearized operator in test of Spitzer conductivity.

Moment form of full, linearized Coulomb collision operators for electrons and ions :

$$C(f_{1a}, f_{0b})+C(f_{0a}, f_{1b}) = \sum_k \frac{f_{0a}}{\sigma_k^1} P_1(v_{||}/v) \left[\nu_{ab}^{1k,0}(s_a) M_{||a}^{1k}(\mathbf{r}, t) + \nu_{ab}^{0,1k}(s_a) M_{||b}^{1k}(\mathbf{r}, t)\right].$$

Solve for perturbed distribution functions, $f_{1e}$and f_{1i}:

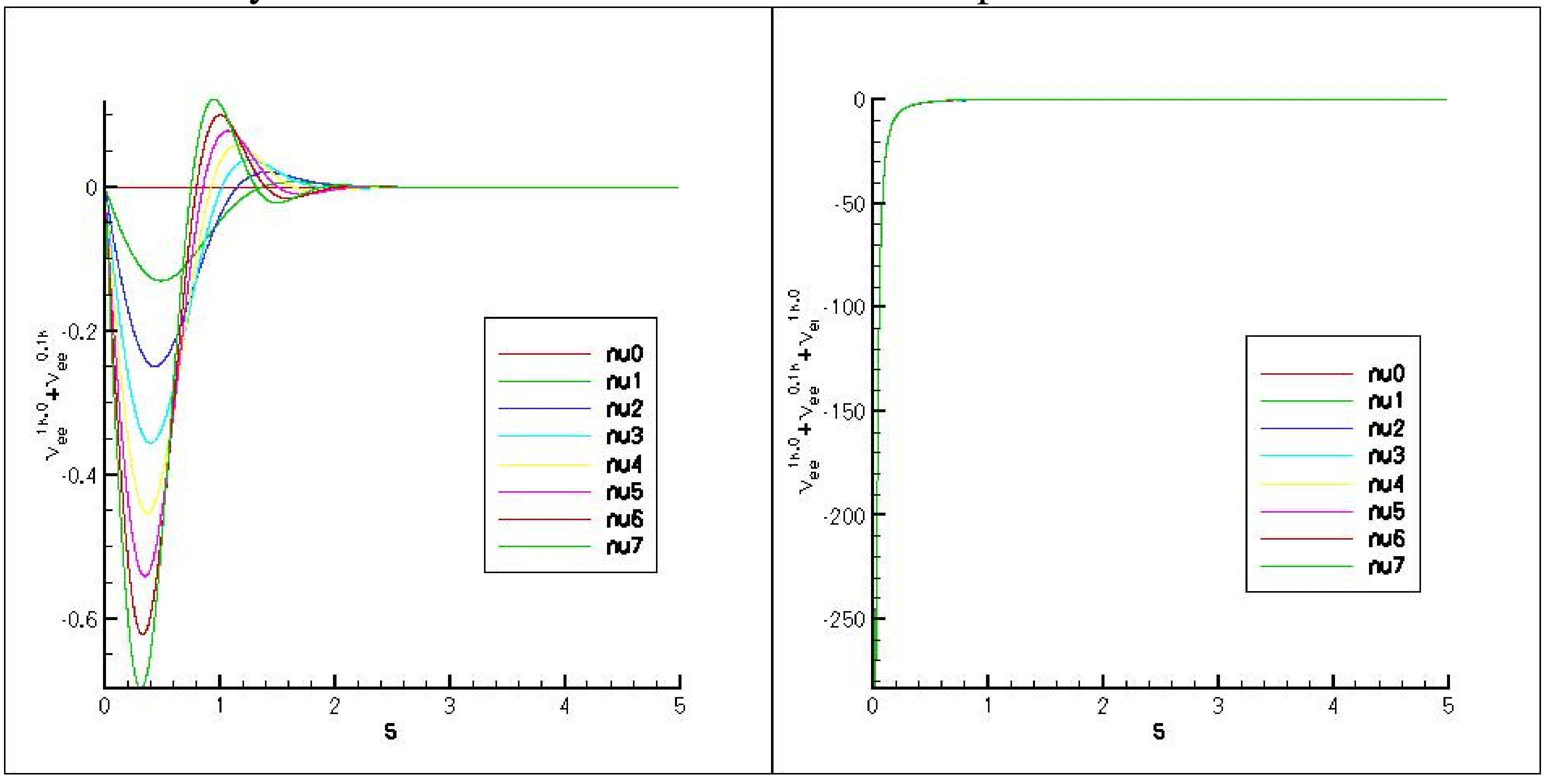
$$\partial_t f_{1e} + C(f_{1e}, f_{0e}) + C(f_{0e}, f_{1e}) + C(f_{1e}, f_{0i}) + C(f_{0e}, f_{1i}) = v_{||}(q_e E_{||}/T_0)\, f_{0e}.$$

$$\partial_t f_{1i} + C(f_{1i}, f_{0i}) + C(f_{0i}, f_{1i}) + C(f_{1i}, f_{0e}) + C(f_{0i}, f_{1e}) = v_{||}(q_i E_{||}/T_0)\, f_{0i}.$$

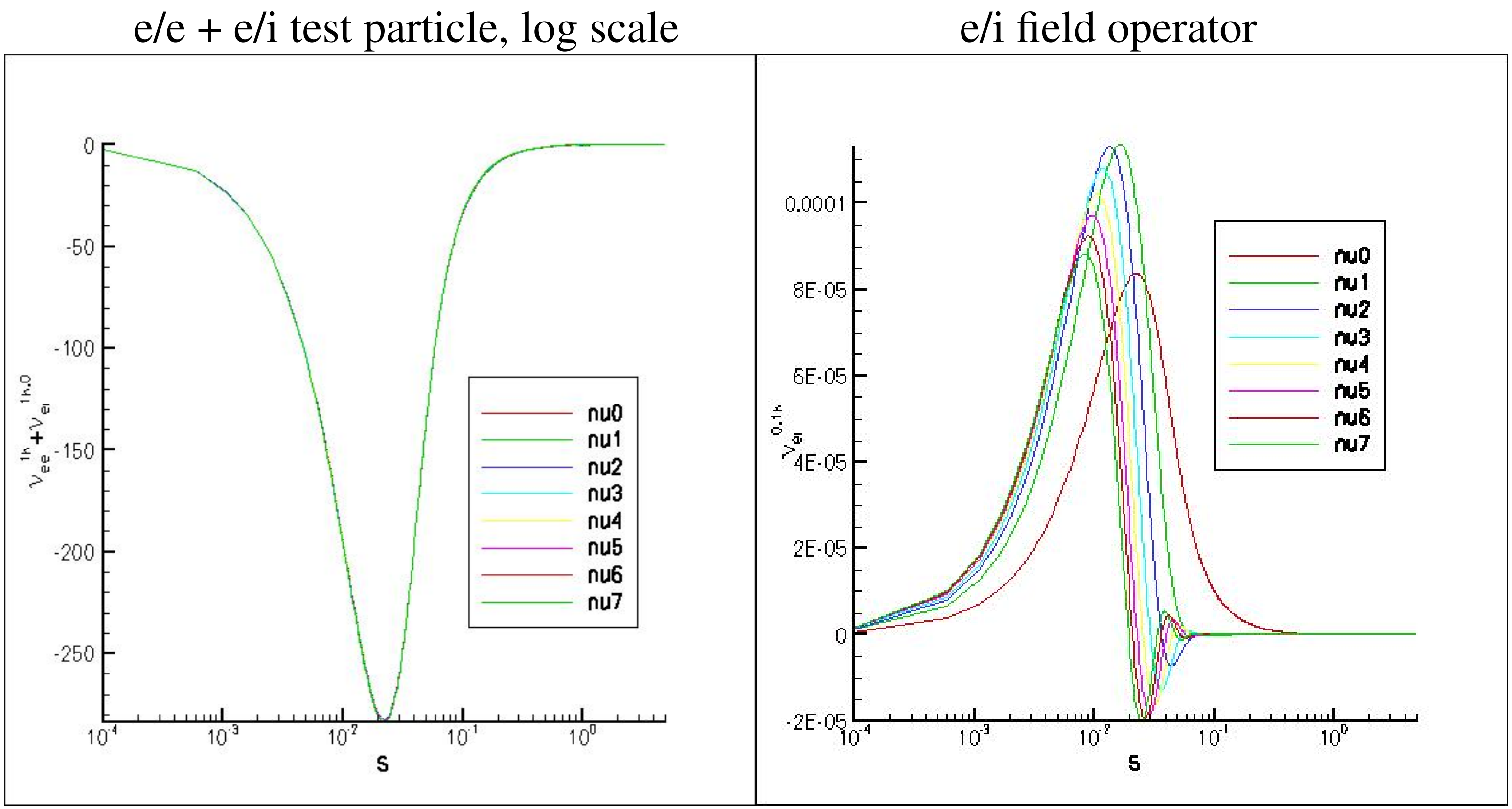
Test momentum conservation for various speed grids, $s_a = v/v_{Ta}$.

0.5 Stong e/i interaction at low electron speeds.

e/e only e/e + e/i test particle



0.6 Stong e/i interaction at low electron speeds.



0.7 Results for Spitzer problem

Evolve kinetic equations for 100 electron collision times.

	conductivity	electron momentum	ion momentum
exact	1.9623169	-2.2256440E-4	2.2256440E-4
32 nodes	1.9623200	-2.2256482E-4	2.2244207E-4
8 nodes	1.9624281	-2.2257996E-4	2.1716122E-4
4 nodes	1.9933479	-2.2613467E-4	1.3289914E-4

Conductivity and electron momentum accurately reproduced.

Ion momentum conservation requires more quadrature points.

Introduce momentum restoring concepts into this discrete approach.

0.8 Progress since Sherwood

1. Data reordering carried out on cel_continuum code in nimdevel.

2. Full, linearized Coulomb collision operator implemented with diagonal-in-s preconditioning.

3. F_e and F_i advanced simultaneously.

4. Time discretization and velocity space representation tested on Spitzer thermalization and conduction problems using CEL approach with $\mathbf{V}_e, \mathbf{V}_i, T_e, T_i$ staggered in time from F_e, F_i.

0.9 CEL formulation of Spitzer test problems

$$
\begin{aligned}
\partial_t F_a + \partial_t s \partial_s F_a &= \sum_b f_{Ma} \left(\nu_{ab}^{00,0} + \frac{2v_{||}}{v_T^2} \left(\nu_{ab}^{10,0} V_{||a} + \nu_{ab}^{0,10} V_{||b} \right) \right) \\
&+ \sum_{lk} \frac{f_{Ma}}{\sigma_k^l} s^l P_l \left(\nu_{ab}^{lk,0} \hat{M}_{||a}^{lk} + \nu_{ab}^{0,lk} \hat{M}_{||b}^{lk} \right) \\
&+ \frac{2}{3} L_1^{1/2}(s^2) \left(Q_a^* - V_{||a} R_{||a}^* / v_{Ta} \right) f_{Ma} - \left(s P_1 - V_{||a} / v_{Ta} \right) R_{||a}^* f_{Ma},
\end{aligned}
$$

$$
\begin{aligned}
\partial_t n_a &= 0 \\
\partial_t V_{||a} &= \frac{q_a}{m_a} E_{||} + v_{Ta} R_{||a}^* / 2 \\
\partial_t T_a &= \frac{2}{3} T_a (Q_a^* - V_{||a} R_{||a}^* / v_{Ta}).
\end{aligned}
$$

0.10 Collisional Moments

$$
\begin{aligned}
R_{||a} &= \int d\mathbf{v} m_a v_{||} C_{ab} \\
&= \frac{8\pi}{3}\frac{p_a}{v_{Ta}}\int_0^\infty dss^3 \left[\sum_k \frac{\bar{f}_{Ma}}{\sigma_k^1}\left(\nu_{ab}^{1k,0}\hat{M}_{||a}^{1k} + \nu_{ab}^{0,1k}\hat{M}_{||b}^{1k}\right)\right] \\
&= \frac{p_a}{v_{Ta}} R_{||a}^*
\end{aligned}
$$

$$
\begin{aligned}
Q_a &= \int d\mathbf{v}(m_a v^2/2) C_{ab} \\
&= 4\pi p_a \int_0^\infty dss^4 \left[\bar{f}_{Ma}\nu_{ab}^{00,0} + \sum_{k>1}\frac{\bar{f}_{Ma}}{\sigma_k^0}\left(\nu_{ab}^{0k,0}\hat{M}_{||a}^{0k} + \nu_{ab}^{0,0k}\hat{M}_{||b}^{0k}\right)\right] \\
&= p_a Q_a^*.
\end{aligned}
$$

0.11 Diagonal-in-s preconditioning

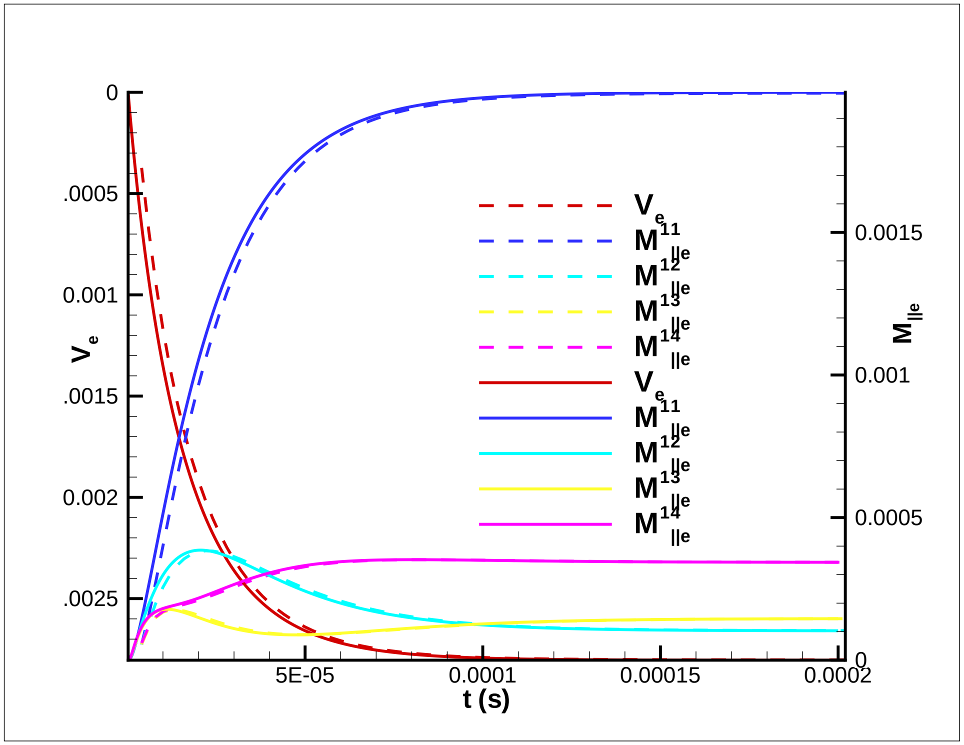
Implicit terms in equation for $l-th$ Legendre coefficient at grid point s_j:

$$
\begin{aligned}
\Delta F_a &- \theta\Delta t \sum_k \frac{f_{Ma}(s_j)}{\sigma_k^l} s_j^l P_l \left(\nu_{ab}^{lk,0}(s_j)\hat{M}_{||a}^{lk}(\Delta F_a^l) + \nu_{ab}^{0,lk}(s_j)\Delta\hat{M}_{||b}^{lk}(F_b^l)\right) \\
&- \theta\Delta t\delta_{l0}\frac{2}{3}L_1^{1/2}(s_j^2)\left(Q_a^*(\Delta F_a^0) - V_{||a}R_{||a}^*(\Delta F_a^0)/v_{Ta}\right) f_{Ma}(s_j) \\
&- \theta\Delta t\delta_{l1}\left(s_j P_1 - V_{||a}/v_{Ta}\right) R_{||a}^*(\Delta F_a^1) f_{Ma}(s_j) \\
&+ \theta\Delta t s_j \sum_m (M_{lm}^S + \nabla\ln B M_{lm}^B)\Delta F_a^m(s_j)
\end{aligned}
$$

where $\hat{M}_{||a}^{lk} = \frac{4\pi l!}{(2l+1)!!}\frac{v_{Ta}^3}{n_a}\sum_i w_i s_i^{l+2} L_k^{l+1/2}(s_i^2) F_a^l(s_i)$.

0.12 Conduction Results

Compare moments, $n_a \hat{M}^{lk}_{||a} = \frac{n!}{(2n-1)!!} \int d\mathbf{v} s^n P_n L_p^{n+1/2} F$, with "exact" calculation.



0.13 Thermalization Results

Time-dependent s term required: $\partial_t s \partial_s F = \sum_{lk} \frac{f_{Ma}}{\sigma_k^l} s^l P_l \Big[(3/2 - s^2 + l/2) L_k^{l+1/2} - s^2 L_{k-1}^{l+3/2} \hat{M}_{||a}^{lk} \partial_t$