Ion temperature gradient driven instabilities in extended MHD

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To evaluate the effects of ion-temperature-gradient (ITG) in extended MHD regimes and applications

- ITG effects found in recent extended MHD simulations of DIII-D experiments using the NIMROD code [Schnack 2010]
- Two-fluid Ohm's law and ion gyroviscosity included in extended MHD model implemented in NIMROD code [Sovinec *et al.* 2004].
- How does ITG affect MHD (in)stabilities in extended MHD model?
  - May help explain unstable modes in MHD-stable configurations.
  - Verification of extended MHD code.
  - Connections between macroscopic and microscopic instabilities.
- Recently theory and NIMROD calculations developed.

Differences from conventional ITG instabilities in slab configuration?

- Conventional slab-ITG modes [Rudakov and Sagdeev 1961] are electrostatic
  - Can have electromagnetic extension but not a prerequisite.
  - Here we focus on extended MHD regimes where $\tilde{\mathbf{B}} \neq 0$.
- Conventional slab ITG modes rely on diamagnetic heat flow
  - Only modes are sound waves in absence of $\mathbf{q}_* = \frac{5nT}{2m_i\Omega}\mathbf{b} \times \nabla T$ [e.g. Sovinec 2010].
  - Here we focus on extended MHD regimes where adiabatic energy evolution applies.
- We have found slab ITG-driven instabilities that are electromagnetic and in absence of diamagnetic heat flow in an extended MHD model.

Single fluid formulation of extended MHD model implemented in NIMROD code is used in both theory and calculation

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{u} + D\nabla^2\rho \quad (1)$$

$$\rho\frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{J}\times\mathbf{B} - \nabla\cdot\boldsymbol{\pi} \quad (2)$$

$$\frac{n}{\gamma-1}\frac{dT}{dt} = -\frac{p}{2}\nabla\cdot\mathbf{u} - \boldsymbol{\pi} : \nabla\mathbf{u} - \nabla\cdot\mathbf{q} + Q \quad (3)$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} \quad (4)$$

$$\mu_0\mathbf{J} = \nabla\times\mathbf{B} \quad (5)$$

$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta\mathbf{J} + \frac{\lambda}{ne}(\mathbf{J}\times\mathbf{B} - \nabla(1-\tau)p) \quad (6)$$

where

$$\boldsymbol{\pi}_i = \frac{\tau p}{4\Omega}\left\{\left[\mathbf{b}\times(\nabla\mathbf{u}+\nabla\mathbf{u}^{\mathrm{T}})\cdot(\mathbf{I}+3\mathbf{bb})\right] + [\text{previous term}]^{\mathrm{T}}\right\} \quad (7)$$

Two-fluid equilibrium in shear-less slab configuration and fluite-like perturbation are assumed

- Two-fluid equilibrium

$$\mathbf{u} = 0 \tag{8}$$

$$\mathbf{B} = B\mathbf{e}_z \tag{9}$$

$$\frac{d}{dx}\left(p + \frac{B^2}{2}\right) = \rho \mathbf{g} \cdot \mathbf{e}_x = \rho g \tag{10}$$

$$ne\mathbf{E} = \nabla p_i - \rho \mathbf{g}. \tag{11}$$

- Flute-like perturbation with local approximation

$$\mathbf{u} = (u_x(x), u_y(x), u_z(x)) \exp\left(-i\omega t + ik_y y + ik_z z\right). \tag{12}$$

where $k_z/k_y \sim \epsilon$, $k_y L_A \sim \epsilon^{-1}$, $k_y d_i \sim \lambda \sim \delta$, $u_y \sim \epsilon u_x$, $u_x \sim u_z$, $\epsilon \ll 1$, where $L_A = |d \ln A/dx|^{-1}$ is the spatial scale of field A in x direction, and $d_i = \sqrt{p_i/\rho}/\Omega$ is the ion Larmor radius.

Electrostatic ITG instability in extended MHD model requires diamagnetic heat flow

With $\mathbf{g} = 0$ and $\mathbf{q} = \mathbf{q}_* = \frac{\chi p_i}{\Omega} \mathbf{b} \times \nabla \frac{p_i}{\rho}$, electrostatic ITG mode dispersion relation has the cubic form

$$c_3 \omega^3 + c_2 \omega^2 + c_1 \omega + c_0 = 0 \tag{13}$$

where

$$c_3 = 4\gamma + \frac{k_y^2 \delta^2 \tau^2}{\Omega^2} \frac{p}{\rho} \tag{14}$$

$$c_2 = -\left[\frac{k_y^3 \delta^3 \tau^3 (1+\beta)}{\Omega^3} \frac{p}{\rho} + \frac{2 k_y \delta \tau}{\Omega} + \frac{4 k_y \tau^2 \chi (\gamma - 1)(1+\beta)}{\Omega} \right] \frac{p'}{\rho} + \ldots \tag{15}$$

$$c_1 = -\frac{k_y^2 \delta^2 \tau^2}{\Omega^2} \frac{p^2}{\rho^2} \left\{ 9\gamma k_z^2 + \left[2\beta(4\gamma - 1) + 4\gamma\beta^2 + 2(2\gamma - 1) \right] \left(\frac{p'}{p} \right)^2 \right\} + \ldots \tag{16}$$

$$c_0 = \frac{k_y^3 \delta^2 \tau^4 \chi (\gamma - 1)(1+\beta)}{\Omega^3} \frac{p^3}{\rho^3} \frac{p'}{p} \left[9k_z^2 + 4(1+\beta)^2 \left(\frac{p'}{p} \right)^2 - 2(1+\beta) \frac{\rho'}{\rho} \frac{p'}{p} \right] \tag{17}$$

Ideal MHD model yields coupling of shear-Alfvén and slow wave modes

- In absence of **g**

$$(\omega^2 - k_z^2 u_{\mathrm{A}}^2)\left(\omega^2 - \frac{k_z^2 c_s^2}{1+\gamma\beta}\right) = 0 \qquad (18)$$

- 3D g mode

$$\omega^4 + \left[\frac{g^2}{u_{\mathrm{A}}^2(1+\gamma\beta)} - \frac{\rho'}{\rho}g - k_z^2 u_{\mathrm{A}}^2 \frac{1+2\gamma\beta}{1+\gamma\beta}\right]\omega^2 + \frac{k_z^2 u_{\mathrm{A}}^2}{1+\gamma\beta}\left(k_z^2 c_s^2 - \frac{g^2}{u_{\mathrm{A}}^2} + \gamma\beta g\frac{\rho'}{\rho}\right) = 0 \qquad (19)$$

Extended MHD model leads to linear dispersion relation with ITG effects [Zhu *et al.* 2011]

To dominant order in ϵ, the dispersion relation takes the quintic form

$$c_5\omega^5 + c_4\omega^4 + c_3\omega^3 + c_2\omega^2 + c_1\omega + c_0 = 0 \tag{20}$$

where

$$c_5 = 1 + \gamma\beta + \frac{k_y^2\delta^2\tau^2}{4\Omega^2}\frac{p}{\rho}\beta \tag{21}$$

$$c_4 = -\frac{k_y^3\tau^3}{4\Omega^3}\frac{pp'}{\rho^2}\beta(1+\beta) + \frac{k_y\lambda}{\Omega}\left[(1+\tau+\gamma\beta)\frac{p'}{\rho} - \tau c_s^2\frac{\rho'}{\rho}\right] + \dots \tag{22}$$

$$c_3 = -k_z^2u_{\mathrm{A}}^2(1+2\gamma\beta) - \frac{k_y^2\lambda^2}{\Omega^2}\left[k_z^2u_{\mathrm{A}}^2c_s^2 + \tau\frac{p'}{\rho}(-\frac{p'}{\rho} + c_s^2\frac{\rho'}{\rho})\right] + \dots \tag{23}$$

$$c_2 = -\frac{k_y\lambda}{\Omega}k_z^2u_{\mathrm{A}}^2\left[(1+\tau+\gamma\beta)\frac{p'}{\rho} - \tau c_s^2\frac{\rho'}{\rho}\right] + \frac{k_y\delta\tau}{\Omega}k_z^2u_{\mathrm{A}}^2(1+\beta)\frac{p'}{\rho} + \dots \tag{24}$$

$$c_1 = k_z^4u_{\mathrm{A}}^2c_s^2 + \frac{k_y^2\lambda^2}{\Omega^2}k_z^2u_{\mathrm{A}}^2\tau\frac{p'}{\rho}\left(-\frac{p'}{\rho} + c_s^2\frac{\rho'}{\rho}\right) + \dots \tag{25}$$

$$c_0 = \frac{k_y^3\lambda^2\delta\tau}{\Omega^3}k_z^2u_{\mathrm{A}}^2(1+\beta)\left(\frac{p'}{\rho} - c_s^2\frac{\rho'}{\rho}\right)\left(\frac{p'}{\rho}\right)^2 \tag{26}$$

Further reduction can be made in low frequency and small Larmor radius regime

- To first order in $(\omega_{*i}, \omega_{*p}) \ll \omega_{\mathrm{A}}$ and $k_y \rho_{\mathrm{L}} \ll 1$, the dispersion can be further reduced to

$$(1+\gamma\beta)\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^4 - \lambda\left[\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta - \frac{2}{3}\right) + \frac{\omega_{*p}}{\omega_{\mathrm{A}}}(1+\gamma\beta)\right]\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^3 - (1+2\gamma\beta)\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^2 + \left\{\lambda\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta - \frac{2}{3}\right) + \frac{\omega_{*p}}{\omega_{\mathrm{A}}}[\lambda(1+\gamma\beta) - \delta\tau(1+\beta)]\right\}\left(\frac{\omega}{\omega_{\mathrm{A}}}\right) + \gamma\beta = 0 \quad (27)$$

where $\eta = d \ln T / d \ln \rho$, $T = p/\rho$, $\gamma = 5/3$, and

$$\omega_{*i} = -\frac{k_y}{\Omega}\frac{\tau p}{\rho}\frac{\rho'}{\rho}, \quad \omega_{*p} = -\frac{k_y}{\Omega}\frac{p}{\rho}\frac{p'}{p} \quad (28)$$

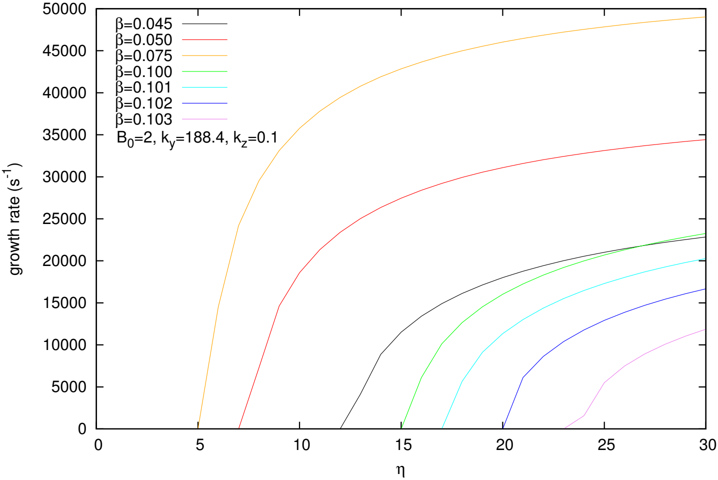
- Formally, the zero β limit gives the cubic form of dispersion relation

$$\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^3 - \lambda\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta - \frac{2}{3} + \frac{\eta+1}{\tau}\right)\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^2 - \frac{\omega}{\omega_{\mathrm{A}}} + \lambda\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta - \frac{2}{3}\right) + \frac{\omega_{*p}}{\omega_{\mathrm{A}}}(\lambda - \delta\tau) = 0 \quad (29)$$

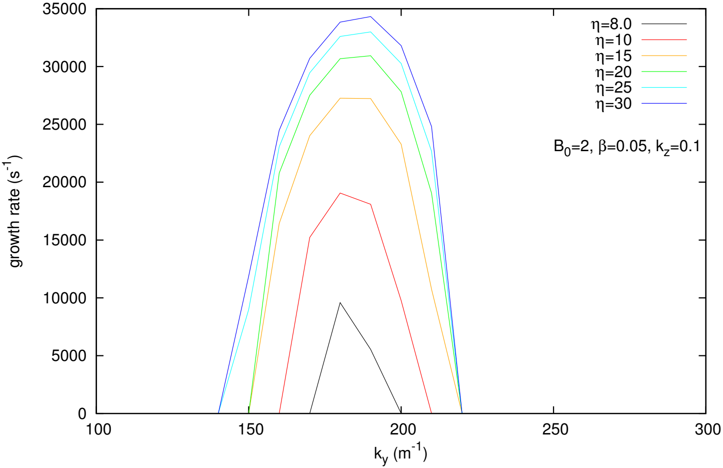
Two cases are considered for the solution of local dispersion relation

- Case 1 [Schnack *et al.* 11]: $B_0 = 2$, $\beta = 0.05$, $\tau = 0.2$ (lower β)
- Case 2 [Zhu *et al.* 07]: $B_0 = 6$, $\beta = 0.1$, $\tau = 0.5$ (higer β)
- Parameters common to both cases:
  $\rho = 2 \times 10^{20} m_i$, density (pressure) gradient scale length L_ρ (L_p).

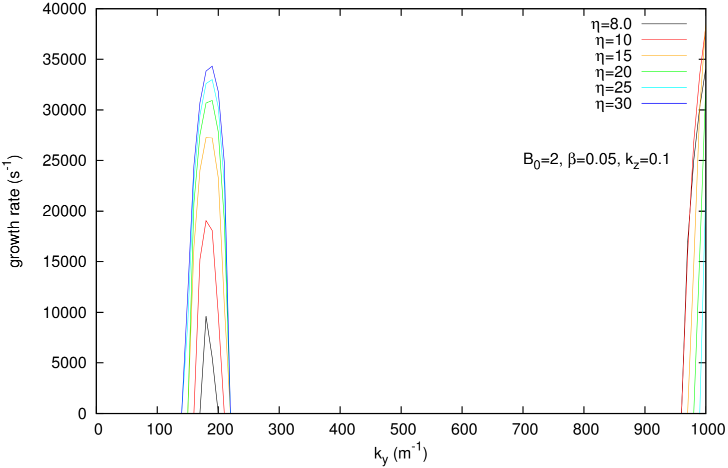
ITG-driven instability exists in extended MHD; η thresholds for onset are sensitive to β



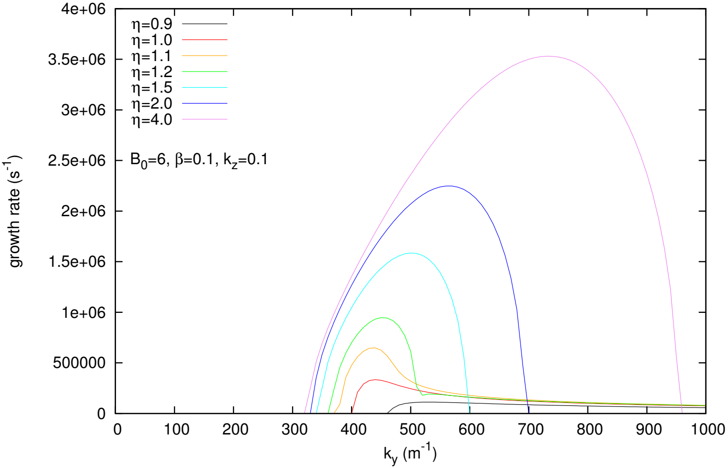
Solutions from analytic dispersion relation show instabilities when $\eta > 8$ in $k_y \rho_L \sim 0.1$ regime



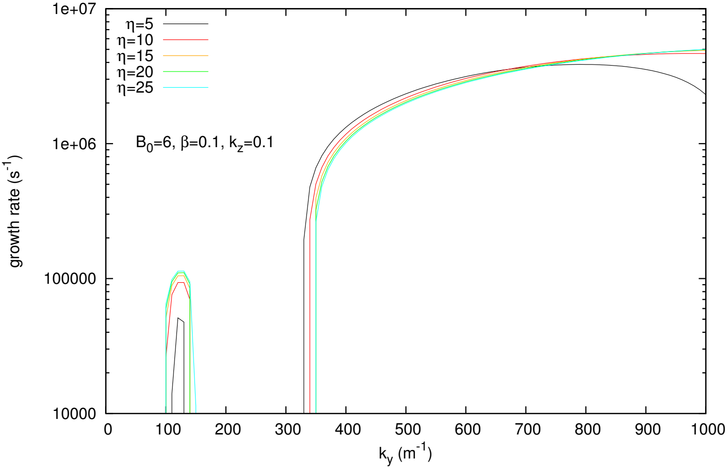
In $k_y \rho_L \sim 0.1 - 1$ regime there are two branches of ITG-driven instabilities



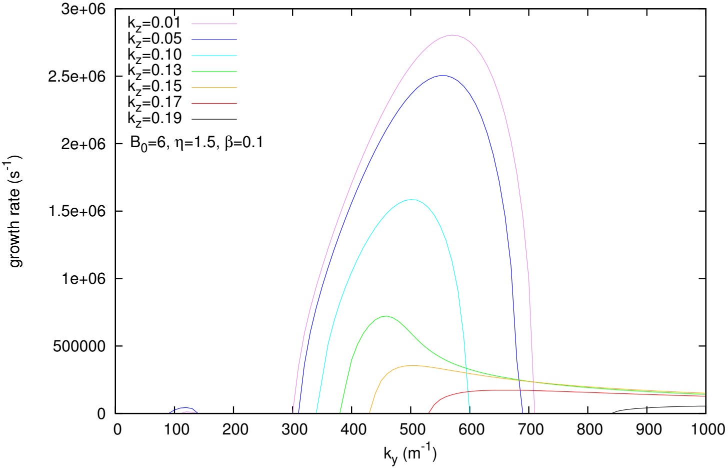
High ITG branch ($k_y \rho_L \sim 1$) more dominant in higher β regime ($\eta_{\rm crit} \sim 0.8$)



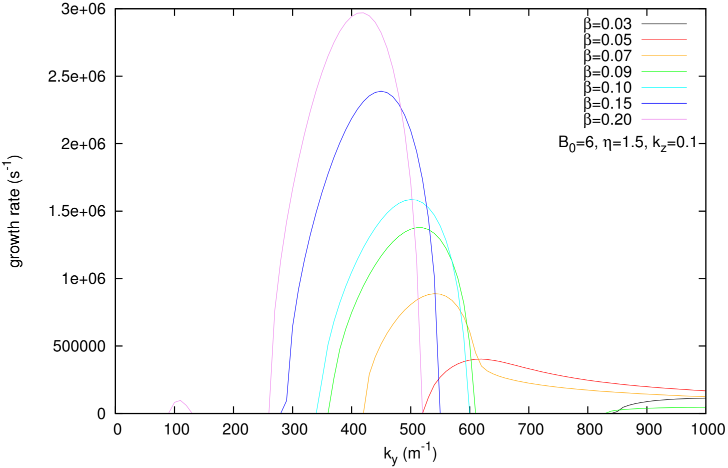
Low ITG branch ($k_y \rho_L \sim 0.1$) becomes less dominant in higher β regime



Instabilities more unstable at smaller k_z and have a cut-off $k_z \sim 0.2$ in high branch ($k_y \rho_L \sim 1$)



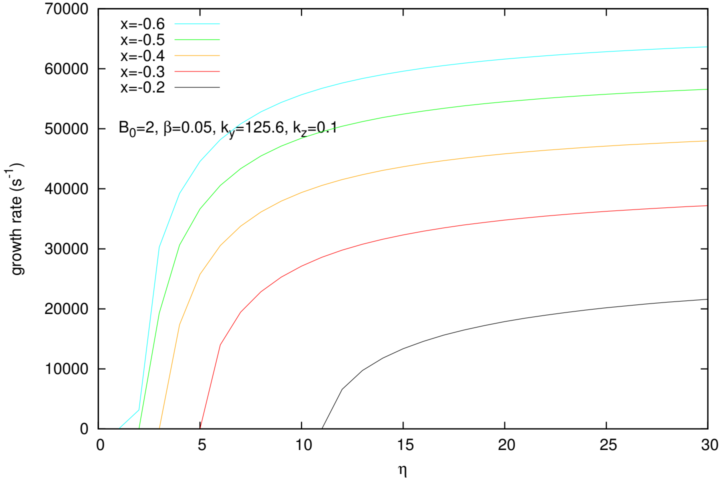
High branch ($k_y\rho_L \sim 1$) growth increases with β ($\beta_{\text{crit}} \sim 0.025$) and shifts towards smaller $k_y\rho_L$



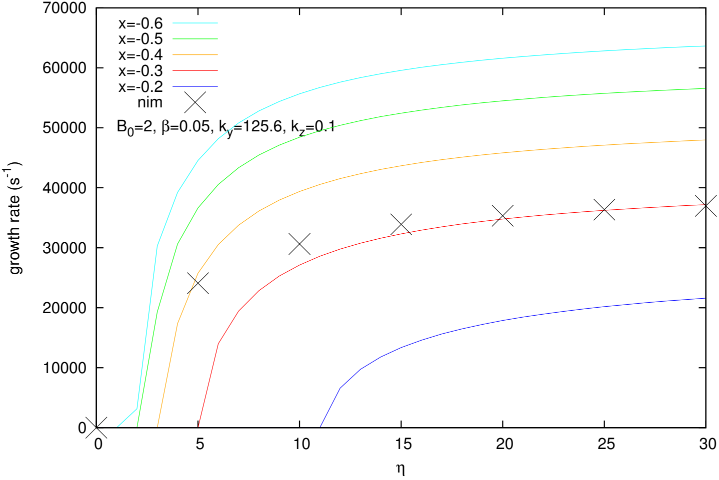
Exponential profiles for both density and pressure are considered in nimrod calculation

- Density and pressure: $\rho(x) = \rho_0 e^{-\frac{x}{L_\rho}}$, $p(x) = p_0 e^{-\frac{x}{L_p}}$, $\eta = \frac{L_\rho}{L_p} - 1$
- Magnetic field $\mathbf{B} = B(x)\hat{\mathbf{Z}}$ is in $R - Z$ plane.
- k_y of perturbation is in periodic direction.
- While η is constant, other key parameters such as β can have large range of variation over x.

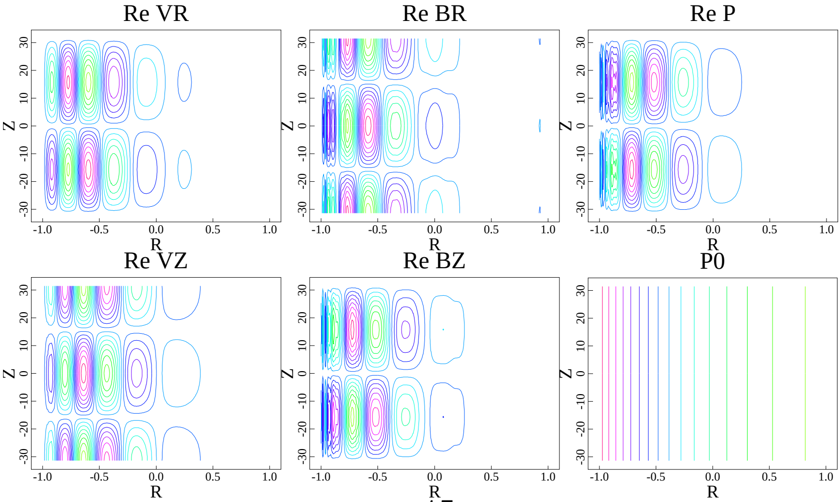
For a fixed k_y ITG growth at each location in x has its own η dependence



Comparison between nimrod calculation and theory show qualitative agreement when spatial profile effects taken into account

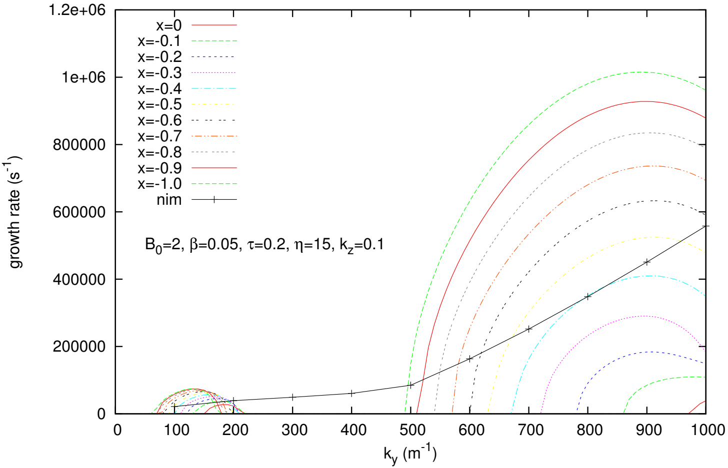


Mode structure of low $k_y = 40\pi$ ITG-driven instability in exponential equilibrium from NIMROD calculation

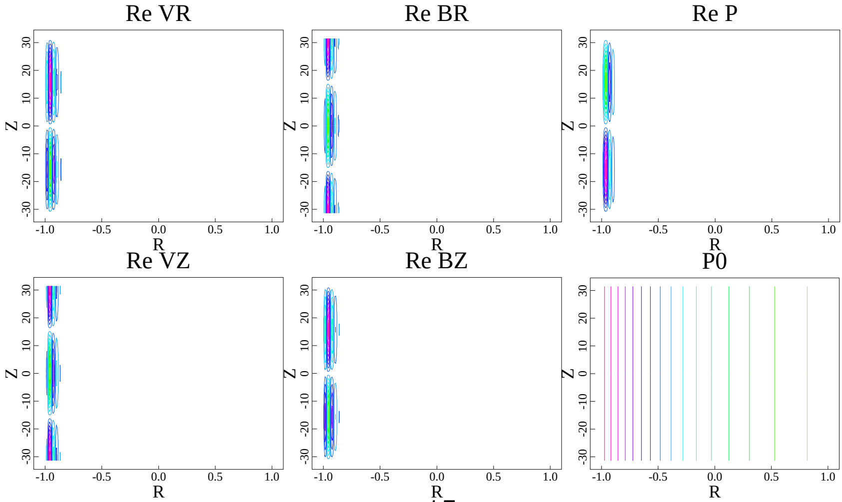


$\eta = 15$

ITG-mode growth rates from NIMROD calculation also show two k_y regimes, consistent with local dispersion relation

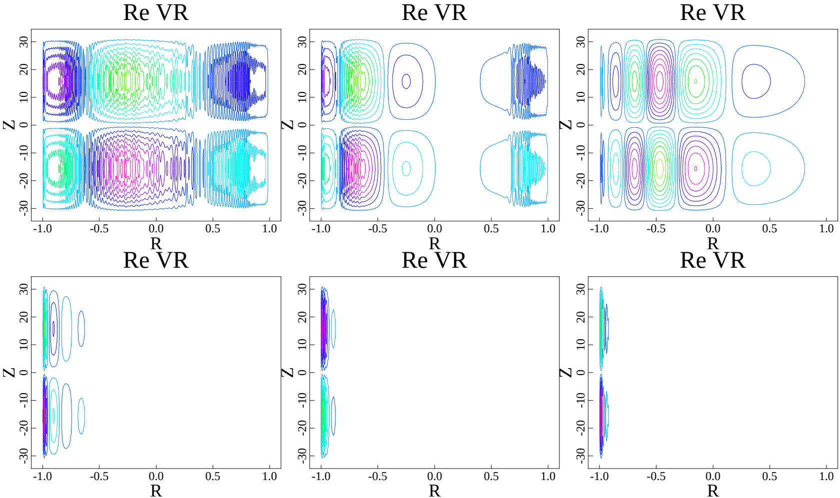


Mode structure of high $k_y = 200\pi$ ITG-driven instability in exponential equilibrium from NIMROD calculation



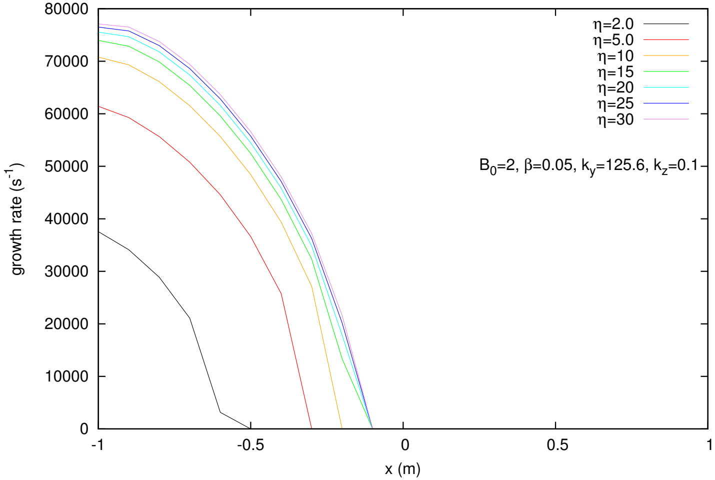
$\eta = 15$

As k_y increases mode structure from calculations becomes more localized and asymmetric in x

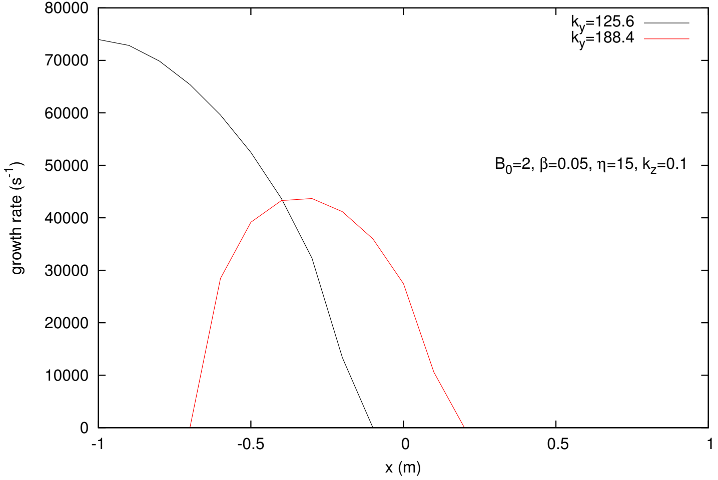


Upper row: $k_y = 100, 200, 400$; Lower row: $k_y = 600, 800, 1000$

For a fixed k_y, the $x < 0$ region is increasingly more unstable from local dispersion relation



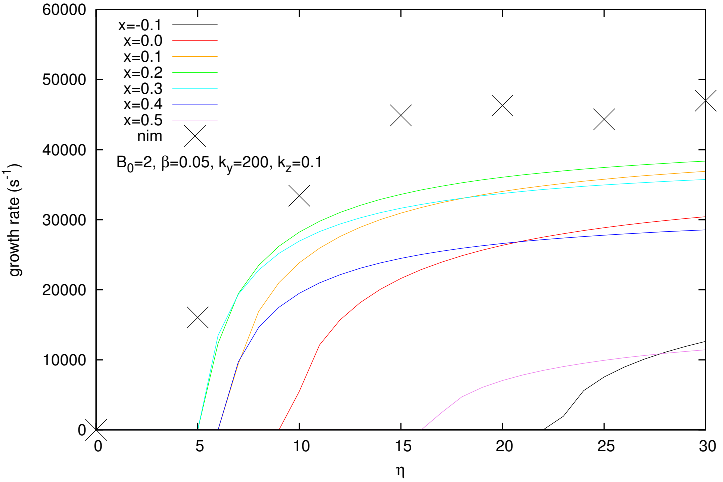
For a fixed η there can be a location in $x < 0$ region that has a maximum growth rate from local dispersion relation



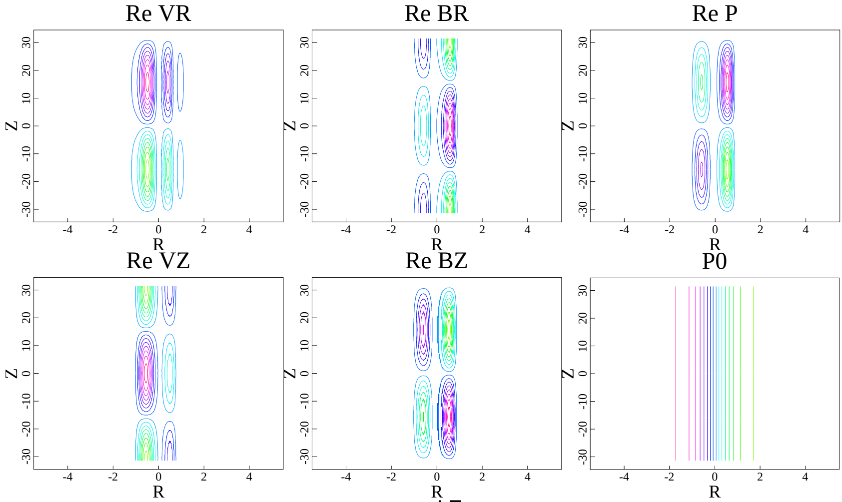
Hypertangential profiles for both density and pressure also considered in nimrod calculation to further examine profile effects

- Density and pressure: $\rho(x) = \rho_0[1 + \tanh(-\frac{x}{L_\rho})]$, $p(x) = p_0[1 + \tanh(-\frac{x}{L_p})]$, $\eta_0 = \eta(0) = \frac{L_\rho}{L_p} - 1$
- Magnetic field $\mathbf{B} = B(x)\hat{\mathbf{Z}}$ is in $R - Z$ plane.
- k_y of perturbation is in periodic direction.
- η no longer constant, peaked at $x = 0$ (i.e. $\eta_{\max} = \eta_0$).
- Other key parameters such as β continue to have large range of variation over x.

Growth from local dispersion is less than nimrod calculation when η profile has a strong variation in x as in hypertangential equilibrium case

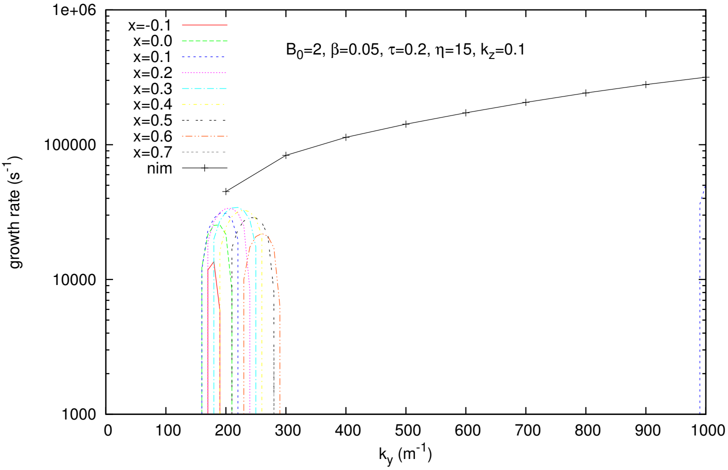


Mode structure of low $k_y = 200$ ITG-driven instability in hypertangential equilibrium from NIMROD calculation

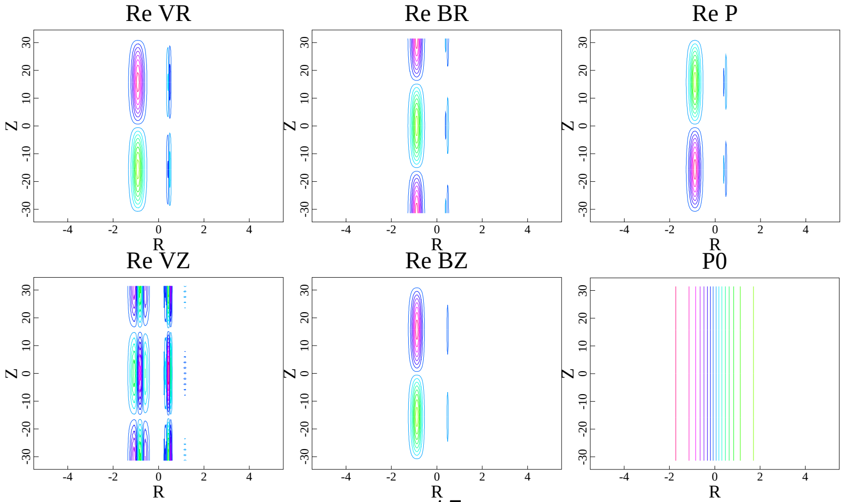


$\eta_0 = 15$

Global ITG mode growth in hypertangential equilibrium from calculation seems out of range from local dispersion relation at higher k_y

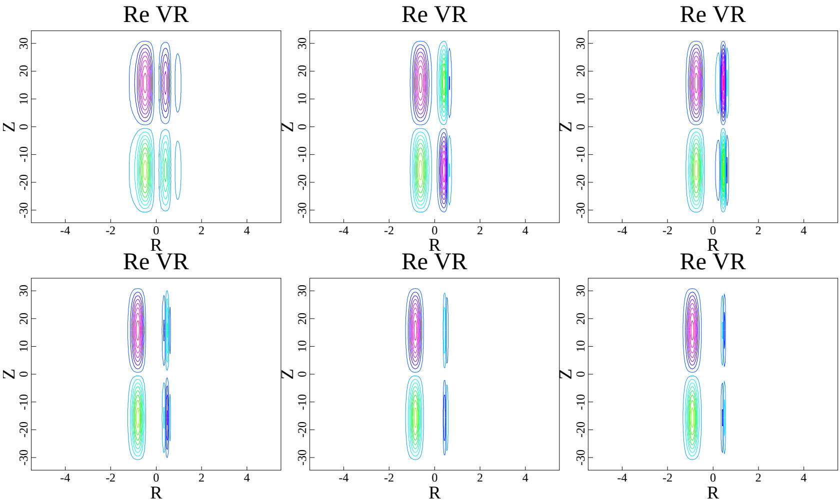


Mode structure of high $k_y = 1000$ ITG-driven instability in hypertangential equilibrium from NIMROD calculation



$\eta_0 = 15$

In hypertangential case, as k_y increases global mode amplitude in $x > 0$ region decreases; mode structure in $x < 0$ region persists for all k_y



Upper row: $k_y = 200, 400, 600$; Lower row: $k_y = 800, 900, 1000$

Summary and Discussion

- Recent theory and NIMROD calculations have identified ITG-driven instabilities in an extended MHD model.
- Difference from conventional slab ITG mode
  - Electromagnetic instead of electrostatic
  - Does not rely on diamagnetic heat flux
- Extended MHD theory based on single-fluid formulation is able to explain observations from NIMROM calculations
  - Two branches of ITG-driven growth in k_y space
  - Asymmetric spatial structure of ITG-mode in exponentially shaped equilibrium
- Spatial profile effects strongly affect global ITG mode growth and structure.
  - Particularly sensitive to η and β profiles.
  - Limitation of local approximation.
- Future work
  - Comparison with ITG results from two-fluid formulation [Schnack *et al.* 2011].