Parallel electron heat flow closure in inhomogeneous magnetic field

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Outline

1. Motivation
   - A general analytical and computational framework for calculating the electron heat flux along magnetic field lines in plasmas of arbitrary collisionality
   - Comparison between the previous theory and the new theory
   - Solving the equation
2. Results
   - Main Results
   - Conclusions

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Close temperature evolution equation

with kinetically derived q

- Species evolution equations and closure moments for five moment model

$\partial_t n + \nabla \cdot n\mathbf{V} = \mathbf{0}$ –> Density

$mn(\partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V}) = en(\mathbf{E} + \frac{1}{c}\mathbf{V} \times \mathbf{B}) - \nabla(nT) - \nabla \cdot \Pi + \mathbf{R}$ –> Flow

$\frac{3}{2}n(\partial_t T + \mathbf{V} \cdot \nabla \mathbf{T}) = -nT\nabla \cdot \mathbf{V} - \Pi : \nabla \mathbf{V} - \nabla \cdot \mathbf{q} + Q$ –> Temperature

$\mathbf{q} \equiv \int d^3v' \frac{1}{2}mv'^2\mathbf{v}'f$ –> Heat flow $\Pi \equiv \int d^3v' m[\mathbf{v}'\mathbf{v}' - \frac{v'^2}{3}\mathbf{I}]f$ –> Stress tensor

- Gyrophased-averaged drift kinetic equation

$$\frac{\partial f}{\partial t} + v_{||}\hat{b} \cdot \nabla f + a_{||} \cdot \nabla_v f = C(f)$$

- Distribution function is written as a superposition of dynamic Maxwellian f_M and a kinetic distortion F

$$f = f_M + F = \frac{n(x,t)}{(\frac{2\pi T}{m})^{3/2}} exp(-\frac{m}{2T}(v - u)^2) + F$$

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Novel treatment of |B| term in the kinetic equation

- The equation we are trying to solve

$$\partial_t \mathbf{F} + \mathbf{v}_{\|}\hat{\mathbf{b}} \cdot \nabla \mathbf{F} - \langle C(F+f_M) \rangle = L_1^{\frac{3}{2}} v(\nabla_{\|} ln T) f_M - f_M L_1^{\frac{1}{2}} (\partial_t ln T)$$

- $[\mathbf{M_t}\partial_t + \mathbf{M}_{\|} v \partial_L + \mathbf{M_B} v \partial_L ln B + C]\mathbf{F} =$ $\delta_{n1} L_1^{\frac{3}{2}} v(\nabla_{\|} ln T) f_M - \delta_{n0} L_1^{\frac{1}{2}} (\partial_t ln T) f_m$
- Expanding the kinetic distortion as $\mathbf{F} = \sum_n F_n(v, \mathbf{x}, \mathbf{t}) P_n \left(\frac{\mathbf{v}_{\|}(\mathbf{x})}{v} \right)$ here $\frac{v_{\|}(x)}{v} = \pm\sqrt{1 - \frac{\mu B(x)}{w}}$
- To lowest order, the magnetic moment μ and kinetic energy w of the particles are conserved

$$\mu = \frac{mv_{\perp}^2}{2B} \text{ and } w = \frac{mv^2}{2} = \frac{mv_{\|}^2}{2} + \mu B$$

- The collision operator $C_a^{(1)} = -\frac{\nu_{L_a}}{2} \sum_n n(n+1) P_n F_n$

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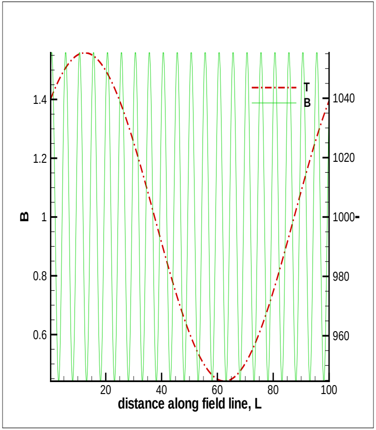
Outline

Comparision between the previous theory and the new theory

Previous Theory	New Theory
Time dependent effects are ignored	Time dependent effects are included
Expansion of F in $C_n(\xi)$	Expansion of F in $P_n(\frac{v_\parallel}{v})$
Multiple scale length expansion	No multiple scale length expansion
Parallel free streaming operator is treated with bounce average	Parallel free streaming operator treated as is
Boundary layer ignored near trapped/passing boundary	Trapped/passing boundary is treated more generally


T
B
B
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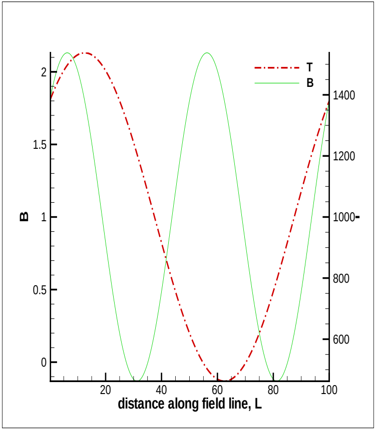
1.2
1
0.8
0.6
1040
1020
1000
980
960
20
40
60
80
100
distance along field line, L


T
B
B
2
1.5
1
0.5
0
1400
1200
1000
800
600
20
40
60
80
100
distance along field line, L

Outline

Solve the system of equations

- The equations we are trying to solve
- The equation at each s

$$[\mathbf{M_t}\partial_t + \delta_{nn}]\mathbf{F} =$$
$$-\mathbf{M}_{||} v\partial_L \mathbf{F} - \mathbf{M_B} v\partial_L lnB \ \mathbf{F} - \delta_{n1} L_1^{\frac{3}{2}} v(\nabla_{||} ln T) f_M - \delta_{n0} f_M L_1^{\frac{1}{2}}(\partial_t ln T)$$
$$\partial_t T = \frac{2}{3n}(S - \hat{b}\cdot\nabla\mathbf{q} + \mathbf{q}\hat{b}\cdot\nabla lnB)$$

Complex Fourier Series

$\mathbf{F}_n = F_{n0} + \sum_{k>0}(F_{nk} e^{ik\phi} + F_{nk} e^{-ik\phi})$ where $\phi = 2\pi L/\triangle Ls$
$\partial_L lnB = \sum_{m>0}(B_m e^{im\phi} + B_m^* e^{-im\phi})$and
$T = T_0 + \sum_{k>0}(T_k e^{ik\phi} + T_k e^{-ik\phi})$

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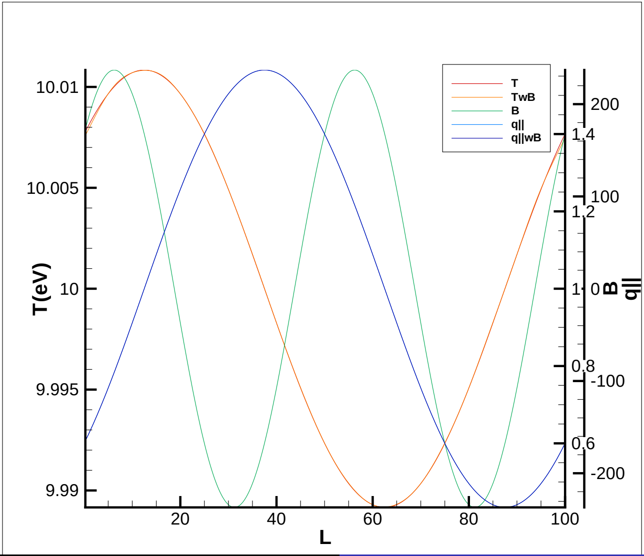
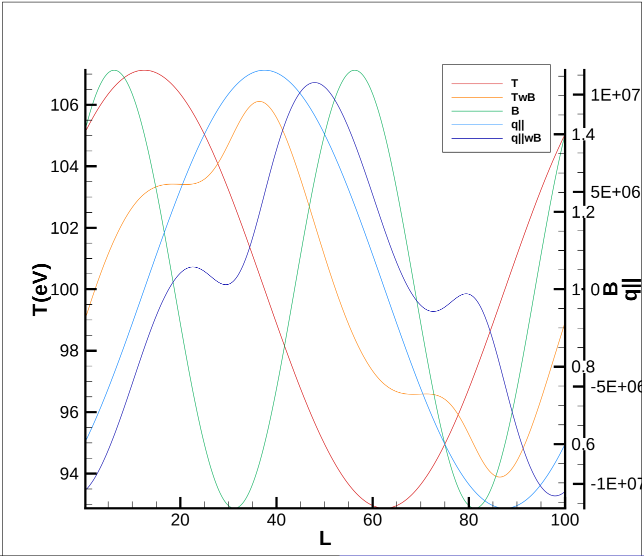
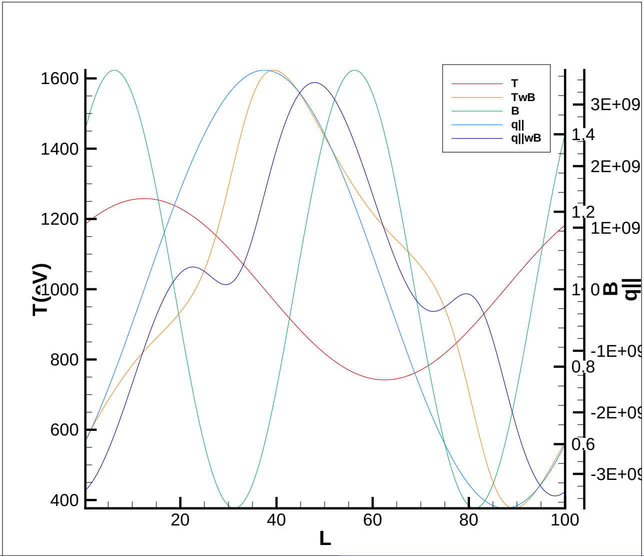
Collisional regime

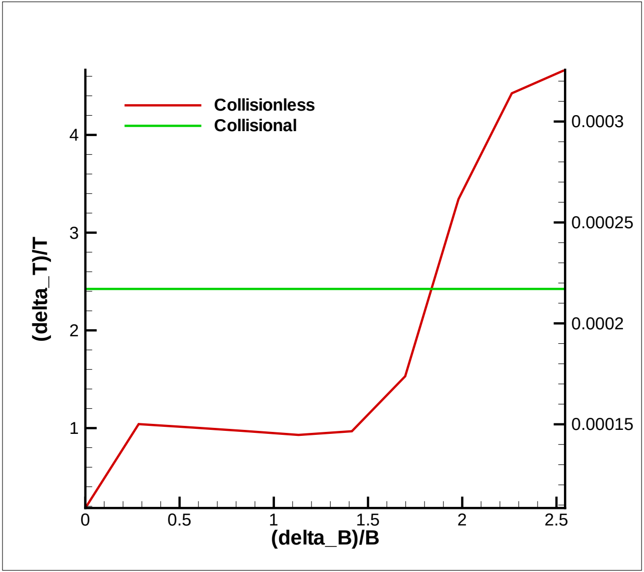


Moderate collsionality



Collsionless regime





Change in T vs change in $|B|$

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Summary

- In nearly collisionless plasmas the heat flow is reduced by the trapped electron fraction
- In moderately collisional plasmas, their is reduction in the heat flow due to the deposition of energy near the trapped/passing boundary
- Change in temperature increases with increasing change in $|B|$ effects in collisionless regime
- Quantative means for calculating parallel electron heat flow along inhomogeneous magnetic fields
- Future work
  - Linearized full collision operator
  - Comparision between linear and non-linear effects

For Further Reading I

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