Results and Improvements for Continuum Kinetic Calculations in NIMROD

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Electron, ion and hot particle DKEs in NIMROD

Hazeltine's first-order drift kinetic equation (DKE):

$$\partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f + \left(\mu \frac{\partial B}{\partial t} + e(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} \right) \partial_\epsilon f = C.$$

Transforming to pitch-angle, $\xi = v_{||}/v$, and normalized speed, $s = v/v_0$:

$$\partial_t f + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f +$$

$$\frac{1-\xi^2}{2\xi} \left[-(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln B + \xi^2 \frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \frac{e}{s^2 T_0} (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + \xi^2 \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B \right] \frac{\partial f}{\partial \xi} +$$

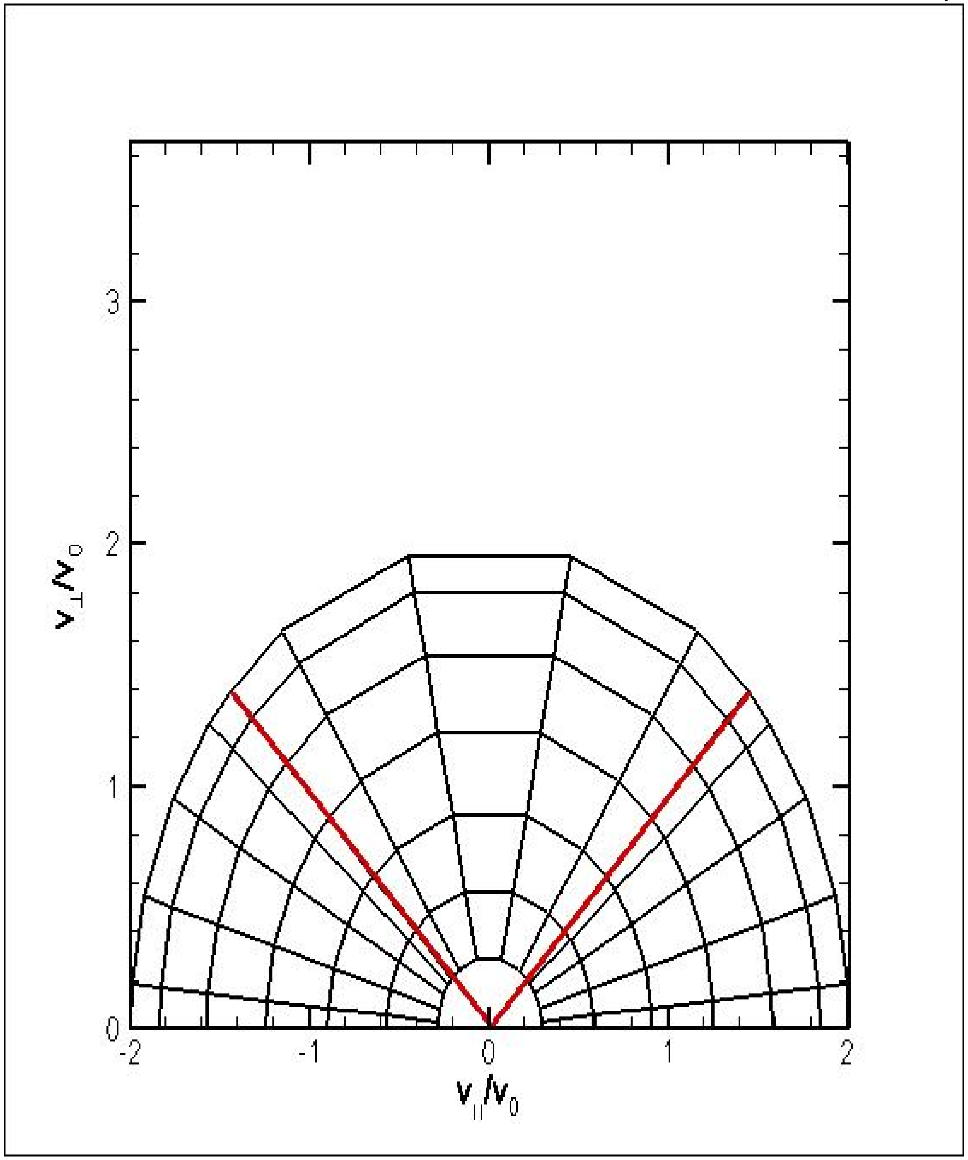
$$\frac{s}{2} \left[-(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln T_0 - (1 - \xi^2) \frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \frac{e}{s^2 T_0} (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + (1 + \xi^2) \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B \right] \frac{\partial f}{\partial s} = C(f),$$

where

$$v_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\epsilon_0 s^2}{eB} \left[\mathbf{b} \times \left((1 - \xi^2) \nabla \ln B + 2\xi^2 \kappa - \frac{v_0 s \xi}{\epsilon_0 B} \nabla \times \mathbf{E} \right) + (1 - \xi^2) \frac{\mu_0 \mathbf{J}_{||}}{B} \right].$$

2D velocity grid at all points in physical space.

Expand distribution functions: $F(s, \xi, \mathbf{x}, t) = \sum_{l=1}^{L} F_l(s, \mathbf{x}, t)\phi_l(\xi)$.



NEO benchmark milestone.

Order $v_D << v_{||}$ and assume weak (relative to Dreicer) electric fields:

$$\partial_t f_0 + \mathbf{v}_{||} \cdot \nabla f_0 - \mathbf{v}_{||} \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s \right] f_0 = C(f_0).$$

This equation is satisfied by a stationary Maxwellian with flux functions n and T.
To next order:

$$\partial_t f_1 + \mathbf{v}_{||} \cdot \nabla f_1 - \left(\mathbf{v}_{||} \cdot \nabla \ln B \right) \frac{1-\xi^2}{2\xi} \partial_\xi f_1 =$$

$$-\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 - \frac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \left(\mathbf{E}^A - \nabla \phi_1 \right) \partial_s f_0 + C^{aa} + C^{ab}.$$

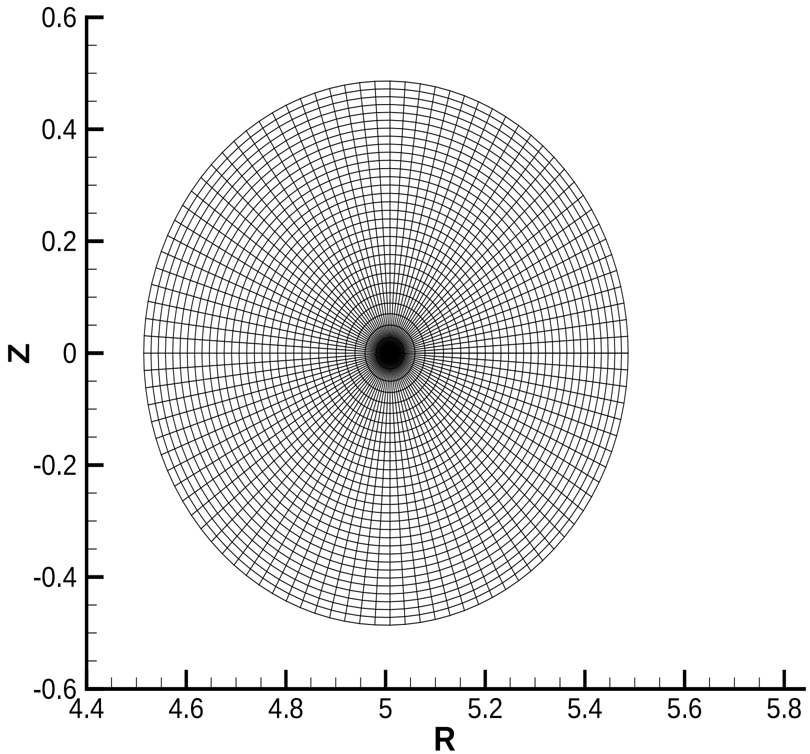
Using $g = f_1 - (e\phi_1 / T_0) f_0$ yields (compare with Eq. 23 of Belli and Candy, 51 PPCF 2009):

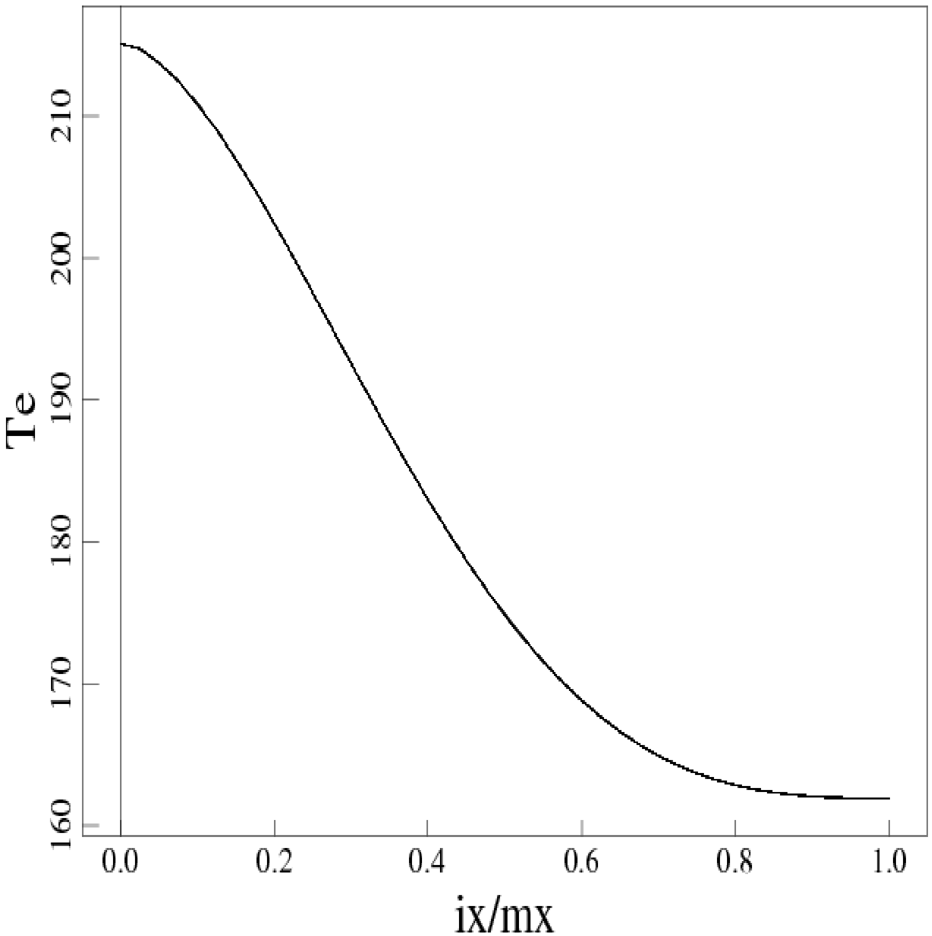
$$\partial_t g + \mathbf{v}_{||} \cdot \nabla g - \mathbf{v}_{||} \cdot \nabla \ln B \partial_\xi g =$$

$$-\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 + C^{aa} + C^{ab} - \frac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \mathbf{E}^A \partial_s f_0 + (e f_0 / T_0) \partial_t \phi_1.$$

Initial NEO benchmark

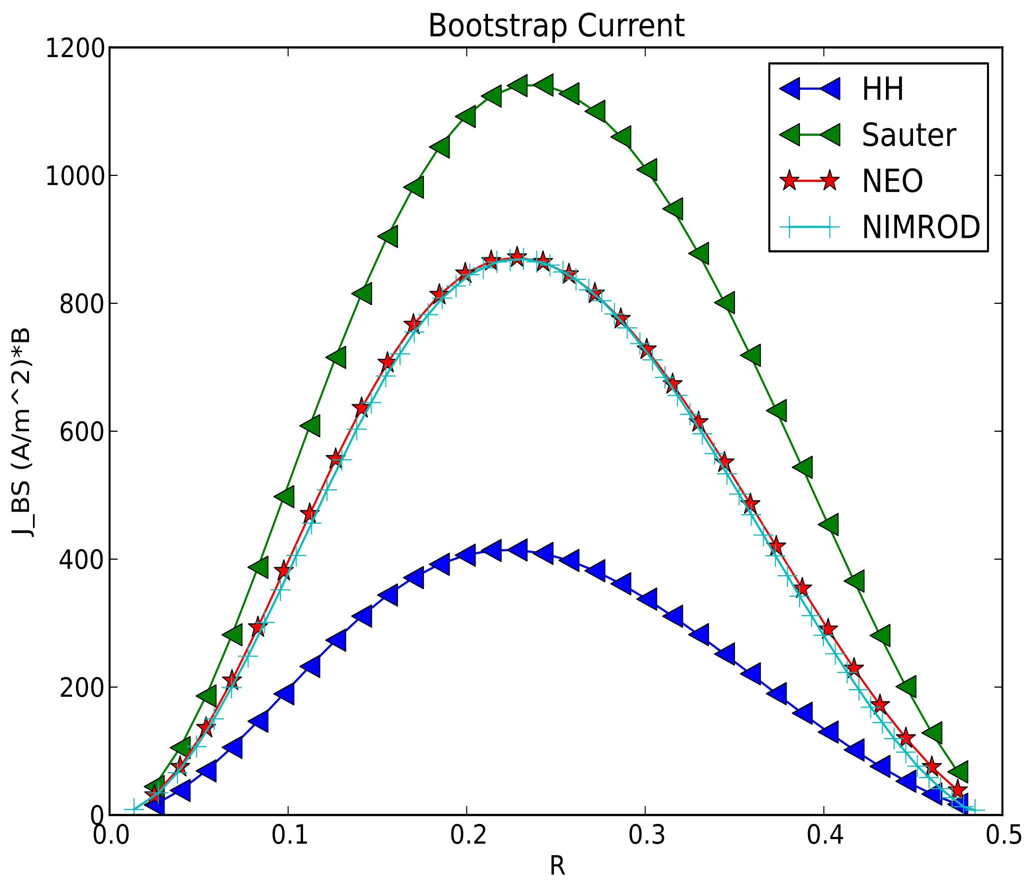
High-aspect ratio Grad-Shafranov equilibrium.





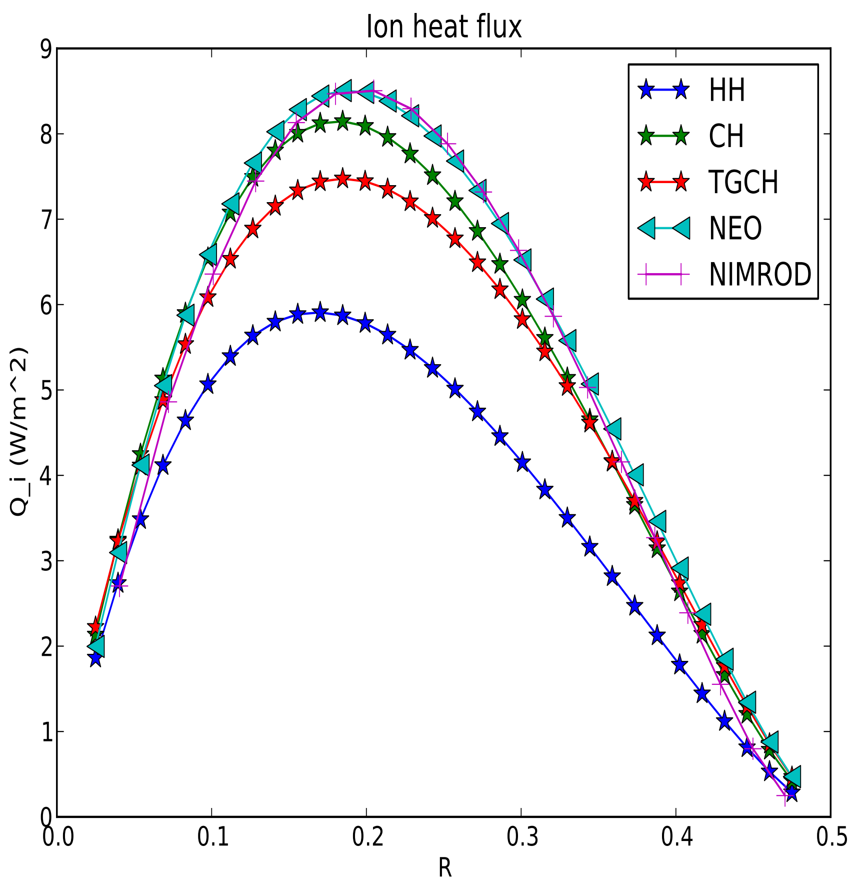
Bootstrap currents agree.

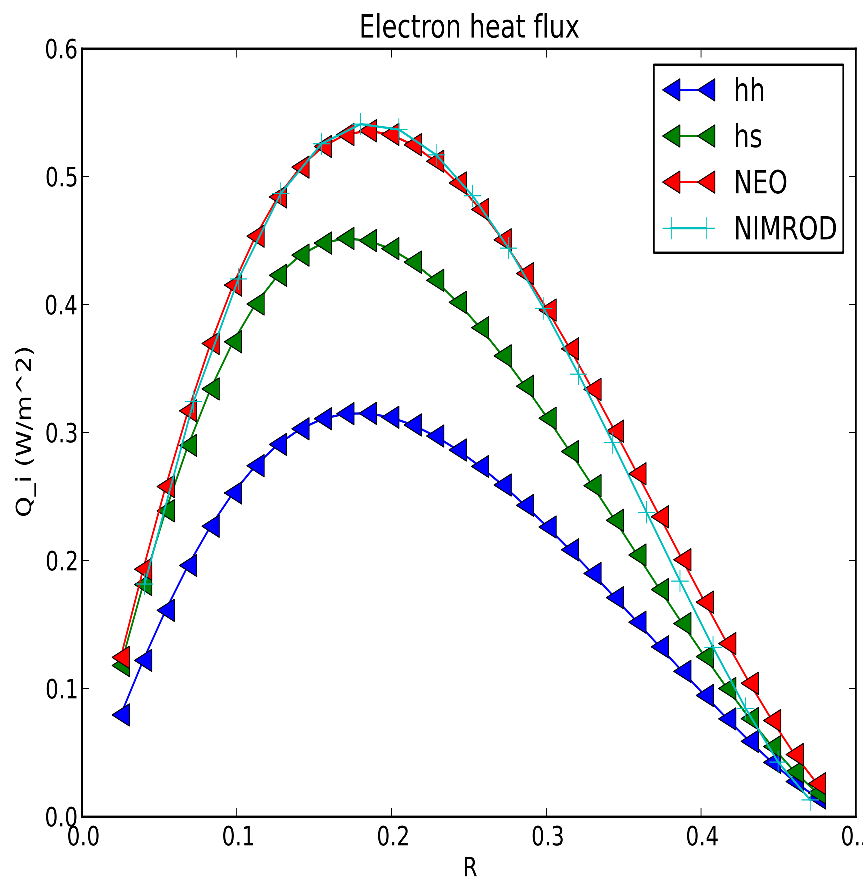
NIMROD and NEO use full linearized Coulomb collision operator.



Ion and electron radial heat fluxes agree.

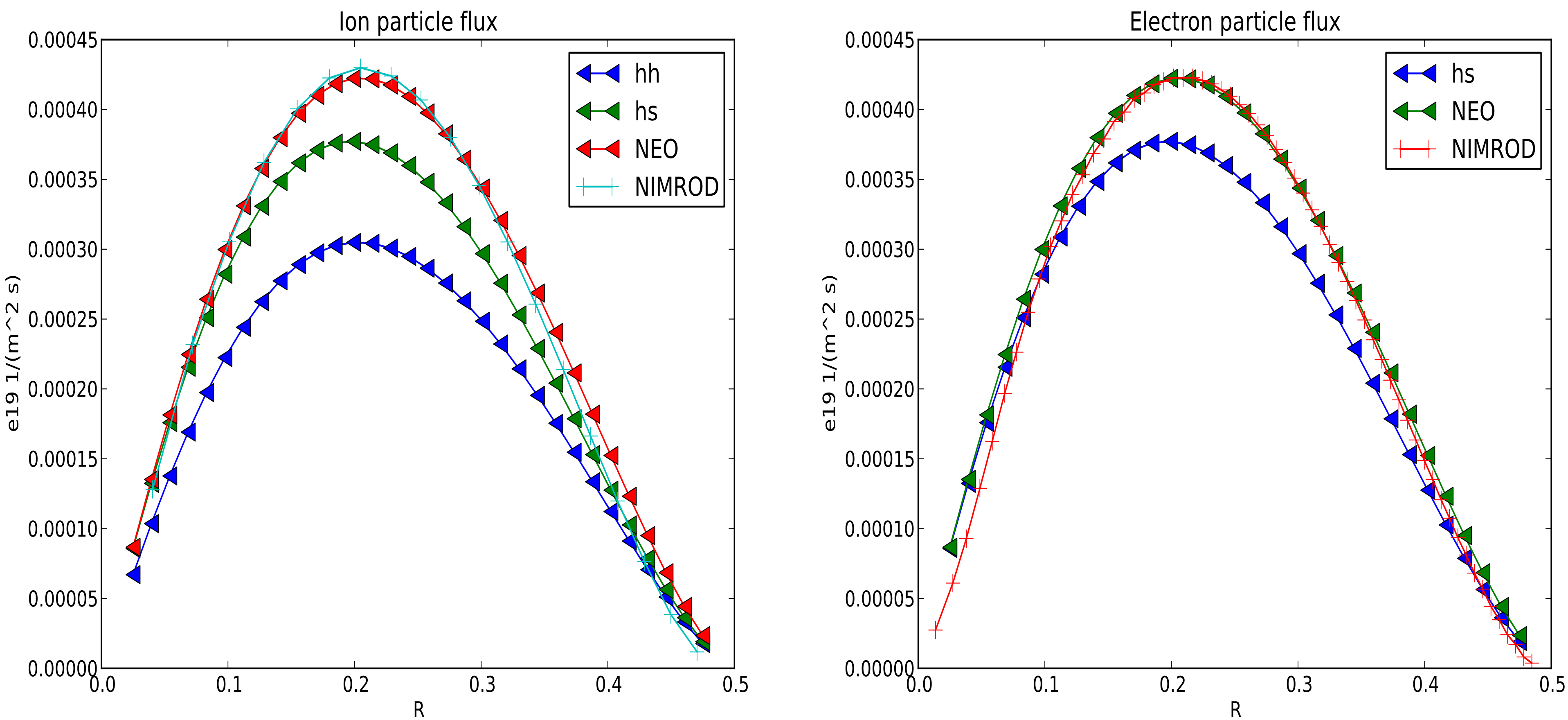
NIMROD and NEO use full linearized Coulomb collision operator.





Ion and electron radial particle fluxes agree.

NIMROD and NEO use full linearized Coulomb collision operator.



Finish NEO benchmark milestone

1. Show quantitative agreement on high-temperature, shaped tokamak equilibria.
2. Parallelize nonlinear, nonsymmetric solves over 2D velocity space to make neoclassical transport calculations efficient and routine.
3. 5D parallel solves important for NTM simulations.

Continuum hot particle milestones

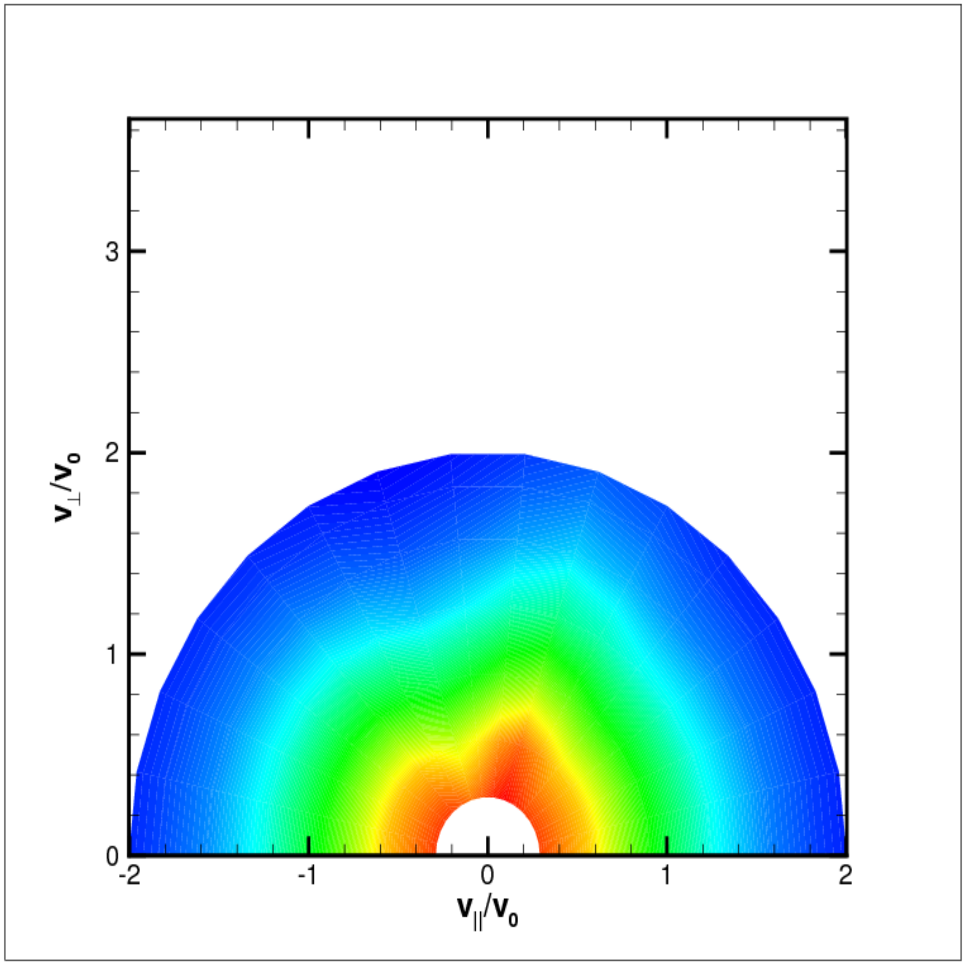
Try hot particle kink benchmark first.
In Fu's formulation, slowing down distribution written as

$$f_0(\psi, \epsilon) = AP_0(\langle\psi\rangle)/(1+s^3)$$

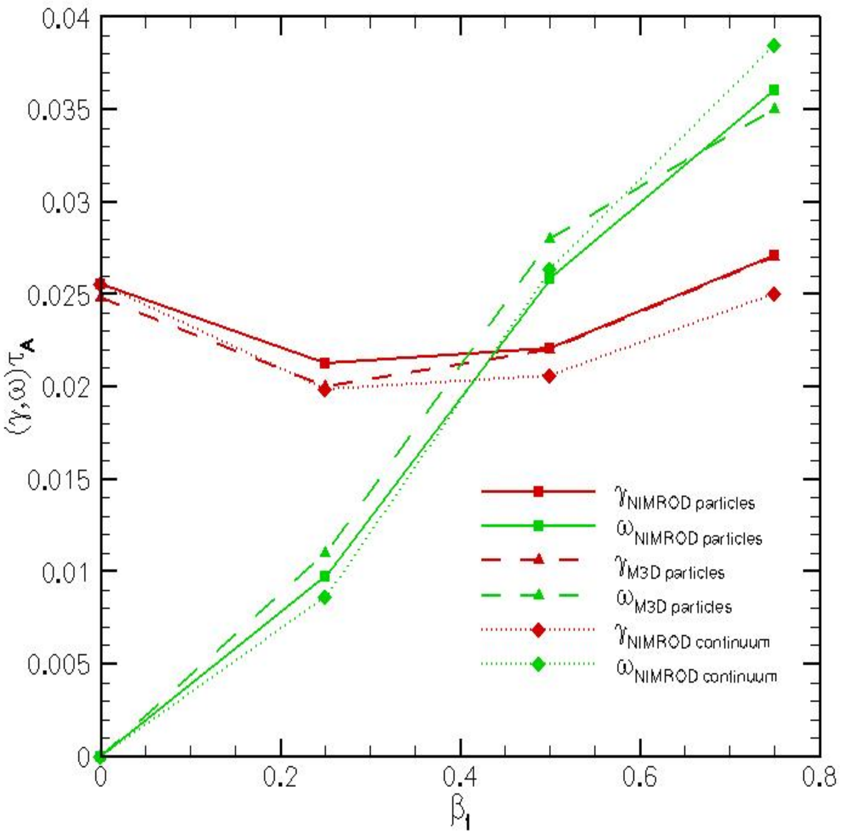
where for trapped and passing particles,

$$\langle\psi\rangle = P_\zeta/e - \frac{m}{e}\langle v_{||} R \frac{B_\phi}{B}\rangle \approx P_\zeta/e,$$

$$\langle\psi\rangle = P_\zeta/e - \frac{m}{e}\langle v_{||} R \frac{B_\phi}{B}\rangle \approx P_\zeta/e - sr_0 R_0 B sign(\frac{v_{||}}{v})\sqrt{1-\mu B_0/\epsilon}.$$



Kink benchmark growth and rotation rates compare favorably.



TAE Benchmark milestone

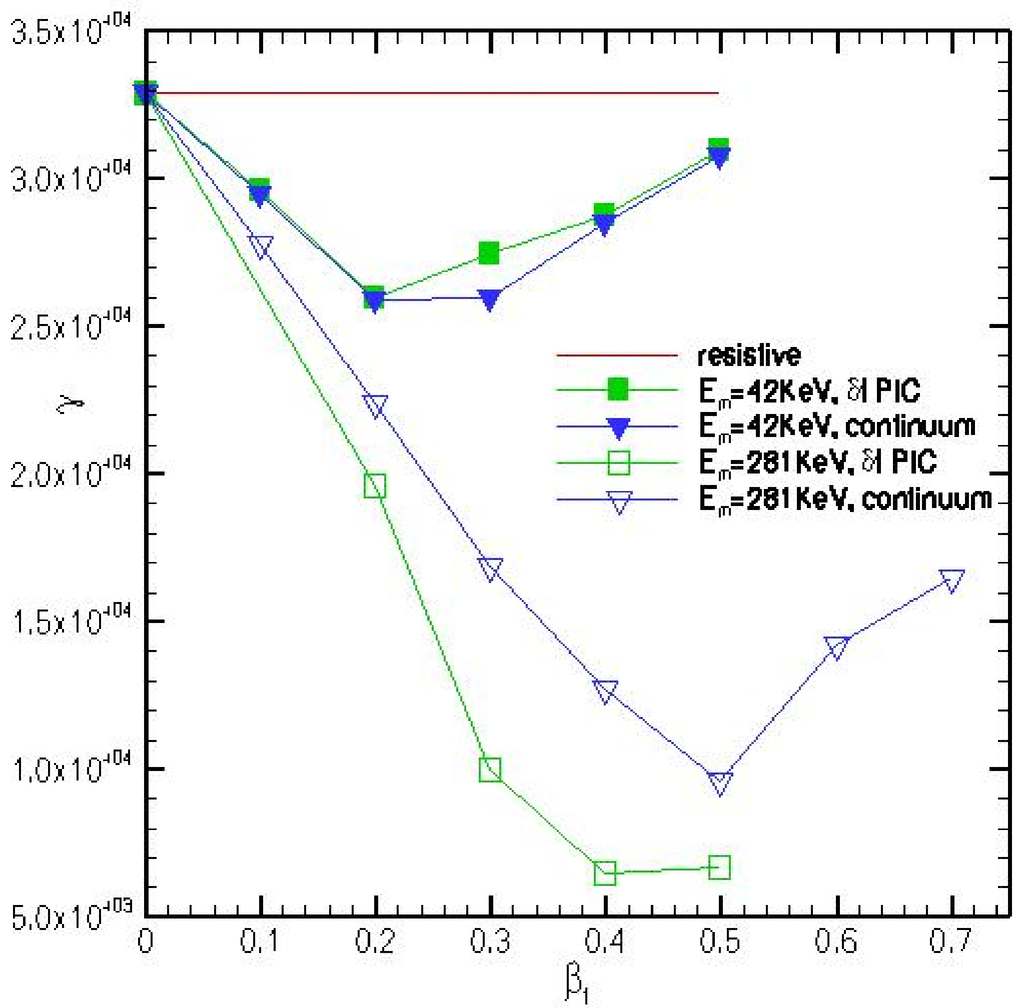
Mischenko *et al.* (*Phys Plasmas*, **16** 082105 (2009)):
energetic Maxwellian of deuterium ions destabilizes a TAE mode.

Spong *et al.* (*Phys. Plasmas* **19**, 082511 (2012)):
compares TAEFL, GTC and GYRO codes with the DIII-D experiment.
Linear stability of a sequence of reverse shear equilibria with decreasing q_{min} is evaluated for reversed shear Alfven eigenmodes (RSAEs).

Continuum calculations for both problems underway.

Giant Sawteeth (GS) milestone

Recent improvements to continuum algorithm have sped up GS scans.
Results below include slowing-down distribution only.
Conclusion: high-energy tail and/or thermal ions needed for stabilization.



GS with high-energy tail

Match energy dependence of tail to Orbit-RF calculation of RF-driven beam ions.

$$F_{hot} = P_0(<\psi>)(\frac{A_{sdd}}{1+s^3} + \frac{A_{tail}}{(1+c_t s)^{p_t}}).$$

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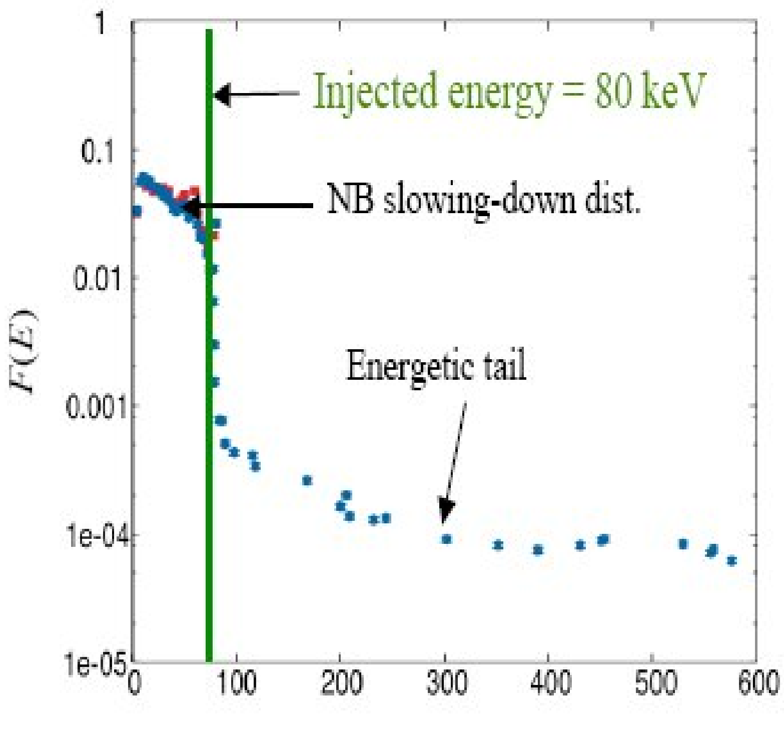
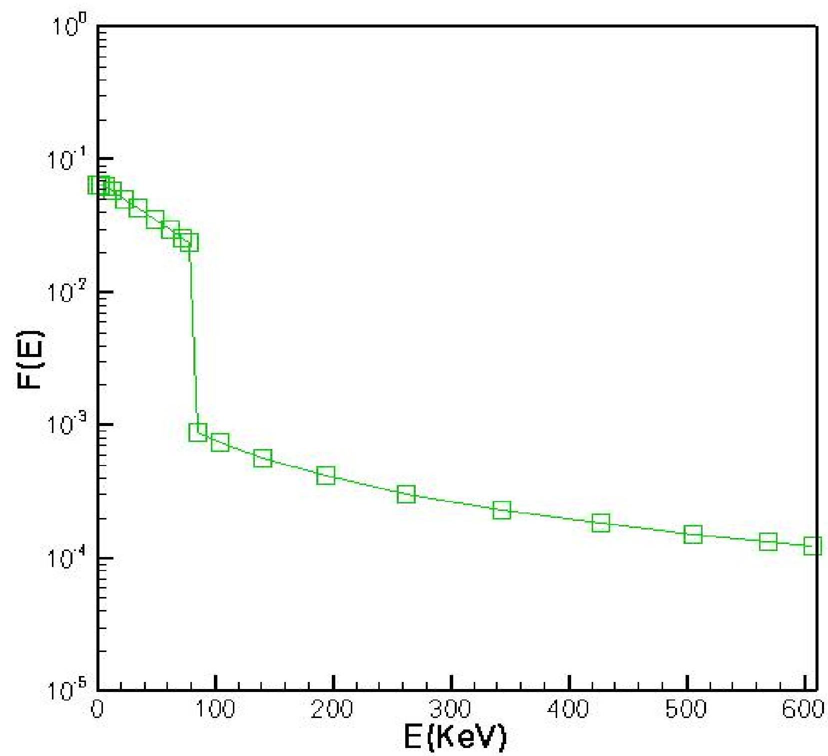
For NIMROD plot at right, critical energy $E_c = 55KeV$, $c_t = 1$ and $p_t = 3$ $((\beta_{sdd} + \beta_{tail})/\beta_{MHD} = 0.55$ and $\beta_{tail}/\beta_{sdd} = 0.6)$.





Improvement 1: Parallelization over s

With $\mathbf{E}_{eq} = 0$ and $C(f) = 0$ solution of linearized DKEs at different s decouple:

$$\partial_t f + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f +$$

$$\frac{1-\xi^2}{2\xi}\left[-(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln B + \xi^2 \frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \frac{e}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + \xi^2 \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B\right] \frac{\partial f}{\partial \xi} +$$

$$\frac{s}{2}\left[-(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \ln T_0 - (1-\xi^2)\frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \frac{e}{s^2 T_0}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E} + (1+\xi^2)\frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B\right] \frac{\partial f}{\partial s} = C(f),$$

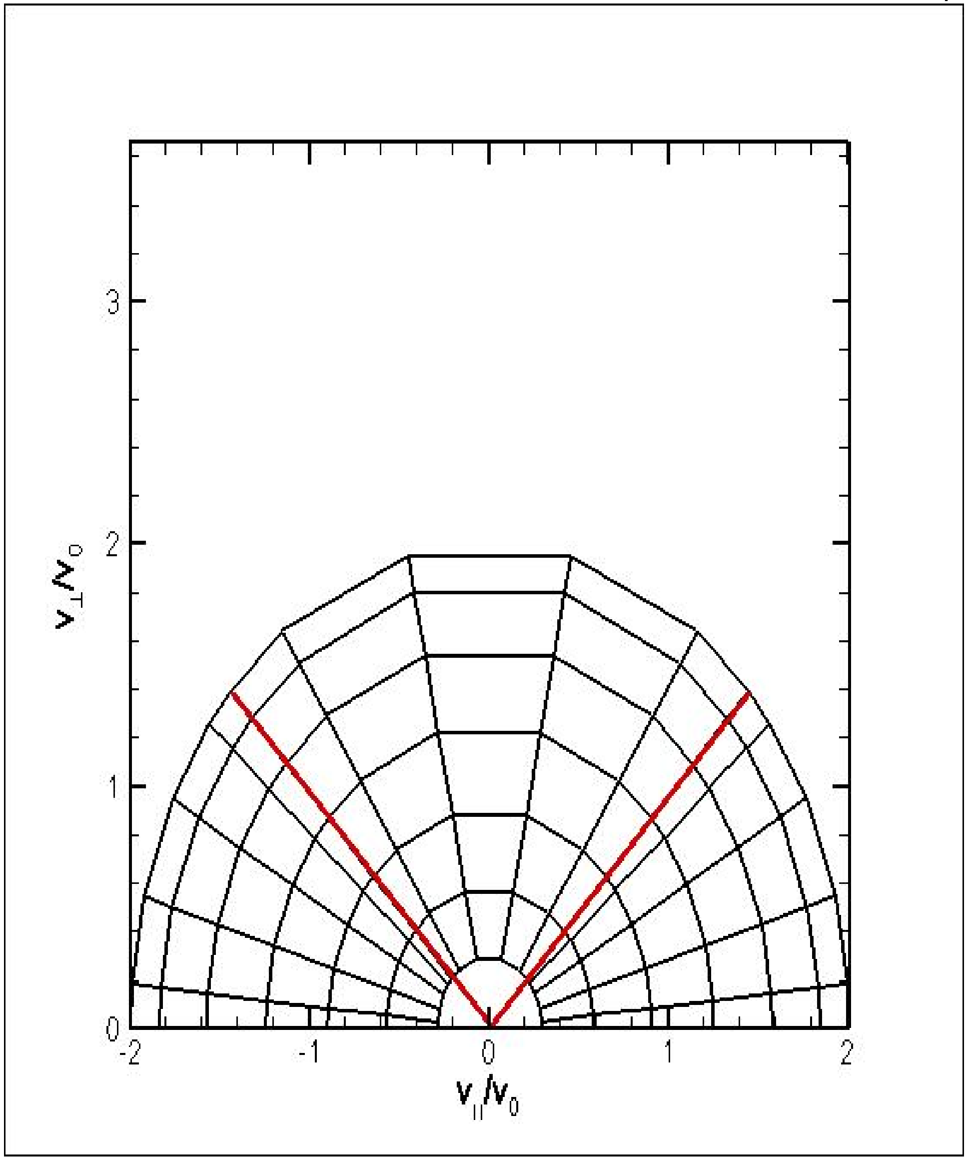
Parallelization over s similar to *nlayers* for Fourier expansion.

Processors communicate to compute closures (integration over s).

Improves parallel scaling and significantly reduces memory.

2D velocity grid at all points in physical space.

Expand distribution functions: $F(s, \xi, \mathbf{x}, t) = \sum_{l=1}^{L} F_l(s, \mathbf{x}, t)\phi(\xi)$.



Improvement 2: Hyper-diffusion

Spatial hyper-diffusion added to the continuum DKEs.
Separate diffusivity shape function added.

$$\frac{\partial \sum_{l=1}^{L} F_l(s,\mathbf{x},t)\phi_l(\xi)}{\partial t} + \nabla \cdot D_h \nabla \sum_{l=1}^{L} g_l(s,\mathbf{x},t)\phi_l(\xi) = 0.$$

$$g_l - \nabla \cdot \nabla F_l = 0, \quad l = 1, ..., L.$$

Particles advect, $(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla f$, into diffusive region and “disappear".
Hit: doubles the number of unknowns.

Improvements for nonlinear solves.

Preconditioned GMRES iterations in NEO benchmark calculations are quite slow.
Parallelize preconditioning so separate groups apply their s-matrix independently.
Parallelize dot routine so *s layers* work with their *s* only and then communicate to form iterate.
Static condensation for smaller GMRES system.