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The semi-characteristic method for the Vlasov equation

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NIMROD meeting

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- Guangye Chen
- ORNL, LDRD
- JCP **230**, 7018 (‘11); **231** , 5374 (‘12); **233**, 1(’12)

Outline

- (1988 – 2012) Adventures in evolving background δf
  - – Wandering the dessert
  - – The “w” formulation
  - – Left as an exercise for the student
- Another noise reduction philosophy
- The semi-characteristic method
  - – Formulation
  - – Initial results

Wandering the dessert

- 198? GK δf method used for ES turbulence
- Could be generalized to EM, …?
- Moment equations + closure (bad design decision)
- “Evolving background” δf method

Moments+Closure

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \bullet n_\alpha \mathbf{u}_\alpha = 0$$

$$m_\alpha n_\alpha \left(\frac{\partial \mathbf{u}_\alpha}{\partial t} + \mathbf{u}_\alpha \bullet \frac{\partial \mathbf{u}_\alpha}{\partial \mathbf{x}} \right) = q_\alpha n_\alpha \left(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B} \right) \quad ,$$

$$- k_B T_\alpha \frac{\partial n_\alpha}{\partial \mathbf{x}} - \frac{\partial}{\partial \mathbf{x}} \bullet \overleftrightarrow{\pi}_\alpha$$

$$f_\alpha = \overline{f}_\alpha + \tilde{f}_\alpha$$

$$\bar{f}_\alpha \equiv n_\alpha \left(\frac{1}{\sqrt{2\pi} v_{T\alpha}} \right)^3 e^{-w_\alpha^2 / 2 v_{T\alpha}^2}$$

$$\overleftrightarrow{\pi}_\alpha \equiv m_\alpha \int d\mathbf{w}_\alpha \mathbf{w}_\alpha \mathbf{w}_\alpha \widetilde{f}_\alpha h_\alpha$$

First attempts < 1996

- Orbit equations
- Weight based on full particle velocity **v**
- Various constraint schemes
- Pretty much an unmitigated bust (sorry, Carl!)
- Wander, find water, fluid calculations, …

The “w” formultaion

- **2006**
  - 28 October, Philadelphia, PA
  - 14-15 August, Seattle, WA
  - 27 January Boulder, CO
- Introduce particular velocity as fundamental particle quantity
  - Eliminated circular equations
  - Symmetry
  - Energy conservation

From NIMROD – Boulder '08

Apply Cloud in Cell rule

$$\frac{\dot{\tilde{h}}_i}{1-\tilde{h}_i} = \int d\mathbf{z}\, s_w\left(\mathbf{w}\ ,\mathbf{w}_i\right) s\ \left(\mathbf{x}\ -\mathbf{x}_i\right)\left[\nabla \bullet \mathbf{u} - \frac{1}{\overline{v}_T^2}\left(\mathbf{ww} : \frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \mathbf{w} \bullet \tilde{\mathbf{A}}\right)\right]$$

$$\frac{\dot{\tilde{h}}_i}{1-\tilde{h}_i} = \sum_g S\left(\mathbf{x}_g - \mathbf{x}_i\right)\left(\frac{\partial}{\partial \mathbf{x}_g} \bullet \mathbf{u}\ - \frac{\mathbf{w}_i \mathbf{w}_i}{\overline{v}_T^2} : \frac{\partial \mathbf{u}}{\partial \mathbf{x}_g}\right)$$

$$\tilde{\mathbf{A}}_g = -\sum_i W_i \tilde{g}_i \left(\mathbf{w}_i \mathbf{w}_i - \overline{v}_T^2 1\right) \bullet \frac{\partial}{\partial \mathbf{x}_g} S\left(\mathbf{x}_g - \mathbf{x}_i\right) \Big/ n\Delta_g$$

- Now <u>symmetric</u> and closure <u>eliminated</u> from weight advance

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From NIMROD – Boulder '08
Symmetry Leads to Energy Integral

$$\frac{d}{dt}\left\{-M\bar{v}_T^2\sum_i W_i\left[\log\left(1-\tilde{h}_i\right)+\tilde{h}_i\right]\right\}=-M\sum_g \Delta_g n_g \mathbf{u}_g \bullet \tilde{\mathbf{A}}_g$$

$$\approx \frac{d}{dt}\left(-\frac{M\bar{v}_T^2}{2}\sum_i W_i\left|\tilde{h}_i\right|^2\right)$$

$$\int d\mathbf{x}\left(\frac{Mnu^2}{2}+Mn\varphi+\frac{B^2}{2\mu_0}+\frac{P_e}{\Gamma_e-1}+M\bar{v}_T^2 n\log n\right)$$

$$-M\bar{v}_T^2\sum_i W_i\left[\log\left(1-\tilde{h}_i\right)+\tilde{h}_i\right]=\mathcal{E}=const.$$

Dimits (Sherwood 2007) has given similar result

D. C. Barnes, J. Cheng, and S. E. Parker. Low-noise particle algorithms for extended magnetohydrodynamic closure. *Phys. Plasmas*, 15(5):055702, 2008.

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A large oasis appears

- ORNL contract with Chacón & Chen
- Use JFNK implicit methods

Langmuir wave — Ion-acoustic wave

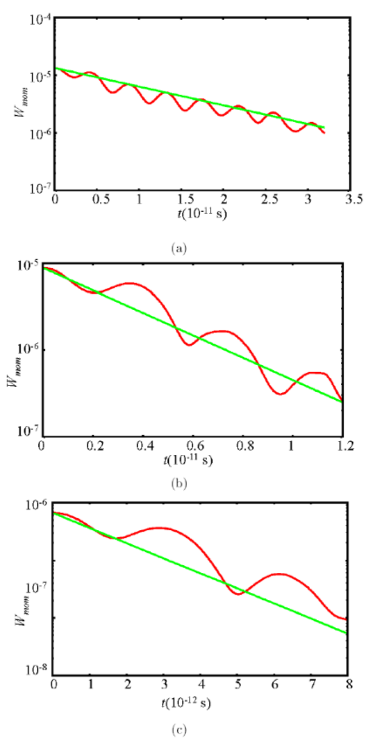

Figure 2: Time history of moment energy (Joules) *vs.* time for $k\lambda_D$: (a) 0.4, (b) 0.6, (c) 0.8.

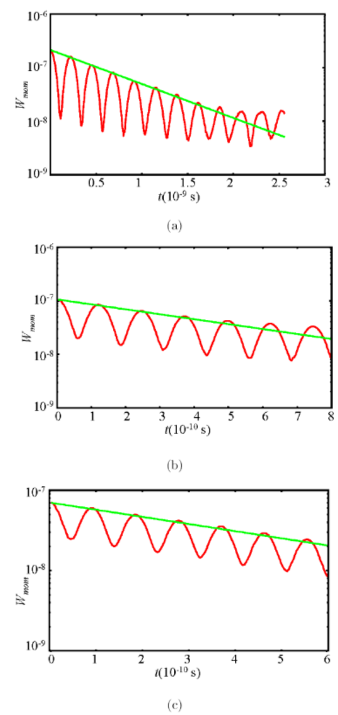

Figure 4: Time history of moment energy for ion-acoustic waves for $k\lambda_D$: (a) 0.25, (b) 0.5, (c) 0.75.

Soon to run dry

2-stream instability

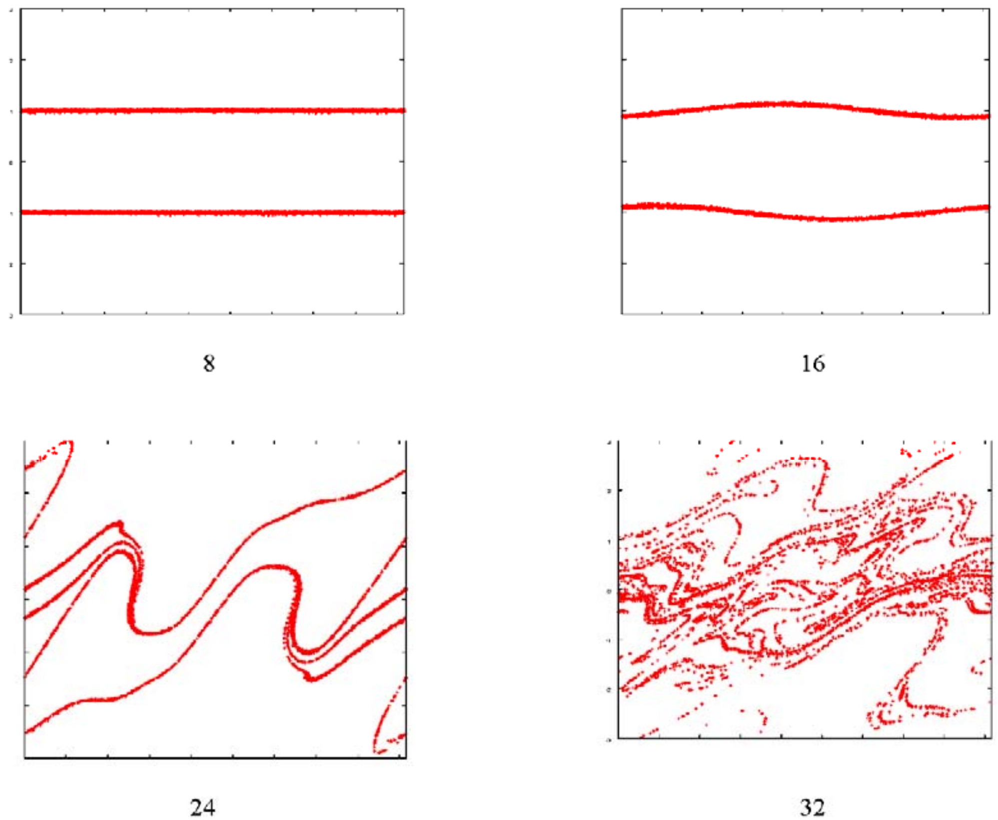

Figure 5: Phase-space evolution of the electron two-stream instability. Numbers are time in units of ω_b^{-1}.

2-stream crashes late in time

- Issue is synchronization
  - Particles give low-order moments $n, \mathbf{u}$
  - Moments de-synchronize
    - Numerical dissipation (viscosity)
- We should synchronize

- A correction electric field is used to maintain synchronization between the low-order moments of the moment and particle represenations.

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Synchronization problem (left as an exercise for the student)

- Have 2 representations of low-order moments
- Fluid is quiet, but not accurate
- Particle is accurate, but noisy
- Simple (linear feedback) remedies not successful
- Kalman filtering problem?

Another approach – perturbed orbits

- Instead of tracking perturbed distribution along orbits, track orbit perturbations

JOURNAL OF COMPUTATIONAL PHYSICS 51, 107–138 (1983)

Direct Implicit Large Time-Step Particle Simulation of Plasmas

A. BRUCE LANGDON, BRUCE I. COHEN, AND ALEX FRIEDMAN

Semi-characteristic method

$$\frac{\partial f}{\partial t} + [H, f] = 0 \qquad \Rightarrow \qquad \frac{\partial f}{\partial t} + [H - F(f,t), f] = 0$$

$$\frac{d\mathbf{P}}{dt} = -\frac{\partial H}{\partial \mathbf{Q}} \qquad\qquad \frac{d\mathbf{P}}{dt} = -\frac{\partial H}{\partial \mathbf{Q}} + \frac{\partial F}{\partial f}\frac{\partial f}{\partial \mathbf{Q}}$$

$$\frac{d\mathbf{Q}}{dt} = \frac{\partial H}{\partial \mathbf{P}} \qquad\qquad \frac{d\mathbf{Q}}{dt} = \frac{\partial H}{\partial \mathbf{P}} - \frac{\partial F}{\partial f}\frac{\partial f}{\partial \mathbf{P}}$$

Equations have same solution, but different characteristics

Semi-characteristic method

- Suppose we have an "underlying" equilibrium

$$\bar{H},\bar{f} \Rightarrow \left[\bar{H},\bar{f}\right]=0$$

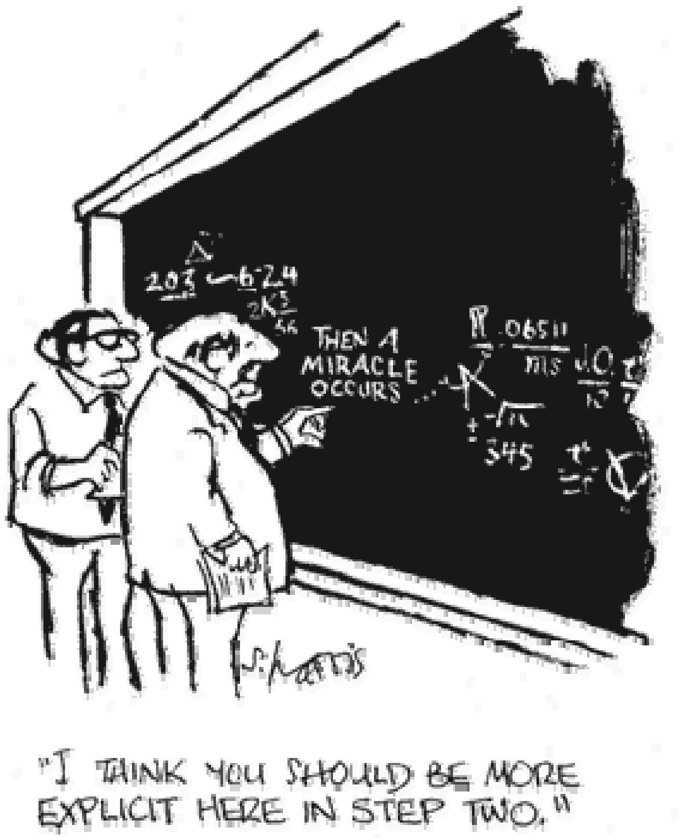
- Then $\bar{H}\left(\bar{f}\right)$ &

$$\bar{H}\left[P_0\left(P,Q,t\right),Q_0\left(P,Q,t\right)\right]=\bar{H}\left[f\left(P,Q,t\right)\right]$$

- So choose

$$F=\bar{H}\left[P_0\left(P,Q,t\right),Q_0\left(P,Q,t\right)\right]$$

Semi-characteristic method

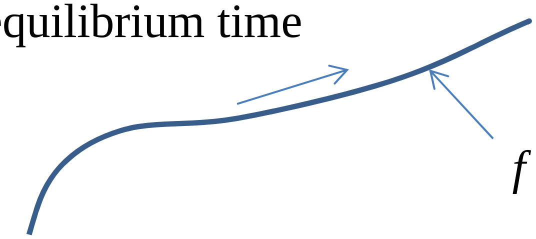


- Then a miracle occurs
- Phase space flow of equilibrium is removed
- Since shot noise constant in time, it can be subtracted

Semi-characteristic method

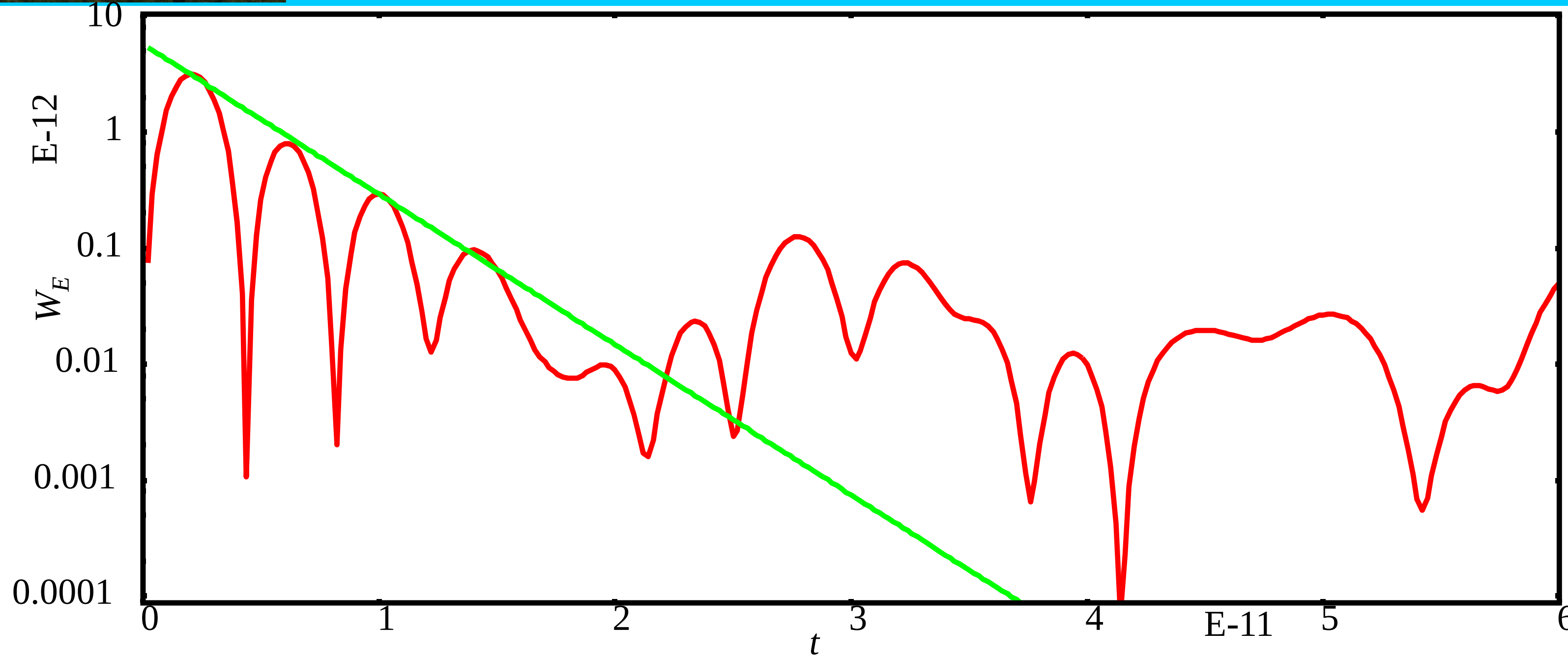
$$\delta\dot{P} = -\partial_Q H\left(P,Q,\mathrm{t},\delta\Phi\right) - \partial_\tau \delta P$$

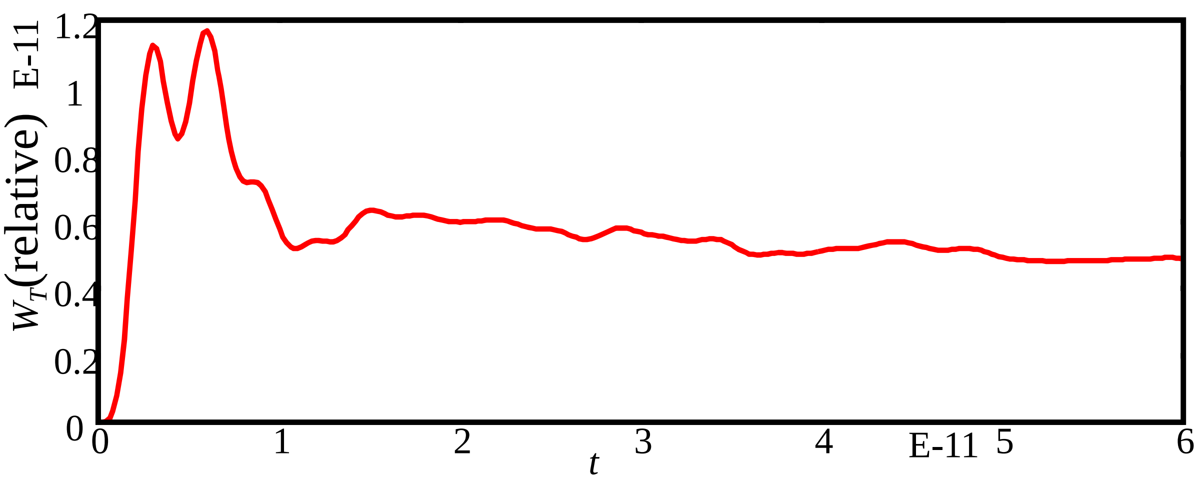
$$\delta\dot{Q} = \partial_P H\left(P,Q,\mathrm{t},\delta\Phi\right) - \partial_\tau \delta Q$$



- Particle load is stepped in eq fields (τ) to generate "*f*-string"
- Markers are moved by standard method
- Then advective mapping applied
  - Allows dissipation (required to solve hyperbolic problem)
  - Can be made energy-momentum conservative
- Perturbed fields found from sources $\left(\delta P, \delta Q\right)$
- All standard PIC tricks can be applied
- All advective tricks can be applied

Example, Landau damping of small amplitude Langmuir wave





Summary

- A cautionary tale
  - Using moments + closure requires synchronization
  - Algorithm for low collisionality largely unknown
  - Other issues solved by particular velocity
- Semi-characteristic method developed
  - PIC shot noise removed
  - Initial experiments successful
  - Remaining issues
    - Non-periodic phase space (advection)
    - Beam-beam instabilities (controlled by randomness)
    - Lyapunov exponents > 0 (is it an issue with SC?)
- Happy to work with anyone on these issues