A Parametric Study of Extended MHD Effects in Spheromak Equilibria

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2014 Summer NIMROD Team Meeting

Outline of this talk

- Extended MHD calculations of interchange modes.
- Difficulties running extended MHD.

Interchange stability is a concern for CHI spheromaks, which have unfavorable magnetic curvature and weak magnetic shear.

- The highest temperatures observed in spheromaks to-date were obtained in SSPX discharges.
  - Optimal performance was obtained by maintaining a safety factor profile with $q_0 < 2/3$ and $q_{min} > 1/2$. [McLean et al, POP 2006]
  - $T_e > 400eV$ were routinely observed in high performance SSPX discharges.
- The traditional mentality of stabilizing ideal interchange modes in high β spheromaks involves maximizing magnetic shear[e.g. Mayo & Marklin, Phys. Fluids 1988].
  - In these studies the safety factor q is bound between $q = 0$ at the wall and $q < 1$ on the magnetic axis.
  - Magnetic shear alone can not completely stabilize resistive interchange.

In this study, we explore the extent to which extended MHD effects can stabilize interchange modes in spheromak equilibria.

- Extended MHD effects introduce drifts which couple to and can stabilize the interchange mode.
  - Simple model shows that stabilization is possible for sufficiently large diamagnetic drifts: $\omega_*^2 > 4\gamma_{mhd}^2$ [Roberts & Taylor, PRL 1962].
  - Complete stabilization is only possible for certain prescriptions of equilibrium density and temperature profiles [Zhu et al, PRL 2008].
- Extended MHD effects also introduces unstable drift waves(e.g. ITG).
- Kinetic effects and 3-D deformations of the equilibrium may also be important.
  - These effects are beyond the scope of this project.

Interchange modes in spheromak equilibria are studied using NIMROD.

$$\rho\left(\partial_t\vec{V}+\vec{V}\cdot\nabla\vec{V}\right)=\vec{J}\times\vec{B}-\nabla P-\nabla\cdot\pi_i$$

$$\pi_i=-\rho\nu_{iso}W+\frac{P_i}{4\Omega_{ci}}\left[\hat{b}\times W\cdot\left(I+3\hat{b}\hat{b}\right)-\left(I+3\hat{b}\hat{b}\right)\cdot W\times\hat{b}\right]$$

$$W=\nabla\vec{V}+\nabla\vec{V}^T-2/3I\nabla\cdot\vec{V}$$

$$\partial_t n+\nabla\cdot(n\vec{V})=\nabla\cdot(D\nabla n-D_h\nabla\nabla^2 n)$$

$$n\left(\partial_t T_s+\vec{V}_s\cdot\nabla T_s\right)=-(\gamma-1)P_s\nabla\cdot\vec{V}_s-(\gamma-1)\nabla\cdot\vec{q}_s$$

$$\partial_t\vec{B}=-\nabla\times\left[\eta\vec{J}-\vec{V}\times\vec{B}+\frac{1}{ne}\left(\vec{J}\times\vec{B}-T_e\nabla n\right)+\mu_0 d_e^2\partial_t\vec{J}\right]+k_{divb}\nabla\nabla\cdot\vec{B}$$

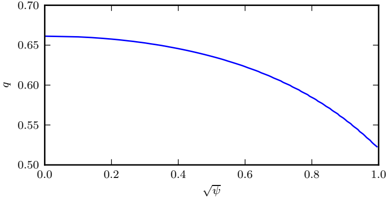
- Artificial particle diffusivity and magnetic divergence diffusions are used to provide numerical stability.
- Gyro-viscosity and two-fluid corrections to Ohm's law are included in extended MHD calculations.

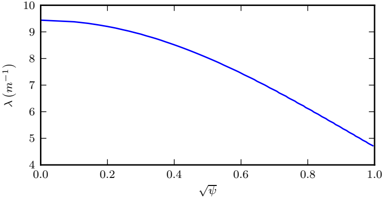
Different extended models of increasing complexity are studied.

Model	Ohm's Law	Gyro-Viscosity	Temperature Eqn	Equilibrium flow
MHD	MHD	No	Single T	None
Gyro	MHD	Yes	Single T	None
2 FL	Two Fluid	Yes	Single T	None
2T	Two Fluid	Yes	T_i and T_e	Ion Diamagnetic

- The heat flux has the general from:
  $\vec{q}_s = -n\chi^s_{\parallel}\nabla_{\parallel}T_s - n\chi^s_{\perp}\nabla_{\perp}T_s - n\chi^s_{\wedge}\nabla_{\wedge}T_s$
  - Unless noted otherwise, single temperature calculations use an isotropic heat flux with $\chi_{\perp} = \chi_{\parallel}$ and $\chi_{\wedge} = 0$
  - Separate temperature calculations include the cross field heat fluxes: $\chi^i_{\wedge} = \frac{5P_i}{2m_i\Omega_i}$ and $\chi^e_{\wedge} = \frac{5P_e}{2m_e\Omega_e}$
- Calculations presented use an uniform equilibrium number density.
  - The equilibrium pressure gradients are entirely due to equilibrium temperature gradients.
  - Calculations that use isothermal equilibrium, where the pressure gradients are due to density gradients, yield qualitatively similar results.

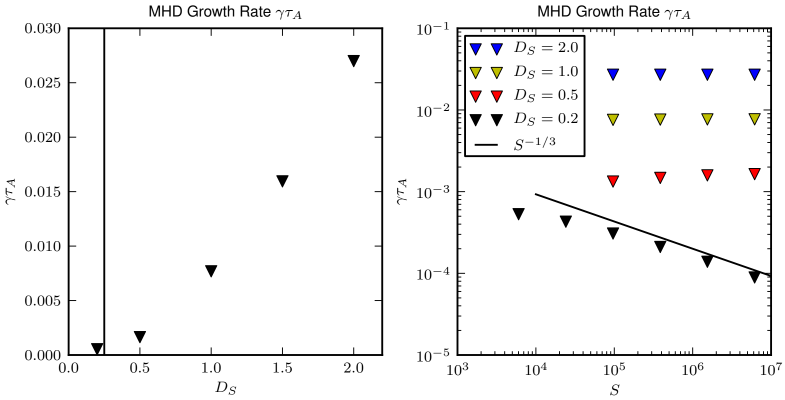
A cylindrical screw pinch is used to approximate SSPX equilibria.





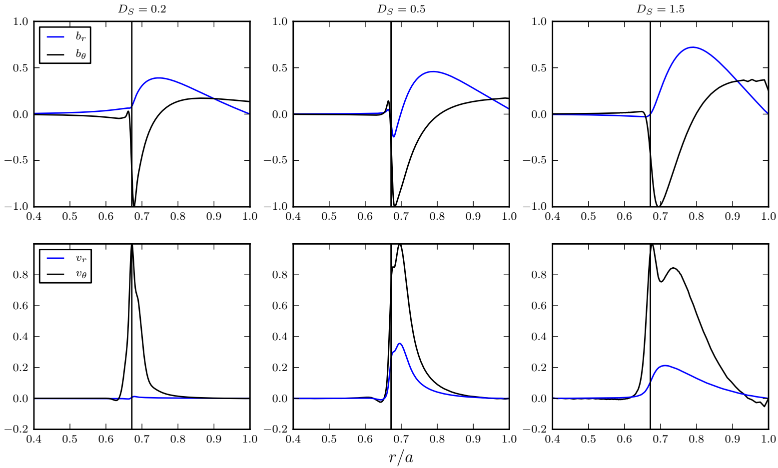
- The pressure profile is calculated to give a uniform Suydam parameter across the equilibrium.
  - $\mu_0 P' = \frac{D_s r B_z^2}{2} \left(\frac{q'}{q}\right)^2$
  - $D_s > \frac{1}{4}$ is ideal interchange unstable.
  - $D_s > 0$ is resistive interchange unstable.
- Equilibria are adapted from [Jardin NF 1982] to allow for finite q at the boundary.
  - A quadratic safety factor profile is: $q = q_0 \left(1 - q_2 \left(\frac{r}{a}\right)^2\right)$.
- Calculated growth rates are for the $n = 5$ $m = 3$ mode. This mode is resonant on $q = 3/5$ surface. The lowest order rational surface.

Linear MHD rates are exponentially small for small D_s.

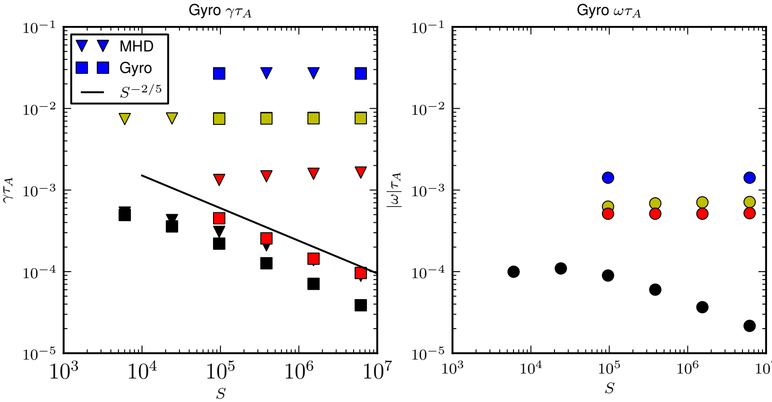


- Growth rates are insensitive to resistivity for $D_s \geq 0.5$.
- Resistive interchange scaling $S^{-1/3}$ is observed for $D_s = 0.2$
- Results are in agreement with [Ebrahimi et al, POP 2002].
- Calculations use $P_m \equiv \frac{\eta\nu}{\mu_0} = 0.1$ and $\nu = D = 10(\gamma - 1)\chi$

The linear mode is concentrated outside of the rational surface.

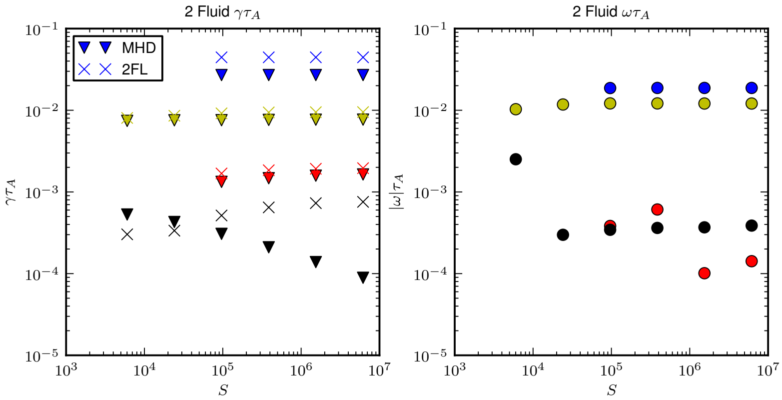


Gyro-viscosity significantly reduces the growth rate for $D_s \leq 0.5$



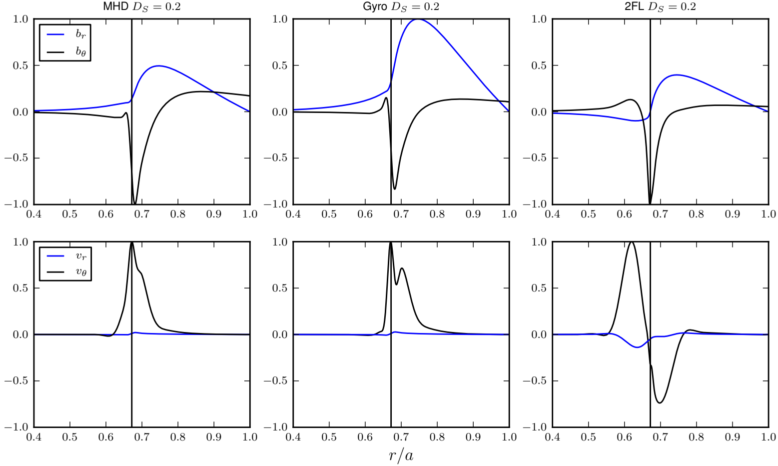
- The linear growth rate scales $S^{-2/5}$ for $D_s \leq 0.5$.
- Gyro-viscosity has a negligible effect at large D_s.

Two fluid effects are destabilizing and increase the linear growth rate.

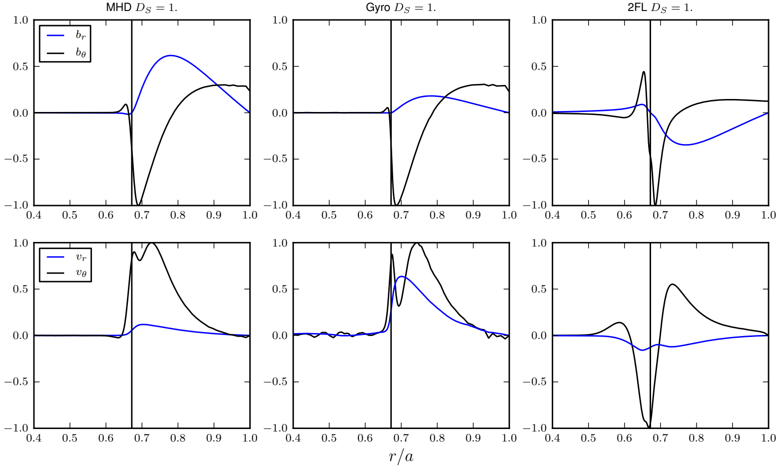


- Growth rates are independent of resistivity at $S \geq 10^5$ for all D_S
- Two fluid effects increase the growth rate of the resistive mode ($D_s = 0.2$) by an order of magnitude at $S = 10^7$.

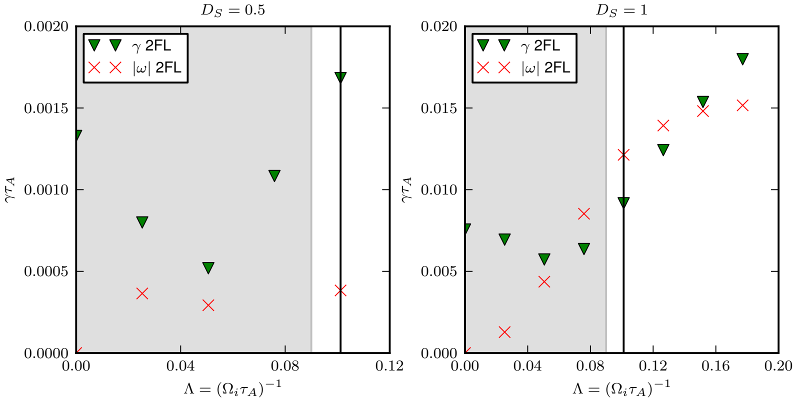
The parity of the V_θ changes when the 2 Fluid Ohm's Law is used.



The change in parity is observed for both ideal and resistive modes.

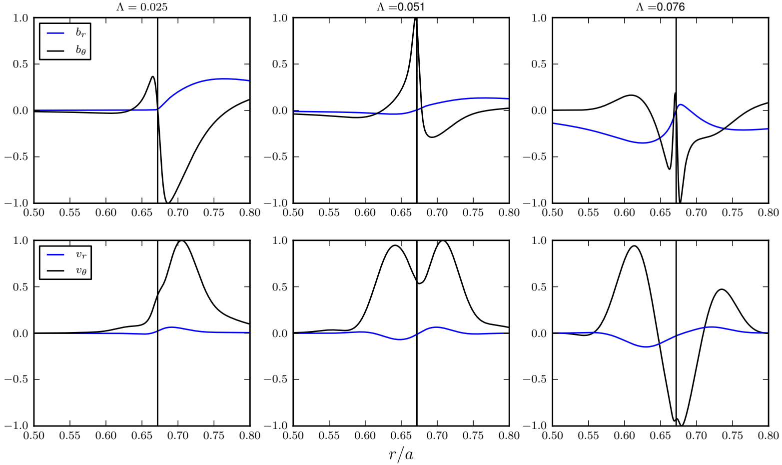


Two distinct branches of the mode are observed as the Hall parameter is increased.

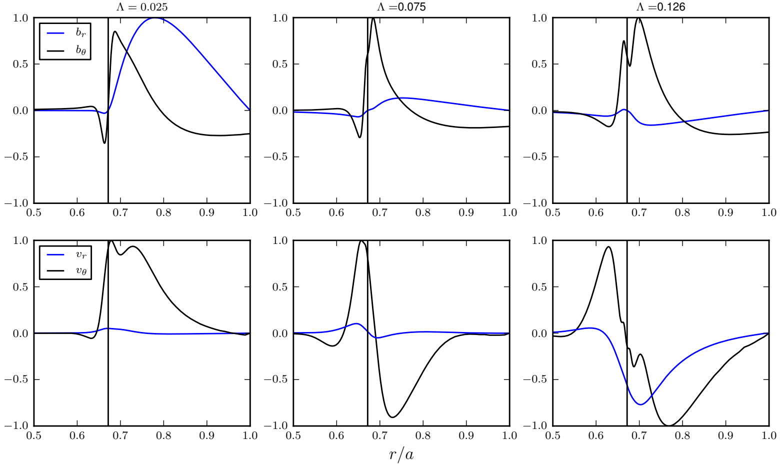


- Two fluid physics are stabilizing for a range of small Λ (gray region).
- A second branch is observed at large Λ, where the growth rate increases with Λ.
- At SSPX relevant conditions ($\Lambda \approx 0.10$) the two fluid effects are destabilizing.

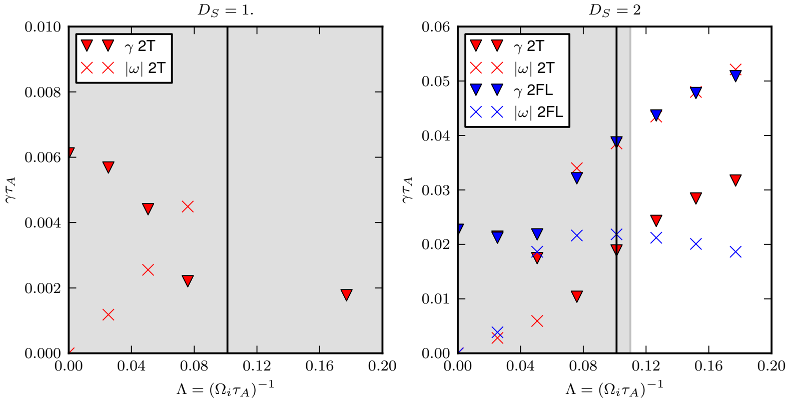
Oscillations in the radial mode structure, characteristic of drift interchange modes, develop with increasing Λ ($D_s = 0.5$).



Oscillations in the radial mode structure, characteristic of drift interchange modes, develop with increasing Λ ($D_s = 1$).



The two temperature model (which includes the two fluid Ohm's Law) is stabilizing for a larger range of Λ.

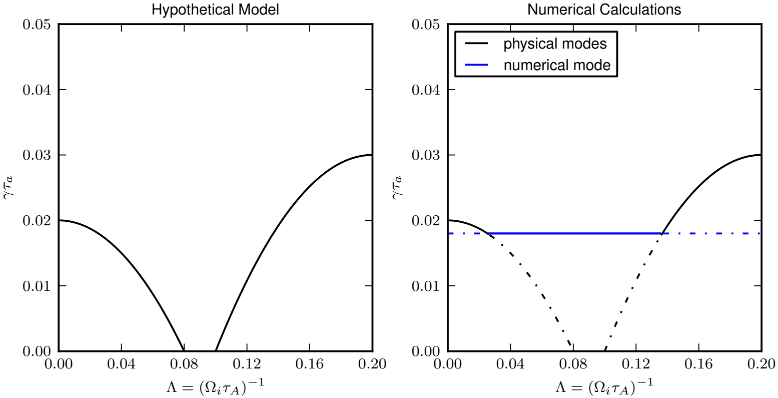


- A 95% reduction in growth rate is observed for $D_S = 1$ at $\Lambda = 0.10$
- These calculations use $P_m = 0.1$ and $D = (\gamma - 1)\chi = 7\frac{\eta}{\mu_0}$
  - The increased particle and thermal diffusivities are required for numerical stability.

Summary of physics observations

- Resistive MHD calculations are in agreement with existing theory.
  - Linear growths scale with D_S.
  - $\gamma \sim S^{-1/3}$ is recovered for $D_S < 1/4$.
- Gyro-viscosity provides significant stabilization for $D_s \leq 0.5$.
  - $\gamma \sim S^{-2/5}$
- The two fluid Ohm's law introduces a second instability at moderate Λ.
  - Two fluid effects are stabilizing at small Λ but destabilizing at large Λ
- Qualitatively similar behavior is observed in the two-fluid two-temperature calculations.
  - Two-temperature model provides the largest degree of stabilization.
  - Significant stabilization is observed at $D_s = 1$ (with large dissipation)

Numerical instabilities are the most problematic where the 2-fluid stabilization is strongest.

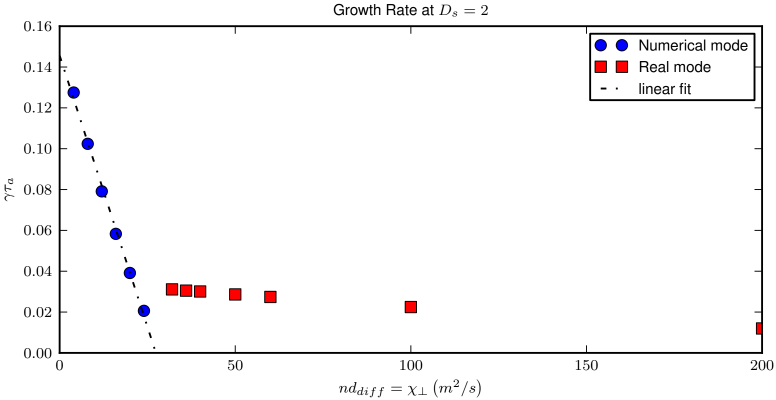


Running with separate temperatures is the most difficult.

Model	Difficulty	Fix
MHD	$\star$	NA
MHD + Gyro-viscosity	$\star$	NA
2 Fluid	$\star\star\star$	reduce δt or add dissipation
2 Fluid + Gyro-viscosity	$\star\star$	reduce δt or add dissipation
Separate Temperatures	$\star\star\star\star\star$	nd_{diff} and $\chi_\perp$

- Numerical instabilities that arise in single temperature two fluid Ohm's law can often be resolved with sufficiently small δt.
  - The necessary time step for numerical stability is restrictively small.
  - Some cases require $\gamma\delta t \sim 10^{-7}$.
- Advection in the electron temperature advance is problematic.
  - Numerical instabilities arise even with the Ohms law and $q_\wedge = 0$.
  - Increasing temporal and spatial resolution does not help.

Significant dissipation is needed to stabilize numerical mode.



- $nd_{diff} = \chi_\perp = 28m^2/s$ is needed for $\Lambda = 0.18$
- "Large" dissipation significantly reduces the linear growth rates.
  - γ_{MHD} is reduced by $\sim 15\%$ at $D_s = 2$.
  - Damping is more significant at smaller D_s.