GIANT SAWTOOTH MODELING WITH PIC CLOSURES: RECENT DEVELOPMENTS

Tom Jenkins
Tech-X Corporation
in collaboration with
Utah State University

Eric Held
Jake King
Scott Kruger
Tech-X Corporation

NIMROD team meeting
August 15, 2015
Boulder, Colorado
Normal sawtooth mode

- Plasma has \( q(0) > 1 \), peaked current density on axis
- Ohmic heating introduced (e.g. 80 keV neutral beam)
- Plasma near axis preferentially heated (higher \( J \)) \( \Rightarrow \) decreased core resistivity (\( \sim T^{-3/2} \)) \( \Rightarrow \) further current peaking, decreased \( q(0) \)
- \((1,1)\) internal kink instability triggered when \( q(0) < 1 \), which rearranges magnetic flux and flattens temperature profile
- Cycle repeats

Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).
Giant sawtooth basics

**Giant sawtooth mode**

- Energetic particle population (e.g. induced by RF heating, or fusion reactions) alters stability of internal kink mode
- Higher temperatures and stored energies achievable even with $q(0) < 1$
- Terminates like a normal sawtooth crash, but with larger amplitude
- Potential trigger for ELMs, NTMs, large heat transfer to vessel wall

“slow leak” description
“soft $\beta$ limit”

DIII-D shot #96043

---

Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas 14, 112517 (2007).
Hot-particle sawtooth stabilization in NIMROD: PIC approach

Momentum equation has an extra term:

\[ \rho \frac{\partial \vec{V}}{\partial t} + \rho (\vec{V} \cdot \nabla)\vec{V} = \vec{J} \times \vec{B} - \nabla \cdot \vec{P} - \nabla \cdot \vec{P}_{\text{hot}} \]

Kinetic PIC:
\[ \vec{P}_{\text{hot}} \] represented by moments of PIC distribution, evolving according to drift-kinetic equation

\[ \vec{P} \rightarrow (1 - \beta_{\text{frac}})\vec{P} \]
\[ \vec{P}_{\text{hot}} = \beta_{\text{frac}}\vec{P} \] comes from energetic particles, via \( T_i \) (energetic particles have low density and high temperature; \( n_{\text{hot}} \ll n \) but \( P_{\text{hot}} \sim P \))

Some fraction of fluid pressure removed; put back in as hot particles
Kinetic PIC formulation

Drift-kinetic equation for hot ions:

\[
\frac{\partial f_{\alpha}}{\partial t} + \vec{u} \cdot \vec{\nabla} f_{\alpha} + a \frac{\partial f_{\alpha}}{\partial v_{\parallel}} = C(f_{\alpha})
\]

parallel free-streaming, E-cross-B drift, grad-B drift, curvature drift, diamagnetic drift, parallel drift, inertia terms

\[
\vec{u} = v_{\parallel} \hat{b} + \frac{\vec{E} \times \hat{b}}{B} + \frac{\mu_{\alpha}}{q_{\alpha}} \frac{\vec{b} \times \vec{\nabla} B}{B} + \frac{m_{\alpha} v_{\parallel}^2}{q_{\alpha} B} \frac{\vec{b} \times \vec{\nabla} B}{B} - \frac{m_{\alpha} v_{\parallel}^2}{q_{\alpha} B^2} \mu_0 \left( \vec{j} - \hat{b} b \cdot \vec{j} \right) + \frac{\mu_{\alpha}}{q_{\alpha} B} \mu_0 \left( \hat{b} b \cdot \vec{j} \right) + \frac{m_{\alpha} v_{\parallel}^2}{q_{\alpha} B} \hat{b} \times \frac{\partial \hat{b}}{\partial t}
\]

\[
a = \frac{\vec{u}}{m_{\alpha} v_{\parallel}} \left( q_{\alpha} \vec{E} - \mu_{\alpha} \nabla B \right)
\]

Collisions determine equilibrium hot-particle distribution function (slowing-down distribution). Ignored at first order?

Standard delta-f approach:

\[
\begin{align*}
    f_{\alpha} &= f_{\alpha 0} + \delta f_{\alpha} \\
    \vec{u} &= \vec{u}_0 + \delta \vec{u} \\
    a &= a_0 + \delta a
\end{align*}
\]

\[
\begin{align*}
    \frac{d}{dt} \left( \frac{\delta f_{\alpha}}{f_{\alpha 0}} \right) &= - \frac{\delta \vec{u} \cdot \vec{\nabla} f_{\alpha 0}}{f_{\alpha 0}} - \frac{\delta a}{f_{\alpha 0}} \frac{\partial f_{\alpha 0}}{\partial v_{\parallel}} \\
    \frac{d\vec{x}}{dt} &= \vec{u}_0 \\
    \frac{dv_{\parallel}}{dt} &= a_0
\end{align*}
\]

Moments of hot-particle PIC distribution form the hot-particle pressure tensor
Equilibrium issues – hot particles

• Formerly, fluxgrid imposed the convention psi>0, even if psi<0 in the EFIT file.
• Now: Sign of psi, J_tor set by EFIT.

• Typical DIII-D operation: J.B < 0 (true for sawtooth equilibria)

• Sign change does not make a difference for resistive MHD runs (no particles), but will change sign of some particle drifts. Have already tracked some of these down; checking the rest.

• Likely also causing issues with initial particle loading – coupled to normalization issues (next slide).

\[
\begin{align*}
\vec{J}_{\text{pol}} &= \frac{F'}{\mu_0} \frac{\nabla \psi \times \hat{\theta}}{R} ; \\
\vec{J}_{\text{tor}} &= -\frac{\Delta^* \psi}{\mu_0} \hat{\theta} \\

P_\xi &= \frac{m_\alpha v_{||} FR}{q_\alpha \sqrt{|\nabla \psi|^2 + F^2}} - \psi \quad \text{Fixes in progress.}
\end{align*}
\]
Present NIMROD implementation has some issues with normalization

- Initial perturbation amplitudes are very large and unphysical.
- These amplitudes cannot be reduced by changing input file parameters.
- Particle pressure is a factor of $\sim 10^{19}$-ish higher than equilibrium pressure, though they should be comparable.
- Only particle weight is evolved substantially in time – no motion in space, little in v-space.
- Still gets a sawtooth mode, if linear.

after 25 timesteps, with current particle normalization
Renormalizing the particle pressure gives more realistic physics

- Empirically, dividing the particle pressure moment by ndens gets things about right – still working out the details of what the right thing to do is (ndens? local density? J to eV factor? something else?)
- Growth rate $3.46 \times 10^4$ is comparable to zero-betafrac result $3.42 \times 10^4$.
- No guarantee that new normalization is correct, but it’s at least in the right ballpark.

After 5000 timesteps, with current particle normalization divided by ndens, betafrac = 0.1
Particle dynamics seem correct in new normalization – issues are with initialization.
We can tell what effect the particles have by comparing with the decoupled results.

From March Sherwood meeting. Fishbone excitation not present; linear decrease in growth rate.

With no particles (just removing pressure via betafrac), decrease in growth rate is also linear. No fishbone expected.
Heuristic renormalization isn’t getting qualitative effects quite right either.

![Graph showing growth rate vs. beta fraction and mode growth rate vs. hot-particle beta fraction.](image)
Growth rates from different equilibria (no particles) agree with Choi’s GATO results.

Figure from M. Choi et al., *Sawtooth control using beam ions accelerated by fast waves in the DIII-D tokamak*, Phys. Plasmas **14**, 112517 (2007).
q profile evolution in the discharge

- Per Steve Jardin’s suggestion: reduce on-axis q and rescale GS solution to alter growth rate

<table>
<thead>
<tr>
<th>q0</th>
<th>growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9619</td>
<td>3.42e4</td>
</tr>
<tr>
<td>0.9300</td>
<td>3.35e4</td>
</tr>
<tr>
<td>0.9000</td>
<td>3.31e4</td>
</tr>
</tbody>
</table>

- Effect is relatively weak
Plan of action going forward

Finish fixing issues discussed on previous slides

and then...

• Continue exploring the extent to which present model can accurately characterize the MHD+hot-particle behavior of linear sawtooth onset – improve model as needed
• Ensure self-consistency between PIC and continuum approaches, in collaboration with Eric.
• Examine the effect of more general hot-particle distribution functions, higher S values
• Code performance improvements for development milestone (further parallelization of hot particles)
• Milestone – DIII-D shot 96043 modeling of hot-particle induced giant sawtooth stabilization.