Electron parallel closures for arbitrary ion charge number

Jeong-Young Ji[*], **Sang-Kyeun Kim**[†], **Eric D. Held**[*],
and **Yong-Su Na**[†]

Presenter

[*]Utah State University
[†]Seoul National University

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Integral parallel closures [Ji and Held, PoP 21, 122116 (2014), signs corrected]

$$\boxed{n_{AB}(\eta) = \int d\eta' K_{AB}(\eta-\eta') g_B(\eta')}$$ where $A, B = h, R, \pi$ and $d\eta = \frac{d\ell}{\lambda_{\rm mfp}}$

$$h_\|(\eta) = -\frac{1}{2} T v_T \int d\eta' K_{hh} \frac{n}{T}\frac{dT}{d\eta'} + T v_T \int d\eta' Z K_{hR} n \frac{V_{{\rm ei}\|}}{v_T} - T v_T \int d\eta' K_{h\pi} (\frac{3}{4} n \tau_{\rm ee} W_\|)$$

$$R_\|(\eta) = -\frac{mn}{\tau_{\rm ei}} V_{{\rm ei}\|} + \frac{m v_T}{\tau_{\rm ei}} \int d\eta' [-K_{Rh} \frac{n}{2T}\frac{dT}{d\eta'} + Z K_{RR} n \frac{V_{{\rm ei}\|}}{v_T} - K_{R\pi}(\frac{3}{4} n \tau_{\rm ee} W_\|)]$$

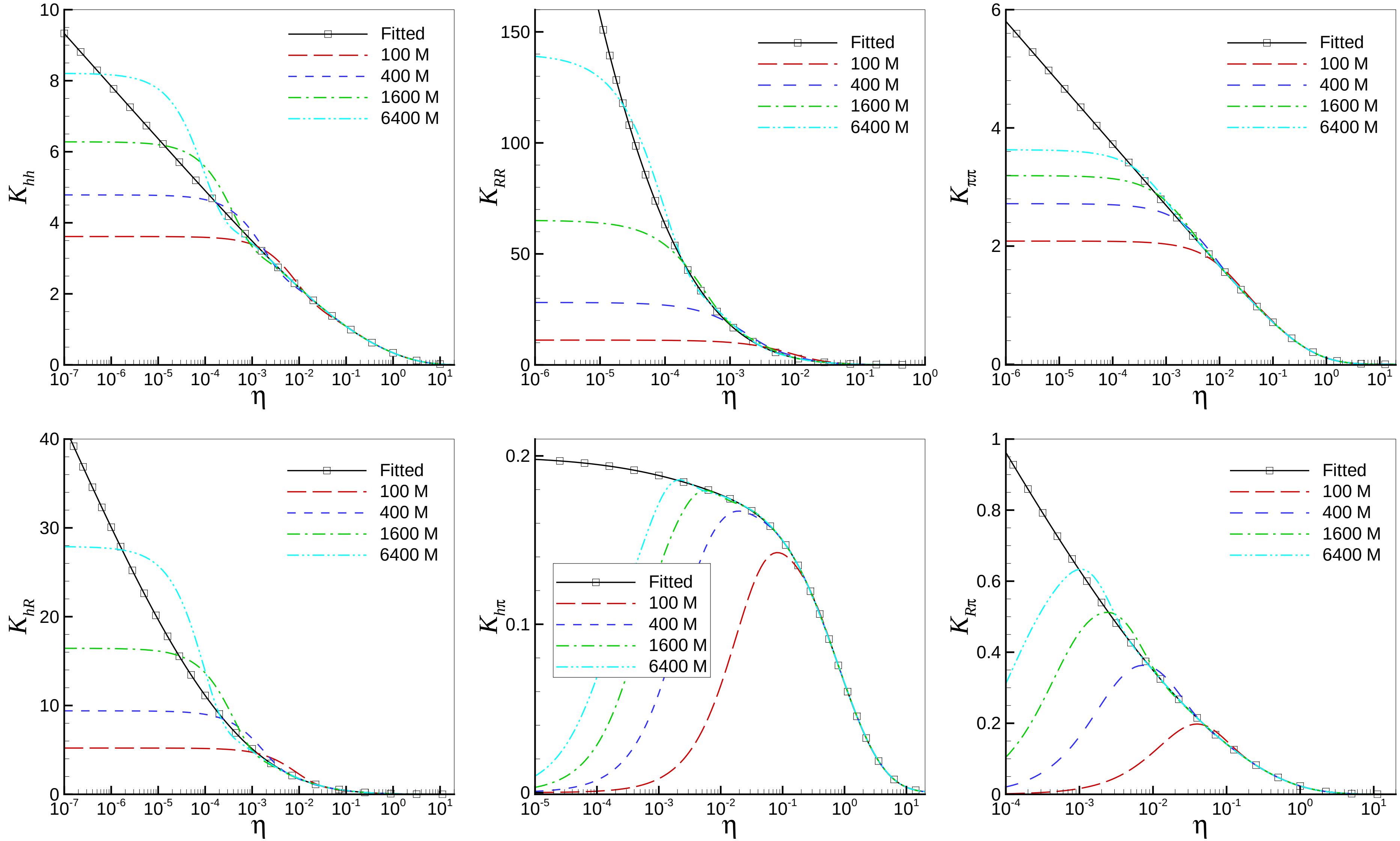
$$\pi_\|(\eta) = -T \int d\eta' K_{\pi h} \frac{n}{T}\frac{dT}{d\eta'} + 2T \int d\eta' Z K_{\pi R} n \frac{V_{{\rm ei}\|}}{v_T} - T \int d\eta' K_{\pi\pi} (\frac{3}{4} n \tau_{\rm ee} W_\|)$$

- Fitted kernel functions $(Z = 1)$

$$K_{AB}(\eta) = -[d + a\exp(-b\eta^c)] \ln[1 - \alpha\exp(-\beta\eta^\gamma)]$$

	a	b	c	d	α	β	γ
K_{hh}	-5.32	0.170	0.646	6.87	1	2.02	0.417
K_{hR}	6.37	5.12	0.160	0.100	1	1	0.583
$K_{h\pi}$	-0.229	2.26	0.594	0.363	0.775	1.49	0.478
K_{RR}	245	8.06	0.147	0.432	1	3.40	0.347
$K_{R\pi}$	-0.226	3.21	0.678	0.696	1	3.40	0.347
$K_{\pi\pi}$	0.724	0.932	0.654	0.195	1	1.60	0.491

Fitted kernel functions for $Z = 1$ (6400 M + collisionless)



Extending to $Z = 2, \cdots, 10$ (K_{hh} only shown)

- $K_{AB}(\eta) = -[d + a\exp(-b\eta^c)]\ln[1 - \alpha\exp(-\beta\eta^\gamma)]$

Z	1	2	3	4	5	6	7	8	9	10
a	-3.85	-3.61	-4.02	-4.50	-5.52	-6.98	-9.58	-14.8	-24.2	-39.0
b	0.248	.387	0.590	0.746	0.796	0.776	0.686	0.528	0.377	0.267
c	0.680	0.551	0.537	0.569	0.581	0.583	0.583	0.583	0.583	0.583
d	5.40	5.47	6.07	6.66	7.74	9.28	11.9	17.1	26.5	41.4
α	1	1	1	1	1	1	1	1	1	1
β	2.02	2.49	2.91	3.20	3.46	3.70	3.93	4.18	4.43	4.65
γ	0.417	0.348	0.316	0.300	0.289	0.281	0.279	0.277	0.276	0.275

- Errors are less than 5 % in the convergent regime ($k \equiv \lambda_{\text{mfp}}/|\nabla^{-1}| \lesssim 80$)
- For $\eta \ll 1$, kernels approach the collisionless asymptote

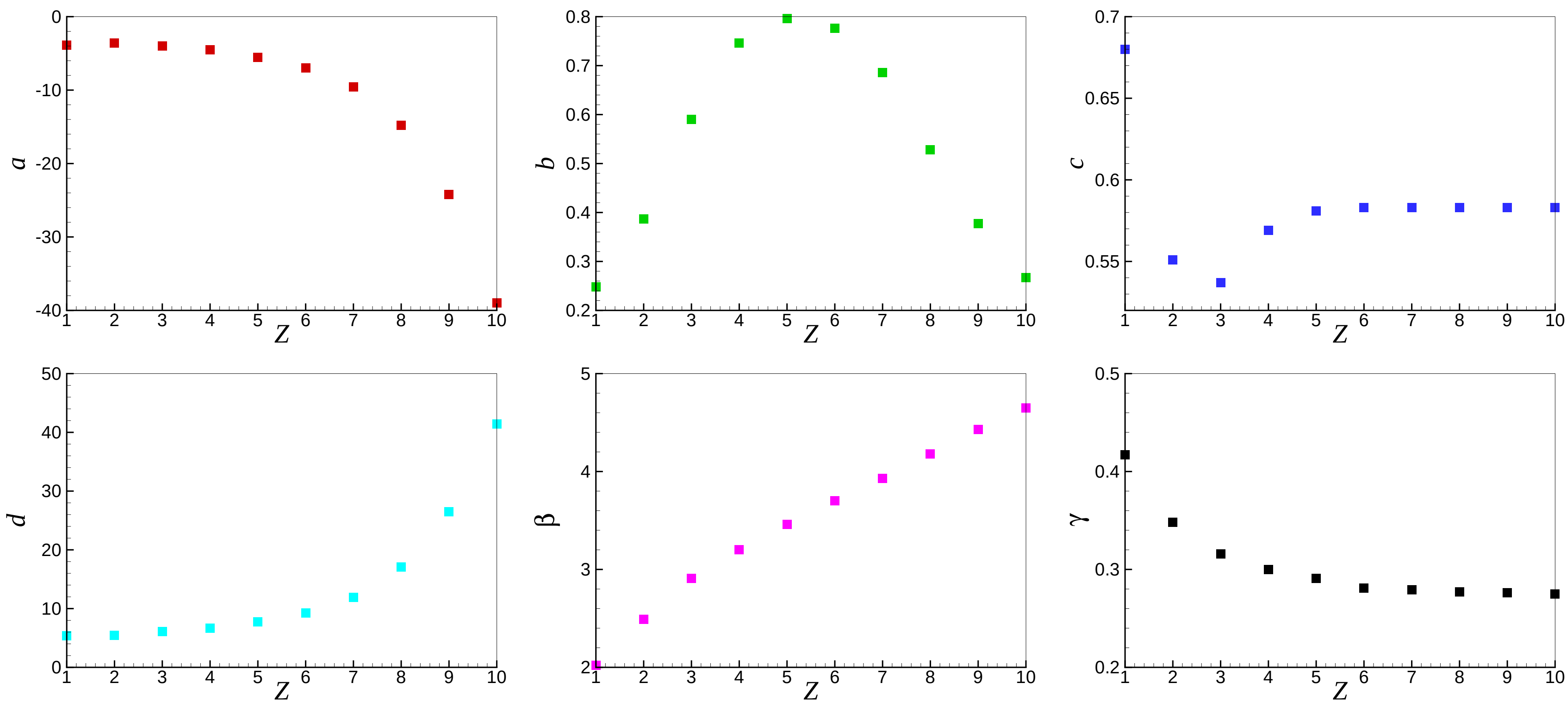
$$K_{hh}(\eta) \approx -\frac{18}{5\pi^{3/2}}(\ln|\eta| + \text{const.})$$

$$a = \frac{18}{5\pi^{3/2}\gamma} - d$$

a is from the analytic theory [Ji, Held, and Jhang, PoP 20, 082121 (2013)]

- $Z = 1$ parameters are slightly modified for better interpolation $1 < Z < 2$

Extending to $Z = 1, 2, \cdots, 10$ (K_{hh} only shown)



- Smooth change in Z
  - ◇ Accurate closures for non integer Z obtained by interpolation

Electron parallel closures for noninteger Z_{eff}

- Linear interpolations for $Z \leq Z_{\text{eff}} \leq Z+1$

$$A_{Z\text{eff}} = (1 + Z - Z_{\text{eff}})A_Z + (Z_{\text{eff}} - Z)A_{Z+1} \text{ for } A = b, c, d, \beta, \gamma$$

$$a = \frac{18}{5\pi^{3/2}\gamma} - d$$

- Maximum errors in the convergent regime $(k \lesssim 80)$

Z	1	1.2	1.4	1.6	1.8	2	2.5
max. error	1.0%	1.5%	2.2%	2.6%	2.8%	2.8%	3.0%

3	3.5	4	4.5	5	5.5	6
4.9%	4.3%	4.8%	4.4%	4.7%	4.2%	4.6%

6.5	7	7.5	8	8.5	9	9.5	10
3.1%	3.1%	2.8%	3.0%	3.7%	3.4%	3.4%	3.4%

- Collisionless limit $(k \to \infty)$
  - ◇ Closures approach exact values

Work in progress

- Ion parallel closures
  - ◇ Ion-electron collision operator included
    - ⋆ Temperature and mass ratios
- Electron parallel closures along an inhomogeneous magnetic field
  - ◇ Collisional to nearly collisionless regimes
    - ⋆ Moment solution
  - ◇ Collisionless limit
    - ⋆ Infinite number of moments required $\Rightarrow$
    - ⋆ A kinetic equation with a Krook-type operator being solved
  - ◇ Fourier representation for arbitrary collisionality