Simulations of MST and RELAX plasmas for validation studies*

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Outline

• Validation review

• Simulations of MST plasmas

• Simulations of RELAX plasmas
Validation is a formal methodology of iterative, quantitative comparison between experiment and computational modeling.

Validation metrics compose a primacy hierarchy of complexity

- Each metric is a cost function for a compared quantity, e.g.
  \[ \chi^2 \sim \sum_i \left( \frac{(y_{i,\text{exp}} - y_{i,\text{sim}})^2}{\sigma_{i,\text{exp}}^2 + \sigma_{i,\text{sim}}^2} \right) \]

- Using multiple levels of hierarchy helps detect and avoid fortuitous agreement

- Need large amounts of high-quality data from comprehensive diagnostics for extensive scans in both experiment and simulation

- Formal validation will be challenging and expensive
Multiple tearing-mode resonances allow complex nonlinear MHD in standard reversed-field pinch (RFP) operation.

- Potential fusion advantages of high $\beta$ and perhaps Ohmic ignition
- Understanding and controlling magnetic fluctuations are key issues
- RFPs provide a platform for validation of extended MHD
Madison Symmetric Torus (MST)

- University of Wisconsin
- $R_0/a = (1.5 \text{ m})/(0.52 \text{ m}) \approx 3$
- $I_p \lesssim 600 \text{ kA}$
- $n_e \sim 10^{19}/\text{m}^3$
- $T_{e,i} \lesssim 2 \text{ kV}$
- Flux-conserving shell
- $\tau_{\text{discharge}} \lesssim 100 \text{ ms}$
Nonlinear NIMROD simulations for MHD validation

- Overarching goal is $S$-scaling of magnetic fluctuations
- Initial simulations are single-fluid
- $R_0/a = 3$, cylindrical geometry
- $P_m = 1$, isotropic viscosity
- Initial paramagnetic pinch equilibrium:

  \[ -\lambda_0 = \mu_0 J(0) \cdot B(0)/B^2(0) \]

- $S$ scan: 1E4, 2E4, 3E4, 4E4, 5E4
For fixed $a\lambda_0$, increasing $S$ results in deeper $F$

- Reversal parameter $F = \frac{B_t(a)}{\langle B_t \rangle}$, pinch parameter $\Theta = \frac{B_p(a)}{\langle B_t \rangle}$
For fixed $S$, increasing initial $a\lambda_0$ results in deeper $F$ once reversed
Empirically fit $a\lambda_0(S, F)$ used to target $F = -0.2$ successfully over range of $S$

- $a\lambda_0 = [4.13, 4.08, 4.05, 4.02, 4.01]$
New scalings produced

Fixed $a \lambda_0$:

\[
\begin{align*}
\tilde{b}_{a,1,5} &\sim S^{-0.28 \pm 0.04} \\
\tilde{b}_{a,1,6} &\sim S^{-0.22 \pm 0.03} \\
\tilde{b}_{a,1,7} &\sim S^{-0.28 \pm 0.02} \\
\tilde{b}_{a,1,8} &\sim S^{-0.24 \pm 0.04}
\end{align*}
\]

Fixed $F$:

\[
\begin{align*}
\tilde{b}_{a,1,5} &\sim S^{-0.20 \pm 0.06} \\
\tilde{b}_{a,1,6} &\sim S^{-0.17 \pm 0.04} \\
\tilde{b}_{a,1,7} &\sim S^{-0.23 \pm 0.03} \\
\tilde{b}_{a,1,8} &\sim S^{-0.25 \pm 0.06}
\end{align*}
\]
Ongoing MST experiments to measure S-scaling of magnetic fluctuation amplitudes

- Low-S pulses enabled by programmable power supply (PPS)
These NIMROD amplitudes do not match MST values

Fixed $a\lambda_0$:
\[
\tilde{b}_{a,1,5} \sim S^{-0.28\pm0.04} \\
\tilde{b}_{a,1,6} \sim S^{-0.22\pm0.03} \\
\tilde{b}_{a,1,7} \sim S^{-0.28\pm0.02} \\
\tilde{b}_{a,1,8} \sim S^{-0.24\pm0.04}
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Fixed $F$:
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\tilde{b}_{a,1,7} \sim S^{-0.23\pm0.03} \\
\tilde{b}_{a,1,8} \sim S^{-0.25\pm0.06}
\]

MST:
\[
\tilde{b}_{a,1,5} \sim S^{-0.45\pm0.14} \\
\tilde{b}_{a,1,6} \sim S^{-0.28\pm0.08} \\
\tilde{b}_{a,1,7} \sim S^{-0.33\pm0.09} \\
\tilde{b}_{a,1,8} \sim S^{-0.36\pm0.10}
\]
Next steps for MST simulations

- Resistivity profile
  - Presently $\eta = \eta_0 \left(1 + (\sqrt{20} - 1) \left(\frac{r}{a}\right)^{20}\right)^2$
  - Trying $\eta = \eta_0 \left(1 + (\sqrt{12} - 1) \left(\frac{r}{a}\right)^{16}\right)^2$ based on fits to neoclassical resistivity profile from Josh Reusch

- Magnetic Prandtl number $P_m = \mu_0 \nu/\eta$
  - Using $P_m = 1$
    - Braginskii perpendicular ion viscosity gives $P_m = 0.25$
    - Previous experiments suggest $55 \lesssim P_m \lesssim 280$
    - MST experiments in progress this month to determine $\nu$ at other plasma currents

- Toroidal flux treatment (conserved flux or external circuit model)
- Two-fluid simulations
RFP with small aspect ratio (A) can have large core region inside innermost \( m = 1 \) resonant surface.

- Core safety factor \( q_0 \approx \frac{2}{a\lambda_0 A} \approx \frac{1}{2A} \) in RFP, where \( \nabla \times \mathbf{B} \approx \lambda \mathbf{B} \)
- Resonant surfaces occur at safety factor \( q = m/n \)
REversed-field pinch of Low-Aspect-ratio eXperiment (RELAX)

- Kyoto Institute of Technology
- $R_0/a = (0.51 \text{ m})/(0.25 \text{ m}) \approx 2$
- $I_P < 125 \text{ kA}$
- $n_e \approx 1E18 \text{ to } 2E19/\text{m}^3$
- $T_e \approx 100 \text{ to } 200 \text{ eV}$
- Resistive wall boundary ($\tau_{RWM} \sim 1 \text{ ms}$)
- $\tau_{\text{discharge}} \lesssim 3 \text{ ms}$
RELAX diagnostics can contribute to MHD validation comparisons

- Two edge toroidal B-probe arrays (each $N = 14$)
- Insertable radial B-probe array
- Single-chord interferometry ($n_e$)
- Single-chord Thomson scattering ($T_e$)
- Double-filter SXR camera

Oki et al., PFR 2012
RELAX experiments evidence $n = 4$ QSH behavior

- $\tilde{B}/B(a) \approx 0.1$ for $n = 4$
Initial NIMROD simulations of RELAX plasmas underway

- Single-fluid, $\beta = 0$
- Cylindrical and toroidal geometry
- $S = 1E4$
- $P_m = 1$
- Uniform resistivity and viscosity
- Conserved toroidal flux
- $m_x = 100, m_y = 16, l_{phi} = 6, \text{poly\_degree} = 3$
Four RELAX cases to be used for comparisons to NIMROD

- Cases
  - B, ultra-low $q$
  - C, nonreversed
  - D, moderate reversal
  - G, deep reversal

- Same cases previously studied with the MIPS MHD code by Mizuguchi et al., TH/P3-26, IAEA FEC, 2012

- MIPS simulations exhibited strong QSH states with $n = 4$, similar to some RELAX experiments
Equilibria span from ultra-low-q pinch to deeply reversed RFP

- Previous nonlinear NIMROD cases started with these G-S states
- Equilibria decayed before tearing modes grew large enough to maintain profiles
- New method adapts paramagnetic pinch initialization for cylindrical cases to toroidal cases
Equilibrium results from initial lam0 scan for cylindrical and toroidal cases close to RELAX data

- Toroidal cases
  - Still evolving slowly in (\(\Theta, F\)) space
  - Expected to match more closely at larger S
One NIMROD toroidal case fairly close to RELAX case D

- NIMROD case has large edge $\lambda$, perhaps due to flat resistivity

- Both code profiles and experimental fits appear to show ‘notches’ in $\lambda$ at midradius
Nearby cylindrical case shows different mode behavior

- Cylindrical equilibria are ‘better behaved’ in plasma edge
- In both cases, mode $W_{\text{mag}}$ is different from $B_{\text{edge}}$
- $B_{\text{edge}}$ values similar to RELAX (detailed comparisons planned)
- No QSH observed in NIMROD results so far
Toroidal case lower $n$ modes largely $m = 0$

$n = 1$

$n = 2$
Toroidal case higher n modes largely $m = 1$

$n = 5$

$n = 6$
Future plans

- Try more realistic resistivity profiles
- Try different $P_m$ values
- Rerun $S = 1E4$ case(s) at higher spatial resolution
- Toroidal lam0 scan at higher $S$ value(s), say $2E4, 5E4, 1E5$
- Choose one or all of RELAX cases B, C, D, G to simulate more precisely in NIMROD
- Make formal validation comparisons!
Future task: compare NIMROD nonlinear runs to RELAX internal probe measurements of magnetic fluctuations

(Oki et al., PFR 2012)
Summary

• We are investigating using the RFP as a formal validation platform for extended MHD

• Main focus of MST validation experiments and NIMROD simulations is the $S$-scaling of magnetic fluctuations

• The RELAX project is studying RFPs at very low aspect ratio, and initial NIMROD simulations of their plasmas in toroidal (and cylindrical) geometry are showing some similarities to experiment
Bonus
Paramagnetic pinch initialization for toroidal geometry

1. Run nimset and then nimrod on geom='lin', lamprof='para' case with chosen be0, lam0.

2. From dump.00000 and discharge.bin, find flux_eq_Z, i_eq_Z, loop_volt (from Ohmic power)

3. For geom='tor', lphi=0 case, set
   be0=flux_eq_Z/(2*pi*xo**2*(1.0 - sqrt(1.0 - (xmax/xo)**2))),
   i_desired=i_eq_Z, loop_volt

4. Run nimset and then nimrod until steady state, remembering to change loop_volt in nimrod.in for restarts.

5. Copy last dump file to, say, 'dump0'.

6. For the geom='tor', finite lphi case, run nimset with nimrod.in from lphi=0 case with reset_file='dump0' in &output_input.