Continuum kinetic physics in NIMROD
NIMROD Summer Meeting, Boulder, CO

E. Held, T. Taylor, B. Adair, J. Jepson, J.-Y. Ji,
and the NIMROD Team

August 17, 2017

Big Ideas

- Coupling solutions of plasma kinetic equations into plasma fluid codes is difficult.
- Solutions to Hazeltine's drift kinetic equation (DKE) are fairly easy in NIMROD but coupling closure moments ($q_{\|}(\delta f)$, $\pi_{\|}(\delta f)$, ...) into NIMROD is not rigorously consistent.
- Ramos formalism for Chapman-Enskog-like (CEL)-DKEs rigorous but requires care when coding and vetting.
- Carrying Hazeltine δf DKE and CEL-DKE capability forward is difficult but useful for verification and vetting.

Existing δf drift kinetic equations (DKEs) in NIMROD

- NIMROD can solve

$$\frac{\partial f}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v_D}) \cdot \nabla f - s\frac{\partial f}{\partial s}\left[\frac{\partial}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v_D}) \cdot \nabla\right] \ln \mathbf{v_0}$$

$$+ \frac{1-\xi^2}{2\xi}\left[-\xi^2\frac{\partial \ln B}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v}_\mathbf{c}^*) \cdot \left(\frac{q}{T_0 s^2}\mathbf{E} - \nabla \ln B\right) + \xi^2 \mathbf{v}_{\mathbf{E}\times\mathbf{B}} \cdot \nabla \ln B\right]\frac{\partial f}{\partial \xi}$$

$$+ \frac{s}{2}\left[(1-\xi^2)\frac{\partial \ln B}{\partial t} + (\mathbf{v}_\parallel + \mathbf{v_c}) \cdot \frac{q}{T_0 s^2}\mathbf{E} + (1+\xi^2)\mathbf{v}_{\mathbf{E}\times\mathbf{B}} \cdot \nabla \ln B\right]\frac{\partial f}{\partial s}$$

$$-\xi(1-\xi^2)\left[\frac{\mu_0}{2B^2}\mathbf{J}_\parallel \cdot \mathbf{E} + \frac{T_0 s^2}{q}\mathbf{b} \cdot \nabla\left(\frac{\mu_0 J_\parallel}{B^2}\right)\right]\frac{\partial f}{\partial \xi} = \mathbf{C}(f)$$

where $\mathbf{v_c} = \frac{\mu_0 s^2 T_0}{qB^2}\left[2\xi^2 \mathbf{J}_\perp + (1-\xi^2)\mathbf{J}_\parallel\right] + \frac{m v_0 s \xi}{qB^2}\mathbf{b} \times \frac{\partial \mathbf{B}}{\partial t}$

and $\mathbf{v}_\mathbf{c}^* = \frac{\mu_0 s^2 T_0}{qB^2} 2\xi^2 \mathbf{J}_\perp + \frac{m v_0 s \xi}{qB^2}\mathbf{b} \times \frac{\partial \mathbf{B}}{\partial t}$.

Aspects of δf DKE implementation

- Relatively easy applications include:
  - solving for electron and ion δf's to predict neoclassical transport in axisymmetric toroidal geometry
  - advancing energetic particle δf and coupling to MHD through closure for anisotropic pressure tensor.
  - Spitzer problem with coupling between electron and ion δf distributions through full linearized, Coulomb collision operator.
  - ion poloidal flow damping with the addition of perturbed electrostaic potential.

- Numerical formulation relatively easy since thermodynamic drives have a simple form.

- Allowed for easy testing of important DKE terms like collision operators, parallel free-streaming and particle trapping.

- But, consistency issues exist and CEL-DKE approach is also desirable.

Chapman-Enskog-like (CEL) DKEs in NIMROD

- Assume $f = f_{\mathrm{M}} + f_{\mathrm{NM}}$ with $\bar{f}_{\mathrm{NMe}} = O(\delta^2 f_{\mathrm{Me}})$ and $\bar{f}_{\mathrm{NMi}} = O(\delta f_{\mathrm{Mi}})$.
- Write CEL-DKE in the fluid frame (Ramos, *Phys Plasmas* **17**, 082502 (2010)):

$$\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial t} + v'_{\parallel}\mathbf{b}\cdot\nabla \bar{f}_{\mathrm{NM}} - \frac{1-\xi^2}{2\xi} v'_{\parallel}\mathbf{b}\cdot\nabla \ln B \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi}$$

$$+\frac{v_0}{2}(\mathbf{b}\cdot\nabla\ln n)[\xi\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} + \frac{1-\xi^2}{s}\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi}] - s[\xi\mathbf{b}\cdot\nabla + \frac{\partial}{\partial t}]\ln v_0 \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} = \langle C(f)\rangle$$

$$+\Bigg[(\frac{5}{2}-s^2)v'_{\parallel}\mathbf{b}\cdot\nabla\ln T + \frac{v'_{\parallel}}{nT}\mathbf{b}\cdot[\frac{2}{3}\nabla\pi_{\parallel} - \pi_{\parallel}\nabla\ln B - \mathbf{F}^{\mathrm{coll}}]$$

$$+2s^2(\frac{3}{2}\xi^2-\frac{1}{2})[\frac{1}{3}\nabla\cdot\mathbf{u} - \mathbf{bb}\cdot\nabla\mathbf{u}] + \frac{2}{3nT}(s^2-\frac{5}{2})[\mathbf{b}\cdot\nabla q_{\parallel} - q_{\parallel}\mathbf{b}\cdot\nabla\ln B - G^{\mathrm{coll}}]$$

$$+\frac{2}{3eB}s^2(\frac{3}{2}\xi^2-\frac{1}{2})[(\frac{5}{2}-s^2)(\nabla\ln B - 2\kappa) + \nabla\ln n]\cdot\nabla T\times\mathbf{b}$$

$$+\frac{4}{3eB}(\frac{s^4}{2}-\frac{5}{2}s^2+\frac{15}{8})(\nabla\ln B+\kappa)\cdot\nabla T\times\mathbf{b}\Bigg] f_M$$

Fluid moments equations

- In Ramos formalism, low-order fluid equations are

$$\frac{\partial n_s}{\partial t} + \nabla \cdot n_s \mathbf{u}_s = 0$$

$$m_s n_s \frac{d\mathbf{u}_s}{dt} + q_s n_s(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \nabla(nT_s) + \nabla \cdot [\pi_{\|s}(\mathbf{bb} - \mathbf{I}/3)] - \mathbf{F}_s^{\text{coll}} = 0$$

$$\frac{3n_s}{2}\frac{dT_s}{dt} + nT_s\nabla \cdot \mathbf{u}_s + \nabla \cdot (q_{\|s}\mathbf{b} + \frac{5nT_s}{2q_s B}\mathbf{b} \times \nabla T_s) - G_s^{\text{coll}}$$

$$-\pi_{\|s}[\frac{1}{3}\nabla \cdot \mathbf{u}_s - \mathbf{bb} \cdot \nabla \mathbf{u}_s] = 0$$

- This rather un-NIMROD-esque form of the fluid equations complicates coupling to NIMROD.
- For CEL-DKE Spitzer problem, separate electron and ion flow evolution equations were implemented.

Aspects of CEL-DKE formulation

- Allows for a tight coupling, *i.e.*, hybrid fluid/kinetic capability that is rigorous and consistent: 1, $v'_{\|}$ and v'^2 moments of $\bar{f}_{\mathrm{NM}}$ vanish.

- DKEs written in moving frame of fluid makes taking moments easy.

- Numerical considerations:
  - time centering of fluid and kinetic variables,
  - enforcing the requirement that fluid moments of $\bar{f}_{\mathrm{NM}}$ vanish,
  - ability to evolve linearized system that expands about an axisymmetric $\bar{f}_{\mathrm{NM}}$ and its closure moments.

Ion poloidal flow damping

- δf DKE approach (Morris, *et al*, *Phys Plasmas*, **3** (1996)):

$$\frac{\partial f_1}{\partial t} + \mathbf{v}_\parallel \cdot \nabla f_1 - C(f_1) = -\mathbf{v}_D \cdot \nabla f_0 - \mathbf{v}_D \cdot q_i \mathbf{E} \frac{\partial f_0}{\partial \varepsilon}$$

  but relax ordering $\mathbf{v}_\parallel \cdot \nabla f_1 >> C(f_1), \partial_t f_1$.

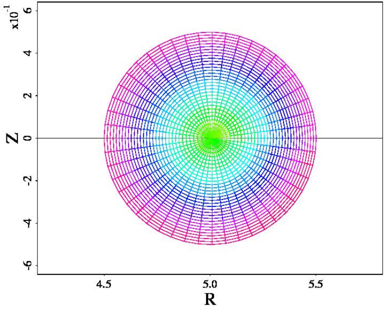
- CEL-DKE approach (Garcia-Perciante, *et al*, *Phys Plasmas*, **12** (2005)):

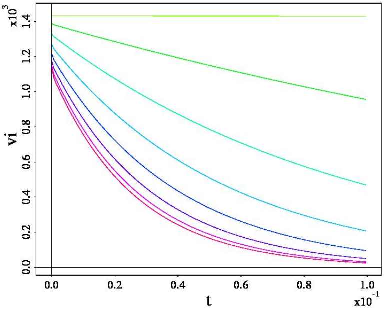
$$\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial t} + (\mathbf{v}_\parallel + \mathbf{u}) \cdot \nabla \bar{f}_{\mathrm{NM}} - C(f) = \frac{f_M}{p} \mathbf{v}_\parallel \cdot \nabla \cdot \pi_\parallel (\mathbf{bb} - \mathbf{I}/3)$$
$$- 2s^2 P_2(v_\parallel / v)[\mathbf{u} \cdot \nabla lnB - \frac{\mathbf{b}}{B} \cdot \nabla \times (\mathbf{B} \times \mathbf{u}) + \frac{2}{3} \nabla \cdot \mathbf{u}]$$

  with $\pi_\parallel \equiv \frac{m}{2} \int d\mathbf{v} \left(3[\mathbf{b} \cdot (\mathbf{v} - \mathbf{u})]^2 - |\mathbf{v} - \mathbf{u}|^2 \right) \bar{f}_{\mathrm{NM}}$, but this misses terms in Ramos' equation.

Damping reflects geometry but quantitatively wrong.

- For δf case, initialize parallel flow using $f_1 = 2s\xi f_M$ and watch it evolve in time
- For banana-regime ($\nu_{ii} \approx 1000s^{-1}$) timescale is way to slow. Need to add radial electric field.





Electron heat transport

- CEL-DKE approach requires modifications:

$$\frac{\partial \bar{f}_{\mathrm{NM}}}{\partial t} + v_\parallel' \mathbf{b}\cdot\nabla \bar{f}_{\mathrm{NM}} - \frac{1-\xi^2}{2\xi} v_\parallel' \mathbf{b}\cdot\nabla \ln B \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial \xi}$$

$$- s[\xi \mathbf{b}\cdot\nabla + \frac{\partial}{\partial t}] \ln v_0 \frac{\partial \bar{f}_{\mathrm{NM}}}{\partial s} - \langle C(f)\rangle =$$

$$(\frac{5}{2} - s^2)\left[v_\parallel' \mathbf{b}\cdot\nabla \ln T - \frac{2}{3nT}\nabla\cdot q_\parallel \mathbf{b}\right] f_M$$

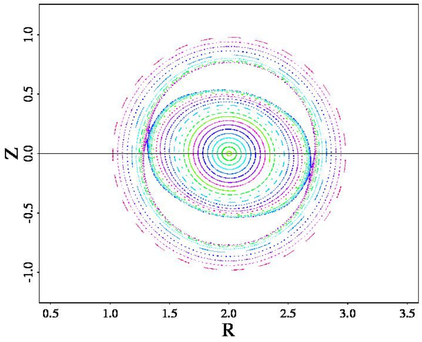
$$+ \left[\frac{2}{3nT}(s^2 - \frac{5}{2})[\nabla\cdot\mathbf{q}_\perp - G^{\mathrm{heat}}]\right] f_M + S^{\mathrm{heat}}$$

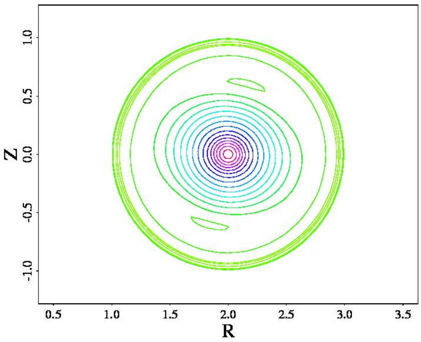
where $G^{\mathrm{heat}} \equiv \frac{m_e}{2}\int d\mathbf{v}\, {v_\parallel'}^2 S^{\mathrm{heat}}$.

- With T_e evolution $\frac{3n}{2}\frac{\partial T}{\partial t} = -\nabla\cdot(q_\parallel \mathbf{b} + \mathbf{q}_\perp) + G^{\mathrm{heat}}$.

Evolve coupled CEL-DKE/T_e to steady state.

- Impose magnetic perturbations on cylindrical equilibria with heating profile (Holzl, *et al*, *Phys Plasmas*, **14** 052501 (2007)) $G^{\text{heat}} = G_0[1 - 75r^2 + 250r^3]$ for $r < 0.2$.
- Anisotropic diffusion $(\chi_{\|}/\chi_{\perp} = 10^8)$, T_e surfaces coincide with magnetic surfaces.





Conclusions

- Further testing underway: neoclassical transport, ion poloidal flow damping, parallel electron heat transport, energetic particles ....
- Apply to NTM problem.
- Apply to RMP-type problems.
- Implement second-order terms in ion CEL-DKE?