Closure theory of partially ionized plasmas

Jeong-Young Ji

Utah State University

NIMROD Team Meeting

August 2, 2018, Logan, UT

General moment equations

- Landau (Fokker-Planck) kinetic equation

$$\frac{\partial f_a}{\partial t} + \mathbf{v}\cdot\nabla f_a + \frac{q_a}{m_a}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\partial_{\mathbf{v}} f_a = \sum_b C(f_a, f_b)$$

- Moment expansion: m^{lk} is a symmetric traceless fluid moment

$$f_a(t,\mathbf{x},\mathbf{v}) = f_a^{\mathrm{M}} \sum_{lk} \mathsf{m}_a^{lk}(t,\mathbf{x}) \cdot \hat{\mathsf{P}}^{lk}(\mathbf{c}_a)$$

$$\mathsf{n}_a^{lk} \equiv n_a \mathsf{m}_a^{lk}(t,\mathbf{x}) = \int d\mathbf{v}\,\hat{\mathsf{p}}_a^{lk} f_a(t,\mathbf{x},\mathbf{v})$$

where $\mathbf{c}_a = (\mathbf{v} - \mathbf{V}_a)/v_{Ta}$

- General moment equations [Ji Held 2006 2008 PoP]

$$\mathsf{L}_a\left[\left(\frac{\partial}{\partial t}, \nabla\right)(T_a, \mathbf{V}_a), \mathbf{E}\right]\mathsf{n}_a + \Omega_a \mathbf{b}\breve{\times}\mathsf{n}_a = \sum_b (\mathsf{A}_{ab}\mathsf{n}_a + \mathsf{B}_{ab}\mathsf{n}_b)$$

(l,k) =(0,0) density (n), (0,1) temperature (T), (1,0) flow velocity $(\mathbf{V})$ (1,1) heat flow $(\mathbf{h})$, and (2,0) viscosity tensor $(\boldsymbol{\pi})$

Maxwellian moment $(n_a, \mathbf{V}_a, T_a)$ equations

(0,0) $d_t n_a + n_a \nabla \cdot \mathbf{V}_a = 0 \, (d_t \equiv \partial_t + \mathbf{V}_a \cdot \nabla)$

(0,1) $\frac{3}{2} n_a d_t T_a + n_a T_a \nabla \cdot \mathbf{V}_a + \nabla \cdot \mathbf{h}_a + \nabla \mathbf{V}_a : \boldsymbol{\pi}_a = Q_a$

(1,0) $m_a n_a d_t \mathbf{V}_a - n_a q_a (\mathbf{E} + \mathbf{V}_a \times \mathbf{B}) + \nabla p_a + \nabla \cdot \boldsymbol{\pi}_a = \mathbf{R}_a$

$$(1,1)\ d_t \mathbf{h} + \Omega \mathbf{b} \times \mathbf{h} + \frac{7}{5}(\nabla \cdot \mathbf{V})\mathbf{h} + \frac{7}{5}\mathbf{h} \cdot (\nabla \mathbf{V}) + \frac{2}{5}(\nabla \mathbf{V}) \cdot \mathbf{h} + \frac{5p}{2m}\nabla T + \frac{T}{m}\nabla \cdot \boldsymbol{\pi} + \frac{7}{2}\frac{\nabla T}{m} \cdot \boldsymbol{\pi} - \mathbf{a} \cdot \boldsymbol{\pi} + \nabla \cdot \boldsymbol{\theta} + \frac{1}{3}\nabla \mathsf{u}^{02} + \nabla \mathbf{V} : \mathsf{u}^{30} = C^1_{10}\mathbf{V}_{\mathrm{ei}} + C^1_{11}\mathbf{h} + C^1_{12}\mathbf{r} + \cdots \ (\mathbf{h} \text{ heat flow})$$

$$(1,2)\ d_t \mathbf{r} + \Omega \mathbf{b} \times \mathbf{r} + \cdots = C^1_{10}\mathbf{V}_{\mathrm{ei}} + \overline{C^1_{21}\mathbf{h} + C^1_{22}\mathbf{r}} + \cdots \quad (\mathbf{r} \text{ heat heat flow})$$

$$(2,0)\ d_t \boldsymbol{\pi} + \Omega \mathbf{b} \breve{\times} \boldsymbol{\pi} + (\nabla \cdot \mathbf{V})\boldsymbol{\pi} + 2\overline{\boldsymbol{\pi} \cdot (\nabla \mathbf{V})} + p\mathsf{W} + \frac{4}{5}\overline{\nabla \mathbf{h}} + \nabla \cdot \mathsf{u}^{30} = C^2_{00}\boldsymbol{\pi} + C^2_{01}\boldsymbol{\theta} + \cdots \ (\boldsymbol{\pi} \text{ viscosity})$$

$$(2,1)\ d_t \boldsymbol{\theta} + \Omega \mathbf{b} \breve{\times} \boldsymbol{\pi} + \cdots = C^2_{10}\boldsymbol{\pi} + C^2_{11}\boldsymbol{\theta} + \cdots \ (\boldsymbol{\theta} \text{ heat viscosity})$$

where $\mathbf{a} = \frac{q}{m}(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - d_t \mathbf{V}$ and $\mathsf{W} = \nabla \mathbf{V} + (\nabla \mathbf{V})^{\mathsf{T}} - \frac{2}{3}\nabla \cdot \mathbf{V}\mathsf{I}$

Closures: express $\mathbf{h}_a(\mathsf{n}_a^{11})$, $\boldsymbol{\pi}_a(\mathsf{n}_a^{20})$, Q_a, $\mathbf{R}_a$ in terms of $n_a, \mathbf{V}_a, T_a$

Electron closures for high collisionality (Braginskii)

$\mathbf{h}_{\mathrm{e}} = (\beta)(\mathbf{V}_{\mathrm{ei}}) - (\kappa_{\mathrm{e}})(\nabla T_{\mathrm{e}})$, $\mathbf{R}_{\mathrm{e}} = -\mathbf{R}_{\mathrm{i}} = -(\alpha)(\mathbf{V}_{\mathrm{ei}}) - (\beta)(\nabla T_{\mathrm{e}})$

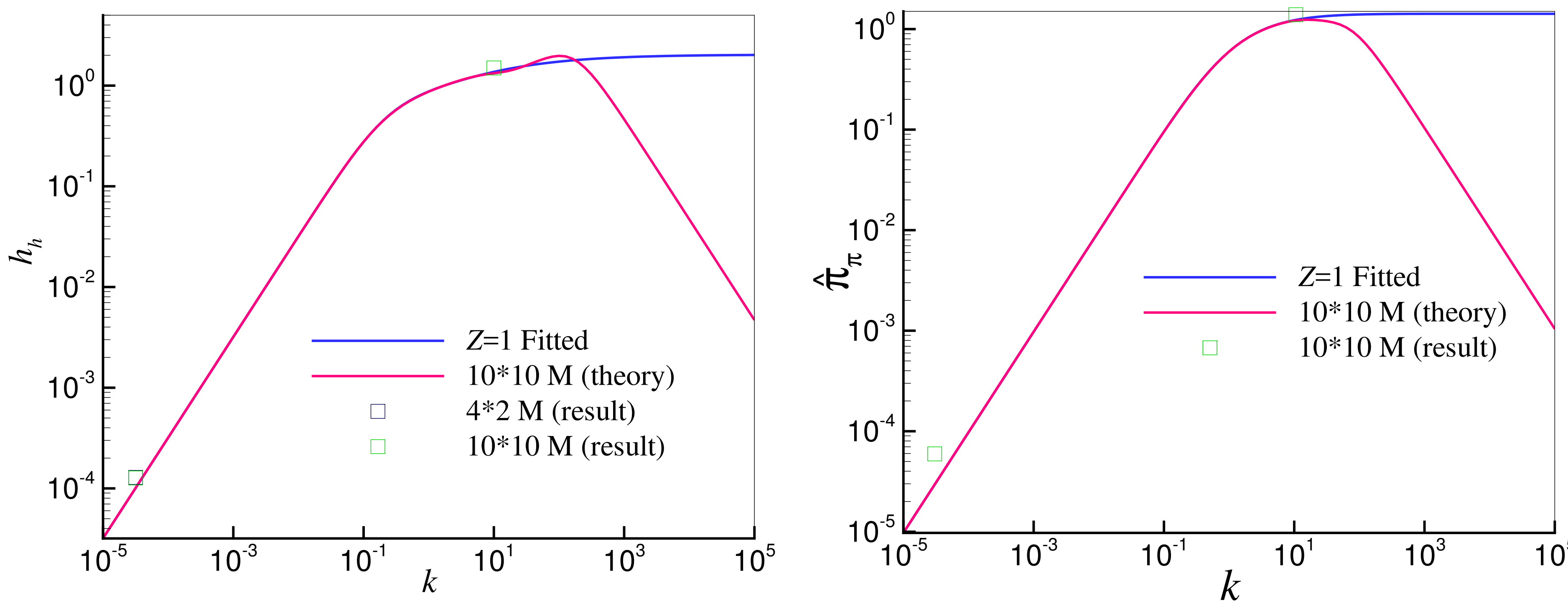
$\mathbf{h}_{\mathrm{i}} = -(\kappa_{\mathrm{i}})(\nabla T_{\mathrm{i}})$, $\boldsymbol{\pi}_{\mathrm{e}} = -(\eta_{\mathrm{e}})(\nabla \mathbf{V}_{\mathrm{e}})$, $\boldsymbol{\pi}_{\mathrm{i}} = -(\eta_{\mathrm{i}})(\nabla \mathbf{V}_{\mathrm{i}})$

Transport: relate flux densities $\mathbf{h}_{\mathrm{e}}$, $\mathbf{J}$ to thermodynamic forces ∇T_{e} and $\mathbf{E}$

$\mathbf{h}_{\mathrm{e}} = (\tilde{\alpha})\mathbf{E} - (\tilde{\kappa})(\nabla T_{\mathrm{e}})$, $\mathbf{J} = (\tilde{\sigma})\mathbf{E} - (\tilde{\alpha})(\nabla T_{\mathrm{e}})$

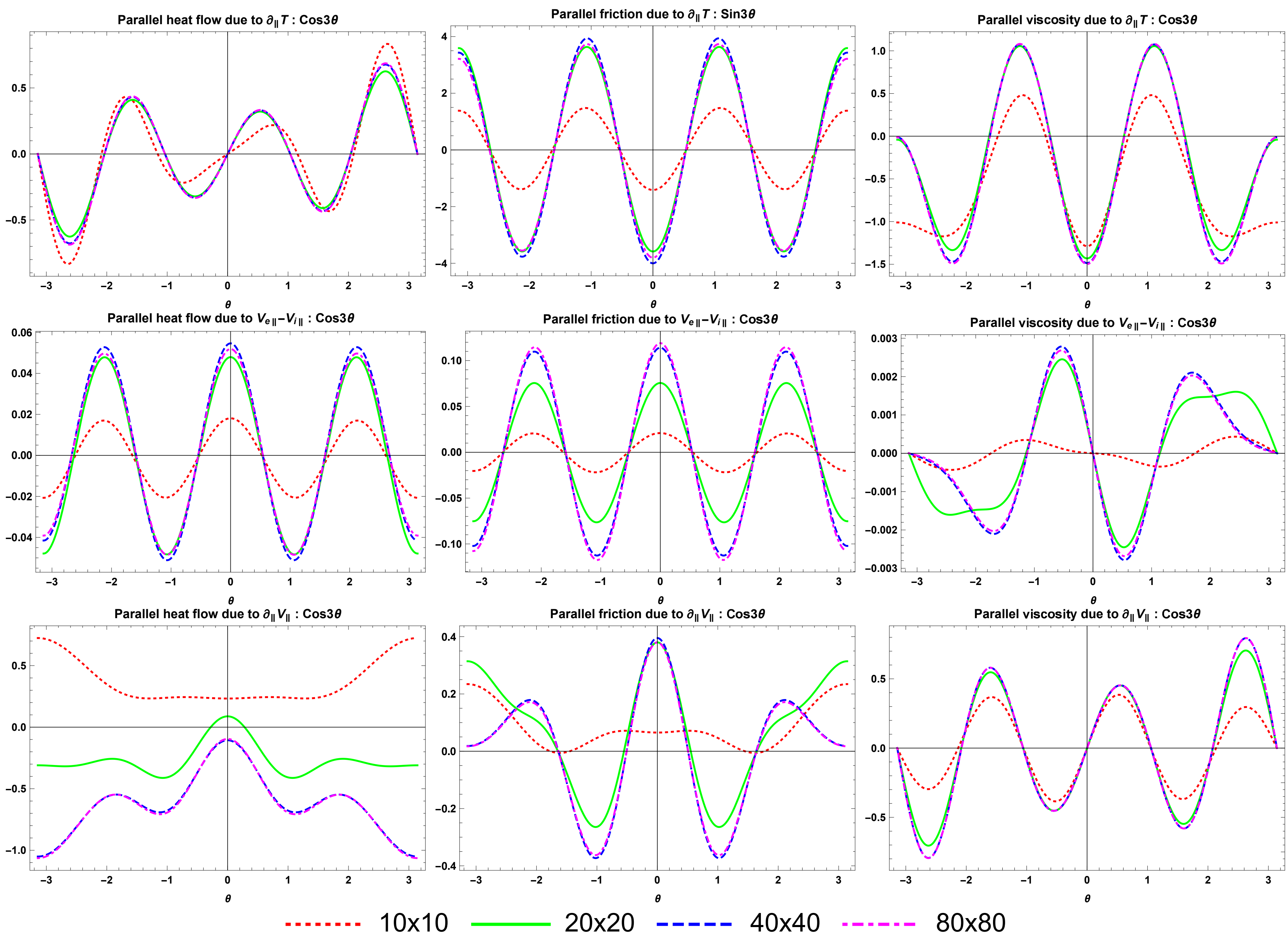
Parallel moment equations in NIMROD

$$\frac{\partial}{\partial t}[n] + v_T \Psi \partial_\| [n] + \underbrace{v_T(\partial_\| \ln B)\Psi_B [n]}_{\text{inhomo. magectic field}} + \underbrace{v_T(\partial_\| \ln T)\Phi [n] + \frac{q}{2T}E_\| \Theta [n]}_{\text{nonlinear coupling terms}} = \frac{1}{\tau} c [n] + [g]$$



Parallel heat flow and viscosity responding to the temperature and flow velocity gradients, respectively. The NIMROD results (squares) are compared to the analytical integral closures.

Parallel closures in an inhomogeneous magnetic field
$(\epsilon = r/R = 0.5)$



Kinetic equations for a partially ionized plasma

$$\frac{\partial f_\mathrm{n}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{n}}{\partial \mathbf{r}} + \frac{\mathbf{F}_\mathrm{n}}{m_\mathrm{n}}\cdot\frac{\partial f_\mathrm{n}}{\partial \mathbf{v}} = X(f_\mathrm{n}, f_\mathrm{i}) + Y(f_\mathrm{i}) - Z(f_\mathrm{n})$$

$$\frac{\partial f_\mathrm{i}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{i}}{\partial \mathbf{r}} + \frac{\mathbf{F}_\mathrm{i}}{m_\mathrm{i}}\cdot\frac{\partial f_\mathrm{i}}{\partial \mathbf{v}} = C(f_\mathrm{i}) - X(f_\mathrm{n}, f_\mathrm{i}) - Y(f_\mathrm{i}) + Z(f_\mathrm{n})$$

$$\frac{\partial f_\mathrm{e}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{e}}{\partial \mathbf{r}} + \frac{\mathbf{F}_\mathrm{e}}{m_\mathrm{e}}\cdot\frac{\partial f_\mathrm{e}}{\partial \mathbf{v}} = C(f_\mathrm{e}) - Y(f_\mathrm{e}, f_\mathrm{i}) + Z(f_\mathrm{e}, f_\mathrm{n})$$

Recom: $Y(f_\mathrm{e}, f_\mathrm{i}) = \int d\mathbf{v}'\sigma_\mathrm{r}|\mathbf{v}-\mathbf{v}'|f_\mathrm{i}(\mathbf{v}')f_\mathrm{e}(\mathbf{v})$

$$Y(f_\mathrm{i}) = Y(f_\mathrm{i}, f_\mathrm{e}) = \int d\mathbf{v}'\sigma_\mathrm{r}|\mathbf{v}-\mathbf{v}'|f_\mathrm{e}(\mathbf{v}')f_\mathrm{i}(\mathbf{v}) \approx n_\mathrm{e}\langle\sigma_\mathrm{r} v_\mathrm{e}\rangle f_\mathrm{i}(\mathbf{v})$$

Ioniz: $Z(f_\mathrm{e}, f_\mathrm{n}) = \int d\mathbf{v}'\sigma_\mathrm{z}|\mathbf{v}-\mathbf{v}'|f_\mathrm{e}(\mathbf{v}_\mathrm{f})f_\mathrm{i}(\mathbf{v}') - f_\mathrm{e}(\mathbf{v})f_\mathrm{n}(\mathbf{v}')] + \frac{\delta f_\mathrm{e}^\mathrm{created}(\mathbf{v})}{\delta t}$

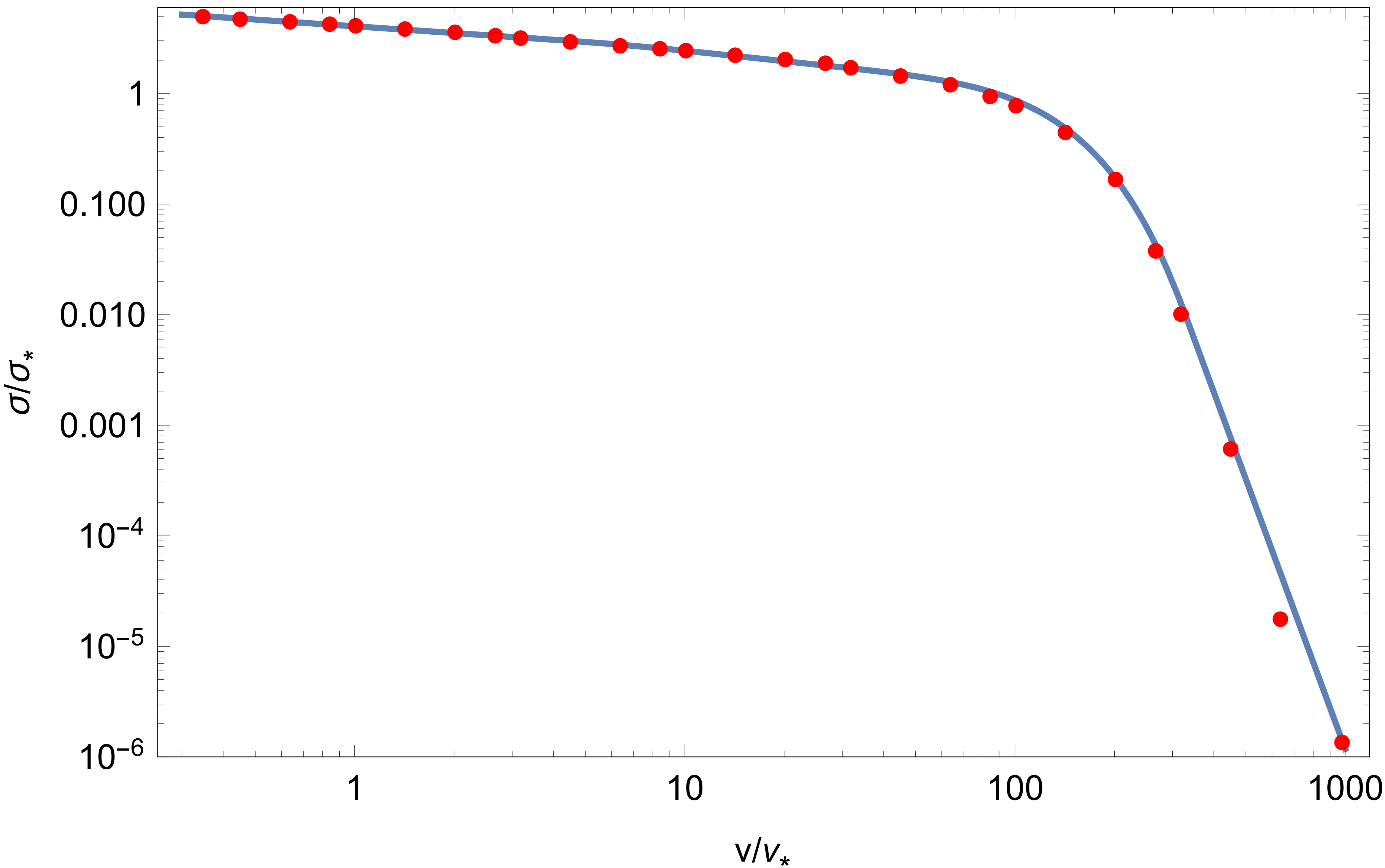
$$Z(f_\mathrm{n}) = Z(f_\mathrm{n}, f_\mathrm{e}) = \int d\mathbf{v}'\sigma_\mathrm{z}|\mathbf{v}-\mathbf{v}'|f_\mathrm{e}(\mathbf{v}')f_\mathrm{n}(\mathbf{v}) \approx n_\mathrm{e}\langle\sigma_\mathrm{z} v_\mathrm{e}\rangle f_\mathrm{n}(\mathbf{v})$$

McWhirter 1965 $\langle\sigma_\mathrm{r} v_\mathrm{e}\rangle = 7.57\times 10^{-18}\sqrt{\frac{13.6\mathrm{eV}}{T_\mathrm{e}}}\mathrm{m}^3/\mathrm{s}$ (not 0.7×10^{-19})

Voronov 1997 $\langle\sigma_\mathrm{z} v_\mathrm{e}\rangle = 2.91\times 10^{-14}\frac{(13.6\mathrm{eV}/T_\mathrm{e})^{0.39}}{0.232 + 13.6\mathrm{eV}/T_\mathrm{e}}\exp\left(-\frac{13.6\mathrm{eV}}{T_\mathrm{e}}\right)\mathrm{m}^3/\mathrm{s}$

Cross section for the charge exchange operator

Atomic data for fusion vol. 1, C. F. Barnett, ORNL 1990



$v_\star = 1.38 \times 10^4 \mathrm{m/s}$ and $\sigma_\star = 10^{-19} \mathrm{m}^2$

Moments of the charge exchange operator $(a,b) = (\mathrm{n},\mathrm{i})$ or (i,n)

$$X(f_a, f_b) = \int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|[f_a(\mathbf{v}')f_b(\mathbf{v}) - f_a(\mathbf{v})f_b(\mathbf{v}')]$$

$$\hat{A}^{lpk}_{ab}\mathsf{N}^{lk}_a = \int d\mathbf{v}\hat{\mathsf{P}}^{lp}_a X(f^0_a\hat{\mathsf{P}}^{lk}_a\cdot\mathsf{M}^{lk}_a, f^0_b)$$

$$\hat{B}^{lpk}_{ab}\mathsf{N}^{lk}_b = \int d\mathbf{v}\hat{\mathsf{P}}^{lp}_a X(f^0_a, f^0_b\hat{\mathsf{P}}^{lk}_b\cdot\mathsf{M}^{lk}_b)$$

$$\int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|f^{lk}_b(\mathbf{v}') = n_b\sigma_\star\mathsf{P}^l(\hat{\mathbf{v}})\cdot\mathsf{M}^{lk}_b S^{lk}_{ab},\ \hat{w}_a = \frac{\frac{1}{2}m_a v^2}{T_a} = s^2_a$$

$$S^{lk}_{ab} = \frac{1}{\pi^{1/2}}\underbrace{\int_0^\infty d\hat{w}'\hat{w}'^{(l+1)/2}L^l_k(\hat{w}')e^{-\hat{w}'}}_{\text{Gauss−Laguerre}}\underbrace{\int_{-1}^{1} d\xi' P_l(\xi')\hat{\sigma}_X|\mathbf{v}-\mathbf{v}'|}_{\text{Gauss−Legendre}}$$

$$\sigma_l A^{lpk}_{ab}\mathsf{M}^{lk}_a = -\int d\mathbf{v}\mathsf{P}^{lp}_a(\mathbf{s}_a)\int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|[f^{lk}_a(\mathbf{v})f^0_b(\mathbf{v}') - f^{lk}_a(\mathbf{v}')f^0_b(\mathbf{v})]$$

$$= -\sigma_l n_b\sigma_\star\mathsf{N}^{lk}_a\frac{2}{\sqrt{\pi}}\int d\hat{w}\hat{w}^{(1+l)/2}L^l_p(\hat{w})e^{-\hat{w}}$$

$$\times\left[\hat{w}^{l/2}L^l_k(\hat{w})S^{00}_{ab} - S^{lk}_{aa}(T^a_b)^{3/2}e^{\hat{w}(1-T^a_b)}\right]$$

$$\sigma_l B^{lpk}_{ab}\mathsf{M}^{lk}_a = -\int d\mathbf{v}\mathsf{P}^{lp}_a(\mathbf{s}_a)\int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|[f^0_a(\mathbf{v})f^{lk}_b(\mathbf{v}') - f^0_a(\mathbf{v}')f^{lk}_b(\mathbf{v})]$$

$$= -\sigma_l n_a\sigma_\star\mathsf{N}^{lk}_b\frac{2}{\sqrt{\pi}}\int d\hat{w}\hat{w}^{(1+l)/2}L^l_p(\hat{w})e^{-\hat{w}}$$

Ion closures: kinetic equation

- Sum of neutral and ion equations

$$\frac{\partial f_\mathrm{i}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{i}}{\partial \mathbf{r}} + \mathbf{a}_\mathrm{i}\cdot\frac{\partial f_\mathrm{i}}{\partial \mathbf{v}} + \frac{\partial f_\mathrm{n}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{n}}{\partial \mathbf{r}} + \mathbf{a}_\mathrm{n}\cdot\frac{\partial f_\mathrm{n}}{\partial \mathbf{v}} = C(f_\mathrm{i})$$

- Set $f = f^\mathrm{M} + f^\mathrm{N}$

  Solve the kinetic equation to express f^N in terms of f^M

  Take closure moments to express closures in terms of thermodynamic drives

- Asymptotic closure scheme

  Adopt the closure ordering: $\dfrac{\partial f^\mathrm{N}}{\partial t} = 0$

  Ignore $\mathbf{v}\cdot\dfrac{\partial f^\mathrm{N}}{\partial \mathbf{r}}$ and $\dfrac{q}{m}\mathbf{E}\cdot\dfrac{\partial f^\mathrm{N}}{\partial \mathbf{v}}$ $\Leftarrow$ only for high collisionality

$$\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}(f_\mathrm{i}^\mathrm{M} + f_\mathrm{n}^\mathrm{M}) - \frac{q_\mathrm{i}}{m_\mathrm{i}}\mathbf{v}\times\mathbf{B}\cdot\frac{\partial f_\mathrm{i}^\mathrm{N}}{\partial \mathbf{v}} = C(f_\mathrm{i}^\mathrm{M}, f_\mathrm{e}^\mathrm{M}) + C_\mathrm{iL}(f_\mathrm{i}^\mathrm{N}) + \mathrm{Maxwellian}$$

Ion closures: moment equations

- Take moments of $C_{\mathrm{iL}}(f_\mathrm{i}^\mathrm{N}) - \frac{q_\mathrm{i}}{m_\mathrm{i}}\mathbf{v}\times\mathbf{B}\cdot\frac{\partial f_\mathrm{i}}{\partial\mathbf{v}} = -\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}(f_\mathrm{i}^\mathrm{M}+f_\mathrm{n}^\mathrm{M}) + C(f_\mathrm{i}^\mathrm{M}, f_\mathrm{e}^\mathrm{M})$

$$-\mathsf{C}_\mathrm{i}^l\mathsf{N}_\mathrm{i}^l + \Omega_\mathrm{i}\mathbf{b}\breve{\times}\mathsf{N}_\mathrm{i}^l = \mathsf{G}_\mathrm{i}^{\nabla,l} + \mathsf{G}_\mathrm{n}^{\nabla,l} \;\Rightarrow\; \nabla(T_\mathrm{i}+T_\mathrm{n}) \text{ and } \mathsf{W}_\mathrm{i}+\mathsf{W}_\mathrm{n}$$

$$l=1:\; -\frac{1}{\tau_\mathrm{ii}}\mathsf{c}_\mathrm{i}\mathsf{N}_\mathrm{i}^1 + \Omega_\mathrm{i}\mathbf{b}\times\mathsf{N}_\mathrm{i}^1 = \mathsf{g}^1\left(\frac{n_\mathrm{i}v_{T\mathrm{i}}}{T_\mathrm{i}}\nabla T_\mathrm{i} + \frac{n_\mathrm{n}v_{T\mathrm{n}}}{T_\mathrm{n}}\mathsf{g}^1\nabla T_\mathrm{n}\right),\; \mathsf{g}^1 \equiv \begin{pmatrix}\frac{\sqrt{5}}{2}\\ 0\\ \vdots\end{pmatrix}$$

- General solution of a set of vector equations
  Define $\mathbf{g}_\| \equiv \mathbf{bb}\cdot\mathbf{g},\; \mathbf{g}_\times \equiv \mathbf{b}\times\mathbf{g},\; \mathbf{g}_\perp \equiv -\mathbf{b}\times(\mathbf{b}\times\mathbf{g})$

$$-\mathsf{c}\mathbf{N} + r\mathbf{b}\times\mathbf{N} = \mathbf{g} \;\Rightarrow\; \mathbf{N} = -\left[\mathsf{c}^{-1}\mathbf{g}_\| + \frac{1}{\mathsf{c}^2+r^2\mathsf{I}}(r\mathbf{g}_\times + \mathsf{c}\mathbf{g}_\perp)\right]$$

$$\mathsf{N}_{\mathrm{i},a}^1 = -\frac{n_a v_{Ta}\tau_\mathrm{ii}}{T_a}[\mathtt{igl}\nabla_\| T_a + \mathtt{igx}\nabla_\times T_a + \mathtt{igp}\nabla_\perp T_a)]$$

$$\mathsf{N}_\mathrm{i}^1 = \begin{pmatrix}\mathsf{N}_\mathrm{i}^{11}(v^2\mathbf{v})\\ \mathsf{N}_\mathrm{i}^{12}(v^4\mathbf{v})\\ \vdots\end{pmatrix} = \mathsf{N}_{\mathrm{i,i}}^1 + \mathsf{N}_{\mathrm{i,n}}^1 \;\Rightarrow\; \begin{matrix}\mathbf{h} = -\frac{\sqrt{5}}{2}Tv_T\mathsf{N}^{11}\\ \mathbf{R}_\mathrm{ni} \propto \sum_{k=1}(\hat{A}_\mathrm{ni}^{10k}\mathsf{N}_\mathrm{n}^{1k} + \hat{B}_\mathrm{ni}^{10k}\mathsf{N}_\mathrm{i}^{1k})\end{matrix}$$

Neutral closures

$$\mathbf{v}\cdot\frac{\partial f_{\rm n}^{\rm M}}{\partial\mathbf{r}} = X(f_{\rm n}^{\rm M},f_{\rm i}^{\rm M}) + X(f_{\rm n}^{\rm N},f_{\rm i}^{\rm M}) + X(f_{\rm n}^{\rm M},f_{\rm i}^{\rm N}) + Y(f_{\rm i}^{\rm N}) - Z(f_{\rm n}^{\rm N})$$

Take moments: $-\mathsf{G}^{l}_{\nabla,\rm n} = \mathsf{G}^{l}_{X,\rm n} + \mathsf{A}^{l}_{\rm ni}\mathsf{N}^{l}_{\rm n} + \mathsf{B}^{l}_{\rm ni}\mathsf{N}^{l}_{\rm i} + \mathsf{Y}^{l}_{\rm ni}\mathsf{N}^{l}_{\rm i} - \mathsf{Z}^{l}_{\rm nn}\mathsf{N}^{l}_{\rm n}$

$$\underbrace{\left(-\mathsf{a}^{1}_{\rm ni}+\frac{\tau_X}{\tau_Z}\mathsf{z}^{1}_{\rm nn}\right)}_{\mathtt{AZ}}\mathsf{N}^{1}_{\rm n} = \frac{n_{\rm n}v_{T\rm n}\tau_X}{T_{\rm n}}\mathsf{g}^{1}\nabla T_{\rm n} + \mathsf{a}^{1.0}_{\rm ni}\frac{\sqrt{2}n_{\rm n}\mathbf{V}_{\rm n}}{v_{T\rm n}} + \mathsf{b}^{1.0}_{\rm ni}\frac{\sqrt{2}n_{\rm n}\mathbf{V}_{\rm i}}{v_{T\rm i}} + \left(\mathsf{b}^{1}_{\rm ni}\frac{n_{\rm n}}{n_{\rm i}}+\frac{\tau_X}{\tau_Y}\mathsf{y}^{1}_{\rm ni}\right)\mathsf{N}^{1}_{\rm i}$$

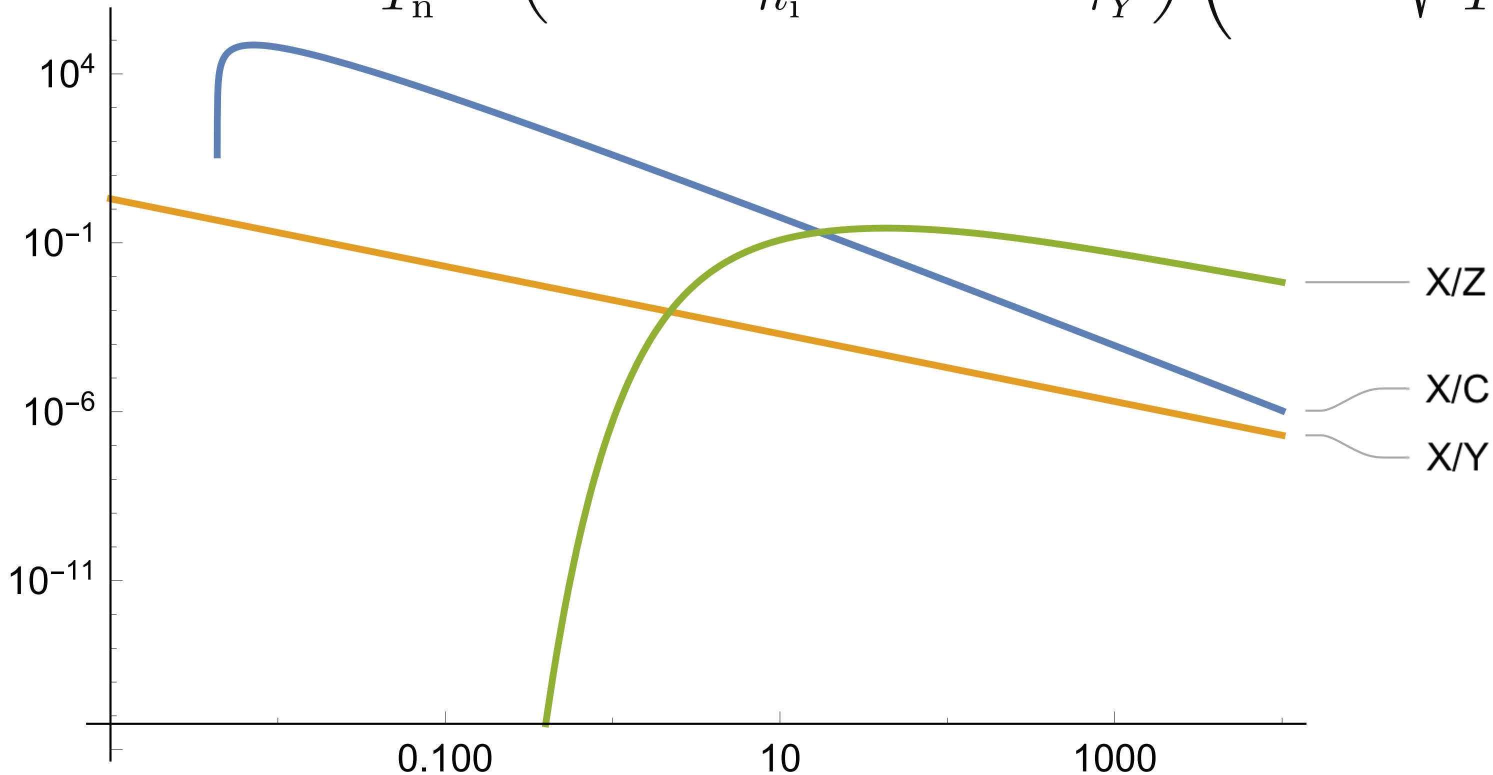
$$\begin{aligned}\mathsf{N}^{1}_{\rm n} \quad = \quad & \frac{n_{\rm n}v_{T\rm n}\tau_X}{T_{\rm n}}\left(\mathtt{ngn}\nabla T_{\rm n} + \mathtt{nvn}\frac{\sqrt{2}n_{\rm n}\mathbf{V}_{\rm n}}{v_{T\rm n}} + \mathtt{nvi}\frac{\sqrt{2}n_{\rm n}\mathbf{V}_{\rm i}}{v_{T\rm i}}\right)\\ & -\frac{n_{\rm n}v_{T\rm n}\tau_{\rm ii}}{T_{\rm n}}\left(\mathtt{ngl}\nabla_\parallel T_{\rm n} + \mathtt{ngx}\nabla_\times T_{\rm n} + \mathtt{ngp}\nabla_\perp T_{\rm n}\right)\\ & -\frac{n_{\rm i}v_{T\rm i}\tau_{\rm ii}}{T_{\rm i}}\left(\mathtt{ngl}\nabla_\parallel T_{\rm i} + \mathtt{ngx}\nabla_\times T_{\rm i} + \mathtt{ngp}\nabla_\perp T_{\rm i}\right)\end{aligned}$$

$$\mathsf{N}^{1}_{\rm n} = \frac{n_{\rm n}v_{T\rm n}\tau_X}{T_{\rm n}}\left(\mathtt{ngn}\nabla_\parallel T_{\rm n} + \mathtt{nvn}\frac{\sqrt{2}n_{\rm n}\mathbf{V}_{\rm ni}}{v_{T\rm n}}\right) - \frac{n_{\rm n}v_{T\rm n}\tau_{\rm ii}}{T_{\rm n}}\mathtt{ngl}\left(\nabla_\parallel T_{\rm n} + \sqrt{\frac{T_{\rm n}}{T_{\rm i}}}\frac{n_{\rm i}}{n_{\rm n}}\nabla_\parallel T_{\rm i}\right) + \ldots$$

where $\mathtt{ngl} = \mathtt{AZ}^{-1}\mathsf{b}^{1}_{\rm ni}\mathtt{igl}\frac{n_{\rm n}}{n_{\rm i}} + \mathtt{AZ}^{-1}\mathsf{y}^{1}_{\rm ni}\mathtt{igl}\frac{\tau_X}{\tau_Y} \equiv \mathtt{nglbigl}\frac{n_{\rm n}}{n_{\rm i}} + \mathtt{nglyigl}\frac{\tau_X}{\tau_Y}$

Neutral closures (cont.)

$$\mathrm{N}_\mathrm{n}^1 = \frac{n_\mathrm{n} v_{T\mathrm{n}} \tau_X}{T_\mathrm{n}} \left(\mathtt{ngn} \nabla_\parallel T_\mathrm{n} + \mathtt{nvn} \frac{\sqrt{2} n_\mathrm{n} \mathbf{V}_\mathrm{ni}}{v_{T\mathrm{n}}} \right) - \frac{n_\mathrm{n} v_{T\mathrm{n}} \tau_\mathrm{ii}}{T_\mathrm{n}} \left(\mathtt{nglbig1} \frac{n_\mathrm{n}}{n_\mathrm{i}} + \mathtt{nglyig1} \frac{\tau_X}{\tau_Y} \right) \left(\nabla_\parallel T_\mathrm{n} + \sqrt{\frac{T_\mathrm{n}}{T_\mathrm{i}}} \frac{n_\mathrm{i}}{n_\mathrm{n}} \nabla_\parallel T_\mathrm{i} \right) + \ldots$$



T (eV)	0.03	1	10	100	1 k	10 k
τ_X/τ_Y	0.067	2.0E-3	2.0E-4	2.0E-5	2.0E-6	2.0E-7
τ_X/τ_Z	0	5.2E-7	0.12	0.23	0.050	0.0069
τ_X/τ_{ii}	1.6E4	40	0.57	0.0074	9.1E-5	1.1E-6

Convergence with Gaussian nodes and moments ($T = 100\text{eV}$)
Convergent results: heat flow vs. temperature

$$\mathrm{N}_\mathrm{n}^1 = \frac{n_\mathrm{n} v_{T\mathrm{n}} \tau_X}{T_\mathrm{n}} \left(\mathtt{ngn} \nabla_\parallel T_\mathrm{n} + \mathtt{nvn} \frac{\sqrt{2} n_\mathrm{n} \mathbf{V}_\mathrm{ni}}{v_{T\mathrm{n}}} \right) - \frac{n_\mathrm{n} v_{T\mathrm{n}} \tau_\mathrm{ii}}{T_\mathrm{n}} \left(\mathtt{nglbigl} \frac{n_\mathrm{n}}{n_\mathrm{i}} + \mathtt{nglyigl} \frac{\tau_X}{\tau_Y} \right) \cdots$$

Gauss-Legendre nodes $= 10$ (or 20), ($n_w =$Gauss-Laguerre nodes), and moments M

ngn			(20)	(20)	nvn				
($n_w = 10$)	(20)	(40)	3M	4M	(10)	(20)	(40)	3M	4M
1.732	1.734	1.735	1.735	1.736	0.0689	0.0685	0.0684	0.0687	0.0687
0.174	0.173	0.172	0.179	0.180	0.0154	0.0150	0.0150	0.0156	0.0157
			0.049	0.052				0.0049	0.0049
				0.018					0.0018
nglbigl					nglyigl				
−1.371	−1.370	−1.369	−1.374	−1.375	−3.133	−3.137	−3.139	−3.140	−3.141
0.508	0.508	0.507	0.543	0.542	0.633	0.634	0.635	0.697	0.696
			−0.060	−0.043				−0.078	−0.054
				−0.021					−0.046

Heat flow results (3 vector moment approximation)

T (eV)	0.03	1	10	100	1 k	10 k
ngn	1.05	1.50	1.59	1.73	3.81	34.7
nvn	0.127	0.127	0.0968	0.0685	0.0391	-0.305
nglbigl	-2.09	-2.09	-1.75	-1.37	-1.74	-1.57
nglyigl	-1.85	-2.63	-2.84	-3.14	-7.01	-76.8

Future work

- Closure coefficients for high collisionality
  - Number of parameters

$$n_{\mathrm{i}},\ n_{\mathrm{n}},\ ,T_{\mathrm{i}},\ T_{\mathrm{n}} \Rightarrow T_{\mathrm{i}},\ T_{\mathrm{n}}$$

  - Obtain functions of T_{i} and T_{n} fitted to closure coefficients
- Low collisionality
  - Combine with collisionless-limit closures

Thanks!

Enjoy Moments and

Team Building Exercise!
(the NIMROD open)