Progress on Integral Parallel Ion Stress in NIMROD

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Momentum diffusion is anisotropic in low collisionality plasmas.

- Closure of interest should be derived from full CEL drift kinetic equation.

- For this talk, consider

\[
\left( \vec{v}_\parallel \cdot \vec{\nabla} \right) \tilde{F} - \left\langle C(\tilde{F} + \tilde{f}_M) \right\rangle = v_\parallel \left( \hat{b} \cdot \vec{\nabla} \cdot \tilde{\Pi}_\parallel - \tilde{R}_\parallel \right) \frac{f^0_M}{p^0} \\
- \frac{m}{T^0}(\hat{b}\hat{b} - \frac{I}{3}) : \nabla \parallel \vec{v} \left( v^2_\parallel - \frac{v^2_{\perp}}{2} \right) f^0_M
\]

- Invert free-streaming + momentum-restoring collision operator to solve for \( \tilde{F} \) assuming:
  1. sheared slab geometry,
  2. \( \partial / \partial t \rightarrow 0 \),
  3. no heat flow terms.
Low collisionality parallel stress takes integral form.

Result is following integral form for $\pi_\parallel$:

$$K_{21}(U_\parallel) + (1 + B_2)\pi_\parallel + K_{22}(\pi_\parallel) =$$

$$\int_0^\infty d\tilde{L} \left( V_\parallel(L + \tilde{L}) - V_\parallel(L - \tilde{L}) \right) \frac{\partial K_2(\tilde{L})}{\partial \tilde{L}},$$

$$K_{11}(U_\parallel) + K_{12}(\pi_\parallel) =$$

$$\int_0^\infty d\tilde{L} \left( V_\parallel(L + \tilde{L}) + V_\parallel(L - \tilde{L}) \right) \frac{\partial K_1(\tilde{L})}{\partial \tilde{L}} + B_1 V_\parallel(L),$$

$\pi_\parallel$ appears on right side of flow evolution equation:

$$\rho \frac{d\vec{V}}{dt} = \vec{J} \times \vec{B} - \vec{\nabla}p - \vec{\nabla} \cdot (\Pi_\parallel + \Pi_{gv}),$$

where $\Pi_\parallel = (\hat{b}\hat{b} - I/3)\pi_\parallel$. 
Need semi-implicit operator to stabilize explicit integral stress.

- NIMROD has used $\Pi = -\rho \nu \nabla \cdot V$ in past to introduce viscous dissipation.

- Upgrade of isotropic form to

$$
\Pi = -\rho \nu (\nabla \cdot V + (\nabla \times V)^T - \nabla \cdot V / 3)
$$

in progress. Full Braginskii stress including gyroviscosity in progress.

- For the purpose of this talk consider evolving:

$$
\left( \rho^n - \Delta t f \nabla \cdot (\rho^n \nu \nabla - \Pi_{si}) \right) \Delta V = \Delta t \nabla \cdot \left( \rho^n \nu \nabla V^n - \Pi_{||} \right),
$$

where $\Pi_{||} = -\rho \nu_{||} (\hat{b} \hat{b} - I / 3) \hat{b} \hat{b} : \nabla \cdot V$ and $\Pi_{si}$ is analogous but acts on $\Delta V$. 

Progress on Integral Parallel Ion Stress in NIMROD – p.4/7
Test fully implicit advance in slab geometry.

Flux of $\phi$ momentum (into page) at left boundary in $\vec{\nabla} R$ direction.
Parallel flow damping depends on ratio $\nu_\parallel / \nu$.

Slight damping evident for $\nu_\parallel / \nu = 10^4$. 
Parallel flow damping depends on ratio $\nu_\parallel / \nu$.

- Damping evident for $\nu_\parallel / \nu = 10^5$. 

![Graph showing parallel flow damping](image-url)
Parallel flow damping depends on ratio $\nu_\parallel / \nu$.

- Large damping for $\nu_\parallel / \nu = 10^6$. 
Incorporate $\Pi_\parallel$ into fluid codes, typically $\nu_\parallel/\nu \geq 10^5$.

- For low collisionality regimes,
  \[ \nu_\parallel = \frac{v_{thi}}{k_\parallel} \approx \frac{10^5 - 10^6}{0.01 - 10} \approx 10^4 - 10^8. \]

- Comparison of low collisionality $\pi_\parallel$ with gyroviscosity yields:
  \[ \frac{\pi_\perp}{\pi_\parallel} = \frac{\rho_i v_{thi} \nabla_\perp V}{v_{thi} k_\parallel^{-1} \nabla_\parallel V} = \rho_i \nabla_\perp \]

- To do list:
  1. Finish implementation of 2-D preconditioner for anisotropic momentum diffusion operator.
  2. Implement integral closure and test effectiveness of semi-implicit stabilization.
  3. Implement viscous heating using integral $\Pi_\parallel$ term in temperature evolution.