Numerical Simulation of Electrostatic Current Drive in Spheromaks and Spherical Tori
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Outline

• Introduction
  – Spheromak background
  – Need for simulation
• SSPX modeling
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  – Comparison with SSPX results
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  – Discussion
• Current filament modeling
• Conclusions

Also see posters by G. A. Cone (HP1.075) and E. B. Hooper (HP1.070) and the invited talk by B. I. Cohen (PI1B.003, Thursday morning).
The most successful spheromak formation scheme uses electrodes impregnated with vacuum poloidal flux.

- Slow formation was a major conceptual breakthrough [Jarboe, et al., PRL 51, 39 (1983)].

- Most theoretical descriptions have been based on relaxation arguments [Taylor, PRL 33, 1139 (1974); Jarboe, PPCF 36, 945 (1994)].
  - No information on fluctuations
  - Sustainment described by global helicity balance and cascades

- During drive, $T_e < 50$ eV, but during decay or partial drive, $T_e >> 100$ eV has been recorded.

Schematic of the SSPX spheromak experiment at LLNL with contours of reconstructed symmetric poloidal flux.
While relaxation theory provides insight, numerical computation is required to solve the time-dependent nonlinear equations that describe macroscopic evolution.


- Simulations of generic spheromaks at 0-\(\beta\) addressed MHD activity underlying formation and sustainment [Finn, PRL 85, 4538 (2000), Sovinec, Phys. Plasmas 8, 475 (2001)].
  - Flux amplification results from \(n=1\) MHD activity.
  - Average parallel current is flattened by a dynamo effect.
  - Chaotic scattering of field-lines occurs during sustainment.
  - Closed flux surfaces form during decay.
  - Confinement was not addressed directly.
Recent computations evolve temperature and number density, in addition to magnetic field and plasma flow velocity, to investigate energy confinement properties.

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \mathbf{J}) \quad \text{Faraday’s/Ohm’s laws}
\]
\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \quad \text{low-}\omega \text{ Ampere’s law}
\]
\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \nu \rho \nabla \mathbf{V} \quad \text{flow evolution}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n \quad \text{particle continuity}
\]
\[
\frac{n}{\gamma - 1} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \mathbf{V} \cdot \nabla + \nabla \cdot n \left[ \chi_{\parallel} \mathbf{b} + \chi_{\perp} (\mathbf{I} - \mathbf{b}) \right] \cdot \nabla T + \frac{\eta J^2}{2} \quad \text{(single) temperature evolution}
\]
\[
\hat{\mathbf{b}} \equiv \mathbf{B}/|\mathbf{B}| \quad \text{local magnetic direction vector}
\]

- Braginskii transport coefficients are used for \( \chi_{\parallel} \) (electron), \( \chi_{\perp} \) (ion), and \( \eta \).
- Heating is Ohmic.
- The NIMROD code [http://nimrodteam.org] evolves the system in 3D.
  - High-order finite elements help resolve anisotropies [JCP 195, 355 (2004)].
Earlier 0-β MHD Simulation Results

- Nonlinear effects from saturation of the \( n=1 \) lead to formation and sustainment.
  - poloidal flux amplification (conversion from toroidal flux)
  - parallel current profile is relaxed
  - MHD dynamo effect sustains toroidal current
  - symmetry-breaking leaves \( \mathbf{B} \) chaotic in most sustained conditions

- There are four different classes of nonlinear states in an \( S \)-Voltage phase diagram.

- Closed flux surfaces with net current form during decay.

- Related numerical studies include:
  - DC injection in RFPs: [Ho, NF 31, 341 (1991); Sovinec, PhD dissertation, UW (1995)]
  - Solar physics: [Lau, PoP 3, 3983 (1996); Lionello, PoP 5, 3722 (1998)]
  - Tokamak current drive: [Sovinec, PoP 3, 1038 (1996); Tang, PoP 11, 2679 (2004)]
Three-dimensional computations in can and gun-driven configurations reproduce flux amplification/spheromak formation.

Two-dimensional computations in a can geometry show pinching without flux conversion.

MHD activity in 3D computations reproduce amplification of the symmetric poloidal flux ($\Psi_0$).
An $n=1$ MHD mode driven by open-field current lies at the heart of the flux conversion.

- This mode can be resistive or ideal depending on how hard the pinch is driven.
- It has been observed during drive in all electrode-driven experiments; thorough measurements were performed on SPHEX [Duck, *et al.*, PPCF 39, 715 (1997)].

Real($v_n$) of the linear $n=1$ mode. Imag($v_n$) of the linear $n=1$ mode.
Saturation of the $n=1$ leads to several important effects in addition to poloidal flux amplification.

1) The symmetric projection of parallel current density ($\mu_0 a J \cdot B / B^2$) is distributed throughout the volume (relaxation).

Examining 3D distributions, the parallel current density is carried by a helical current channel, reminiscent of the “doughhook” observed in SPHEX. [Duck, PPCF 39, 715 (1997).]
Effects from Saturation, Continued

2) When sustained, the fluctuations generate MHD dynamo effects that maintain toroidal current against resistive dissipation.

- Dynamo was measured in SPHEX, each $n$ was thought to represent separate activity [al-Karkhy, et al., PRL 70, 1814 (1993)].
- NIMROD simulations show $n>1$ to be nonlinearly generated by $n=1$ during sustainment.
Effects from Saturation, Continued

3) As the MHD fluctuations reconnect magnetic field-lines and break the toroidal symmetric of the pinched open-field current column, they ensnarl field-lines throughout the volume.

B-field traces before saturation.  
B-field traces after saturation.

- The entanglement and reconnection of injected toroidal flux with the bias poloidal flux leads to poloidal flux amplification.
- There is some confinement on the open fields (discussed later).
Nonlinear behavior at different sustained voltages and resistivity values can be summarized in a phase diagram.

Phase diagram of “can” results. Symmetric state exists at low $S$, $V_a$.

\[
\begin{align*}
V_a & \equiv \frac{\tau_r \int E_a \cdot dL}{\Psi_e} \\
S & \equiv \frac{\tau_r}{\tau_A}
\end{align*}
\]

Limit cycle behavior of magnetic fluctuation energies in a sustained “can” configuration at $S=5000$. 

- Steady, closed flux
- Steady, chaotic field
- Unsteady, limit cycle
When drive is removed from sustained conditions, closed flux surfaces often form.

- Magnetic fluctuations decay faster than the mean poloidal flux generated by the flux conversion.

- Slow decay of mean poloidal flux acts as an Ohmic drive for toroidal current on the flux surfaces.

- These combined transient effects were proposed as the reason underlying temperature increases during decay in experiments [PoP 8, 475 (2002)].
Simulation of SSPX 4620-4644 Shot Series

**INPUT** (Collisional coefficients are based on Hydrogen and Z=1):

- \( n = 5 \times 10^{19} \text{ m}^{-3} \)
- \( \frac{\eta(T)}{\mu_0} = 411 \left( \frac{1 \text{ eV}}{T} \right)^{3/2} \text{ m}^2/\text{s} \)
- \( \chi_{\parallel}(T) = 387 \left( \frac{T}{1 \text{ eV}} \right)^{5/2} \text{ m}^2/\text{s} \)
- \( \chi_{\perp} = 0.50 \left( \frac{1 \text{ eV}}{T} \right)^{1/2} \left( \frac{1 \text{ T}}{B} \right)^2 \text{ m}^2/\text{s} \)
- \( T_{\text{wall}} = 0.1 \text{ eV} \)
- \( \psi_{\text{vacuum}} \) specified
- \( I_{\text{inj}}(t) \) via boundary conditions on \( B \)
- Heat sink controls boundary layer
- \( \nu = D = 2000 \text{ m}^2/\text{s} \)

**OUTPUT:** Everything else

Initial (vacuum) poloidal flux distribution and the NIMROD mesh of bicubic finite elements representing SSPX (upside down).

- Fourier comps. in \( \phi \): \( 0 \leq n \leq 2 \) during initial formation, \( 0 \leq n \leq 5 \) thereafter.
Validity of Collisional Transport

- 0-β MHD results suggest that open-field transport governs confinement during driven conditions.
  - Collisional 3D transport modeling is appropriate for a chaotic B topologies if the effective mean-free-path is sufficiently small.
  - We can confirm *a posteriori* that collisionless conditions only exist when and where closed flux surfaces form.

- At $n = 5 \times 10^{19} \text{ m}^{-3}$ and $T = 1 \text{ eV}$,
  \[ \nu_{Te} \approx 6 \times 10^5 \text{ m/s} \]
  \[ \tau_e \approx 7 \times 10^{-10} \text{ s} \]
  \[ \lambda_e \approx 4 \times 10^{-4} \text{ m} \ll L \]

- Scaling $\lambda_e$ with $T^2$ indicates that $\lambda_e$ reaches the chamber radius at approximately 35 eV.

- From this we infer that anisotropic thermal conduction is a good model for sustained (open field) conditions and for the transition to closed flux during decay.
Comparison of Numerical and Experimental Results

The simulated injector current is programmed to approximate the series of SSPX discharges reported in [McLean, et al., PRL 88, 125004-1 (2002)].

INPUT: A strongly driven phase is followed by decay and then a second, partial drive. [SSPX Data courtesy of H. S. McLean.]

- The peak instantaneous power input reaches ~1 GW in the formation stage.

During formation, the applied potential reaches a few kV. During the second pulse, the potential is ~200 V in SSPX [including 100-150 V of sheath, Hooper, Stallard] and 20 V in the simulation.
Toroidal current and magnetic energy evolution from the simulations are similar to results found by CORSICA fits to laboratory observations [Hooper et al., NF 39, 863 (1999)] during the second current pulse.

$I_{\text{tor}}$ resulting from the series of NIMROD simulations is compared with $I_{\text{tor}}$ from CORSICA equilibrium fits of SSPX data.

- In the simulation, the second current pulse provides 4 MW of power, and the decay of magnetic energy provides an additional 1.4 MW.
The SSPX simulations show amplification of poloidal flux via the current-driven $n=1$ mode, similar to the $0-\beta$ can geometry simulations.

Contour plots of the poloidal magnetic flux function for

(a) the initial vacuum distribution,

(b) the pinched state at $t=0.08$ ms before becoming unstable,

(c) the relaxed state with $I_{\text{inj}}=400$ kA at $t=0.12$ ms, and

(d) the partially driven state at $t=1.2$ ms.

Dashed contour levels indicate poloidal flux converted from toroidal flux by MHD activity.
Both simulation and experiment show a quiescent phase when the second current pulse is applied after a brief period of decay.

Relative poloidal magnetic field fluctuations at the outboard mid-plane position.

- The second current pulse forces fluctuations to smaller amplitude and postpones the emergence of the $n=2$ mode.
- A simulation without the second current pulse shows much larger fluctuation levels, particularly in the $n=2$ component.
Plasma temperature within a toroidally shaped region increases significantly as the magnetic fluctuation level is reduced.

Comparison of temperatures measured with Thomson scattering in SSPX and simulation results with and without the second current pulse.

- The less dramatic response in the simulation may be due to limitations in the collisional transport model as confinement improves.
- The simulation response of 76 eV is also substantial, however. Without the second pulse, the peak temperature is only 49 eV.
Three-dimensional plots of temperature during formation and during the quiescent phase help illustrate changes in confinement.

With chaotic magnetic field-lines, parallel conduction transports heat to the walls, and $T$ is essentially uniform with a maximum of $\sim 35$ eV.  

- Analytical estimate is $\sim 30$ eV. [Hooper, J. Nucl. Materials 278, 104 (2000).]

During the quiescent phase, a ring of hot plasma is surrounded by cold plasma near the wall and along the geometric axis.
Poincaré surfaces of section prove that the magnetic field develops structure, including flux surfaces, during the quiescent phase.

• During formation, the magnetic field shows chaotic scattering as also seen in $0$-$\beta$ simulations of sustained conditions [Finn et al., PRL 85, 4538 (2000)].

• Beginning with the current ramp-down after formation, open field-lines lengthen, and a closed toroidal structure forms during the quiescent phase.
The second current pulse keeps the $q$-profile from falling significantly below 1/2, thereby reducing the impact of the $(m=1,n=2)$ and $(m=2,n=4)$ perturbations.

Safety factor profiles, computed as

$$ q = \frac{d \langle \Phi \rangle}{d \langle \Psi \rangle} $$

in the amplified-flux region, show that the second current pulse maintains a flatter $q$-profile.

Contours of $RB_\phi$ with poloidal flux overlaid from the extended free decay simulation (left) and a two-pulse simulation (right) at $t=1$ ms illustrate how the second pulse maintains larger edge $q$-values.

- The correspondence of MHD activity and resonances in MHD equilibria fitted to SSPX measurements has been noted [Woodruff et al. BAPS 48, 150 (2003)].
Importance of Transients

• Although the second pulse injects half of the current of the formation pulse, dynamo activity is orders of magnitude smaller.

Here we consider the dynamo power density \(-\langle \mathbf{v} \times \mathbf{b} \rangle \cdot \langle \mathbf{J} \rangle\) which contributes to the rhs of

\[
\frac{1}{2\mu_0} \frac{\partial \langle B \rangle^2}{\partial t} + \nabla \cdot \langle \mathbf{E} \rangle \times \langle \mathbf{B} \rangle = -\langle \mathbf{E} \rangle \cdot \langle \mathbf{J} \rangle
\]

At the center of the amplified flux region:

\[
\langle \mathbf{v} \times \mathbf{b} \rangle \cdot \langle \mathbf{J} \rangle \quad \eta \langle \mathbf{J} \rangle^2
\]

Formation \(1 \times 10^9\) W/m\(^3\) \(2 \times 10^7\) W/m\(^3\)

Quiescent \(~10^5\) W/m\(^3\) \(3 \times 10^6\) W/m\(^3\)

Dynamo power density along the midplane during formation and during the quiescent phase.

• Without any drive, the symmetric current would require \(~8\) ms to decay resistively at 50 eV.

• The quiescent phase with closed flux surfaces is not representative of sustainment.
Discussion on SSPX Modeling

- An interplay of transient effects and temperature-dependent transport coefficients produces the low-fluctuation, high-confinement states.
  - The $n=1$ mode of the open-field current channel decays rapidly when the drive is removed—the open-field plasma cools, and the pinch current subsides.
  - Low-resistivity plasma within the hot flux surfaces retains toroidal current associated with the $n=1$-generated poloidal flux.
  - The influence of the MHD activity on the magnetic topology during drive and decay are consistent with earlier 0-$\beta$ simulation results (Finn, et al., PRL 85, 4538, 2002 and Sovinec, et al., PoP 8, 475, 2001).

- The realistic parameters and collisional temperature dependencies make the MHD results consistent with SSPX results:
  - Magnetic fluctuations of $\sim$1 % during partial drive
  - Magnetic energy decay during partial drive
  - Temperature evolution

- The resonant fluctuations during partial should be analyzed with respect to their impact on confinement and not as a mechanism for current drive.
Current Filament Modeling

- Electrostatic current drive through biased probes and miniature plasma guns has been accomplished on CDX, CCT [Darrow, et al., PFB 2, 1415 (1990)], and MST.
- Plasma guns also have potential as a means for non-inductive startup in spherical tori and are now being tested on Pegasus [See poster PP1.020 by Eidietis et al. Thursday afternoon].
- NIMROD simulations are being applied to study the filamentary current channels produced by local sources in vacuum magnetic fields with a large toroidal component.
- Based on the spheromak results, temperature-dependent resistivity is expected to be important, so the combined MHD/collisional energy transport modeling is used.
The first set of current filament simulations model the recent injection tests on Pegasus.

- The mesh represents a 1 m radius chamber with a 5 cm radius center stack.
- Bicubic elements are packed along the current path.
- A finite Fourier series with $0 \leq n \leq 21$ represents the toroidal direction.
- A local ‘hot spot’ of 2 eV imposed as a boundary condition on the bottom surface represents a miniature plasma gun [Fiksel] source.
- Cooling along the bottom surface outside a thin shell containing the hot spot helps maintain a distinct current channel.

- The simulation has the chamber filled with cold but ionized hydrogen at $n=10^{18}$ m$^{-3}$.
- A potential distribution on the bottom surface, similar to the surface temperature distribution, is specified through tangential electric field.
Anisotropic thermal conduction and Ohmic heating produce a spiraling current filament that crosses the chamber axially.

The $2 \text{ eV}$ isosurface extends along the magnetic field from the hot spot on the lower boundary to the top surface.  The $J_{||}/B$ isosurface at $-0.2 \text{ m}^{-1}$ (violet) extends along the warm plasma spiral.  The applied surface electric field also produces a small amount of return current shown at $+0.2 \text{ m}^{-1}$ (red).
The initial results show the expected ratio of toroidal current to injected current.

- With 140 kA of ‘rod’ current and 0.02 T of vertical field, the winding ratio at the location of the current source is

\[ \frac{\Delta z B_\phi}{2\pi R_{inj} B_z} = 3 \]

- With an applied potential of 25 V (~after sheath loss), the simulation shows a net extracted current of 189 A and 555 A of toroidal current. The \( I_\phi / I_{inj} \) ratio matches the winding number.

- Evolution of toroidal current from initial application of the electric potential and hot spot of temperature.

- Although we have not yet observed interesting current amplification effects in the filament simulations, reproducing the magnetic winding at low injector current is a benchmark.

- Scans over a range of applied voltage with a larger winding number will be performed to study current amplification.
Conclusions

• Evolving the complete system with temperature-dependent transport coefficients allows us to assess confinement quality during different stages of spheromak operation.
• Transients play a crucial role in spheromak experiments; thus, modeling injector current programming is necessary for detailed comparison of theory and experiment.
• Relating the $0-\beta$ simulation results to the recent results and from the comparison of the experimental observations with the simulations, we see that only the initial phase of the standard two-stage SSPX operation has the characteristics of full sustainment.
• Initial results on current filament modeling show promise with respect to studying current amplification and non-inductive startup in spherical tori.
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