Neoclassical Tearing Mode simulations with NIMROD. [1]

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Thesis

- **Viscous-stress tensor closures for the simulation of neoclassical tearing modes (NTM's) as implemented in *NIMROD* will be presented and preliminary results presented.**

Outline

- Test problems.

  Poloidal flow damping of an initial equilibrium parallel flow.

  Perturbed bootstrap current generation and a neoclassical tearing mode.

- Closure classification and implementations:

  - CGL-forms

    1) The "Flow and Current" (FC) model.

    2) The "Current Only" (CO) model. (or Flow Only FO).

  - Bootstrap current forms (can be CGL or Flow-damping based.)

    3) The "Perturbed Pressure" (PP) model.

    4) The "Hole" model.

  - Flow-damping forms.

    5) The "Co-variant Poloidal Flow Damping" (PFD_θ) model.

    6) The "Contra-variant Poloidal Flow Damping" (PFD^θ) model.

NIMROD formulation based on single fluid equations.

- The generalized Ohm's law can be expressed as

$$\vec{E} = -\vec{v} \times \vec{B} + \eta \vec{J} + \frac{1}{ne}\frac{1-\nu}{1+\nu}\left(\vec{J} \times \vec{B} - \frac{1}{2}\nabla p\right) + \vec{E}_{neo}$$
$$+\frac{1}{\epsilon_0 \omega_{pe}^2 (1+\nu)}\left[\frac{\partial J}{\partial t} + \nabla \cdot \left(\vec{v}\vec{J} + \vec{J}\vec{v} - \frac{1}{ne}\frac{1-\nu}{1+\nu}\vec{J}\vec{J}\right)\right],$$

E is the electric field, v is the center of mass-velocity,
B is the magnetic field, $\nu = Zm_e/m_i$, Z is the charge of the ion species,
n is the plasma density which is assumed to be constant,
$p = p_e + p_i = 2p_e$ is the scalar pressure,
$\omega_{pe}^2 = ne^2/\epsilon_0 m_e$ is the plasma frequency,
$\vec{E}_{neo} = \frac{-1}{ne(1+\nu)}\nabla \cdot (\vec{\vec{\pi}}_e - \nu\vec{\vec{\pi}}_i)$ is the anisotropic part of the stress tensor.

- The momentum equation is

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} + \frac{1}{n^2 e^2}\frac{\nu(1+Z\nu)}{(1+\nu)^3}\vec{J} \cdot \nabla \vec{J}\right) =$$
$$\vec{J} \times \vec{B} - \nabla P - \mu \nabla^2 \vec{v} + \vec{F}_{neo}$$

ρ is the plasma density,
μ is the kinematic viscosity,
$\vec{F}_{neo} = -\nabla \cdot (\pi_e + \pi_i)$ is the anisotropic portion of the pressure tensor.

- The pressure evolution equation is given by

$$\frac{\partial P}{\partial t} + \vec{v} \cdot \nabla P + \gamma p \nabla \cdot \vec{v} = \chi_\perp \nabla^2 P + \chi_{||}\vec{B} \cdot \nabla\left(\frac{\vec{B} \cdot \nabla P}{B^2}\right),$$

$\chi_\perp$ is the a perpendicular diffusivity,
$\chi_{||}$ is a parallel diffusivity.

- The MHD equations are completed with the non-relativistic Maxwell equations neglecting displacement currents.

The closures must reproduce two physical effects.

- Poloidal flow damping.

  In the absence of diamagnetic/2 fluid effects poloidal flows damp to zero.

  Conservation of toroidal angular momentum.

- Generation of perturbed bootstrap currents.

  Equilibrium bootstrap current buried in equilibrium current.

  For NTM's sole interest is the perturbed bootstrap current.

  Localized pressure flattening in island vicinity should produce perturbed bootstrap current.

1. Flow and Current (FC) model of stress tensor.

- Stress tensor must be evaluated either via a kinetic or higher-order fluid moment approach.
- FC model uses a stress tensor based on a neoclassical closure appropriate for the limit of long collisional mean free path length (or low collision frequency).
- Closure accounts for the viscosity between trapped and untrapped particles.
- The stress tensor, $\vec{\vec{\pi}}$, is represented in a Chew—Goldberger—Low form as

  $$\vec{\vec{\pi}}_\alpha \simeq \vec{\vec{\pi}}_{||\alpha} = \left(\frac{\vec{B}\vec{B}}{B^2} - \frac{\vec{\vec{I}}}{3}\right)(p_{||} - p_\perp)_\alpha,$$

  where

  $$f_\alpha = (p_{||} - p_\perp)_\alpha = -2m_\alpha n_\alpha \mu_\alpha \frac{\langle B^2 \rangle}{\left\langle \left[\frac{\vec{B}\cdot\nabla B^2}{B^2}\right]^2 \right\rangle} \frac{\vec{v}_\alpha \cdot \nabla B^2}{B^2};$$

  the subscript alpha indicates electron's or ions and μ is a poloidal flow damping frequency.
- The closure is linearized in terms of $f = f_0 + f_1$ and $\vec{B} = \vec{B}_0 + \vec{B}_1$.

NIMROD formulation based on single fluid equations.

- The stress tensor in the Ohm's law is

$$\vec{E}_{neo} = \frac{-1}{ne(1+\nu)} \nabla \cdot (\vec{\vec{\pi}}_e - \nu \vec{\vec{\pi}}_i),$$

- A single pressure anisotropy is implied $f = f_e - \nu f_i$ that is expressed in terms of a flow contribution and a current contribution.

where

$$f = N_{p1}^{EQ} \vec{v} \cdot \nabla(B^2) + N_{p2}^{EQ} \vec{J} \cdot \nabla(B^2),$$

$$N_{p1}^{EQ} = \frac{2\langle B^2 \rangle}{\langle \left[\frac{\vec{B} \cdot \nabla B^2}{B^2} \right]^2 \rangle B^2} (\nu m_i \mu_i - m_e \mu_e) n$$

$$N_{p2}^{EQ} = \frac{2\langle B^2 \rangle}{\langle \left[\frac{\vec{B} \cdot \nabla B^2}{B^2} \right]^2 \rangle B^2} \frac{m_e \mu_e + \nu^2 m_i \mu_i}{e(\nu + 1)}$$

The superscript EQ implies evaluated based on equilibrium quantities.

- Stress tensor also appears in momentum balance with the same form but a different $f = f_e + f_i$.

$$f = F_{p1}^{EQ} \vec{v} \cdot \nabla(B^2) + F_{p2}^{EQ} \vec{J} \cdot \nabla(B^2),$$

where

$$F_{p1}^{EQ} = -\frac{2\langle B^2 \rangle}{\langle \left[\frac{\vec{B} \cdot \nabla B^2}{B^2} \right]^2 \rangle B^2} (m_i \mu_i + m_e \mu_e) n$$

$$F_{p2}^{EQ} = \frac{2\langle B^2 \rangle}{\langle \left[\frac{\vec{B} \cdot \nabla B^2}{B^2} \right]^2 \rangle B^2} \frac{m_e \mu_e - \nu m_i \mu_i}{e(\nu + 1)}$$

Four terms appear in the FC closure.

- **The main term is a damping term:**

$$\left[\nabla\cdot\vec{\vec{\Pi}}\right]_{f_1} = f_1\left\{-\frac{\vec{B}_0\vec{B}_0\cdot\nabla B_0^2}{B_0^4}+\frac{\nabla B_0^2}{2B_0^2}+\frac{\mu_0\vec{J}_0\times\vec{B}_0}{B_0^2}\right\} \quad (1)$$

- **The second important term is an an advective like term**

$$\left[\nabla\cdot\vec{\vec{\Pi}}\right]_{\nabla f_1} = \frac{\vec{B}_0\vec{B}_0\cdot\nabla f_1}{B_0^2}-\frac{1}{3}\nabla f_1 \quad (2)$$

- **The other two terms appear to be less important.**

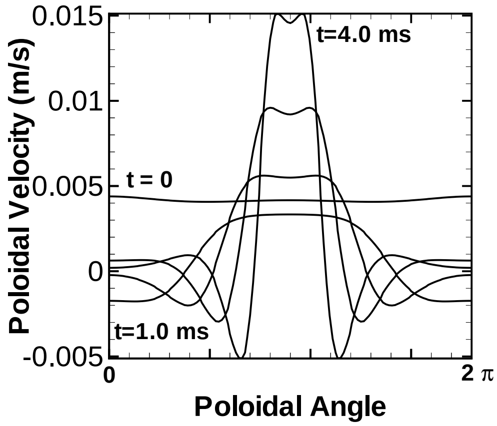
$$\left[\nabla\cdot\vec{\vec{\Pi}}\right]_{\nabla f_0} = \frac{\vec{B}_0\vec{B}_1\cdot\nabla f_0}{B_0^2}+\frac{\vec{B}_1\vec{B}_0\cdot\nabla f_0}{B_0^2} \quad (3)$$

$$\left[\nabla\cdot\vec{\vec{\Pi}}\right]_{f_0} = f_0\left\{-\frac{\vec{B}_0\vec{B}_0\cdot\nabla(2\vec{B}_1\cdot\vec{B}_0)}{B_0^4}-\frac{\vec{B}_1\vec{B}_0\cdot\nabla B_0^2}{B_0^4}-\frac{\vec{B}_0\vec{B}_1\cdot\nabla B_0^2}{B_0^4}+\frac{\nabla\vec{B}_1\cdot\vec{B}_0}{B_0^2}+\frac{\mu_0\vec{J}_1\times\vec{B}_0}{B_0^2}+\frac{\mu_0\vec{J}_0\times\vec{B}_1}{B_0^2}\right\} \quad (4)$$

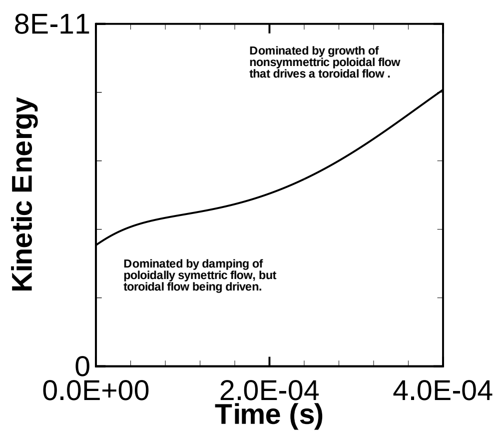
- **Where the f appropriate to either the momentum or the Ohm's law is used.**

FC closure produces unphysical advection

- **Prompt damping of the poloidal flow is observed, but an advective-like action causes peaking at the high-field side.**



- **Term 2 is advective-like and appears to drive an instability.**
- **Coupling into the toroidal direction produces a flow that saturates when the poloidal flow vanishes.**



2. FO closure retains only the damping term.

- The closure essentially reduces to

$$\frac{\partial \vec{v}}{\partial t} \propto -\vec{v} \cdot \nabla B^2 F_{p1}^{EQ} \nabla \cdot \frac{\vec{B}\vec{B}}{B^2}$$

$$\frac{\partial \vec{v} \cdot \nabla B^2}{\partial t} \propto -\vec{v} \cdot \nabla B^2$$

- Simulations indicate:

  No damping in regions where ∇B_0^2 vanishes.

  Generation of toroidal flow that increases kinetic energy.

3. PP model makes ideal MHD approximation.

- The pressure anisotropy is simplified to

$$f = N_{p2}^{EQ}\vec{J}_1 \cdot \nabla(B_0^2),$$

- Ideal MHD is assumed to hold to lowest order

$$\vec{J}_1 \simeq \frac{\vec{B}_0 \times \nabla P_1}{B_0^2}$$

- The perturbed f is then related to the pressure gradient

$$f = -N_{p2}^{EQ}(\vec{B}_0 \times \nabla B_0^2) \cdot \nabla P_1$$

- The ∇f contributions are neglected so that

$$\vec{E}_{neo} \simeq -N_{p2}^{EQ}\frac{\vec{B}_0 \cdot \nabla \vec{B}_0}{B_0^2}(\vec{B}_0 \times \nabla B_0^2) \cdot \nabla P_1$$

$$\vec{E}_{neo} \propto \frac{\partial P_1}{\partial \psi}$$

- This is usually labeled the perturbed bootstrap current.
- The implication is that the bootstrap current vanishes in regions where the pressure gradient vanishes.
- Equilibration of pressure on perturbed surfaces important for the evaluation of the bootstrap current.
- Threshold effects due to insufficient flattening.

4. “Hole” Model of stress tensor.

- The “Hole” model assumes a poloidal flow damping form

$$\vec{E}_{neo} = \frac{\vec{B}_p}{|\vec{B}_{poloidal}|}\eta\frac{\mu_e}{\nu_e}RB_T\frac{dP}{d\psi_{eq}}$$

- The magnitude of the perturbed bootstrap current is determined by assuming pressure flattening inside the island.

- Problem reduces to estimating the island width from the magnitude of the magnetic perturbation at the resonant surface.

  Compute the helical flux function assuming single helicity island.

  Determine three dimensional domain inside the island seperatrix.

  Compute toroidal harmonics of the bootstrap current.

  Add appropriate projections to the Ohm’s law.

- Procedure full of perils.

  Large island width the O-point and X-point deviate from the resonant surface and the evaluation of psi-helical breaks down.

  Secondary islands move O-point and X-point and generate significant stochasticity.

- Advantages

  Linear MHD is sufficient.

  Close tie to analytic theory.

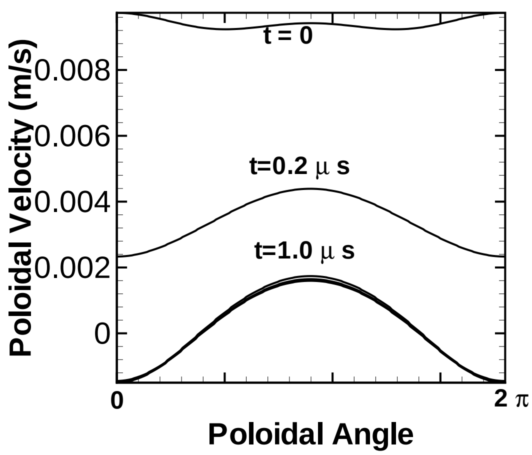
  No need for pressure equation.

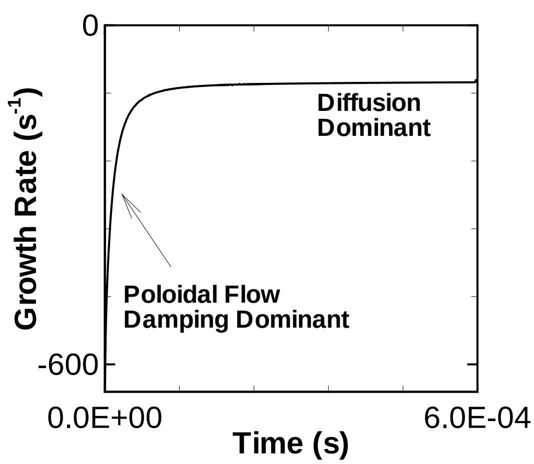
5. "PFD$^\theta$" model of stress tensor reproduces flow damping.

- **Ansatz made that**

$$\nabla \cdot \vec{\vec{\Pi}}_\alpha = m_\alpha n_\alpha \mu_\alpha \langle B_0^2 \rangle \frac{\vec{v}_\alpha \cdot \nabla\theta}{(\vec{B}_0 \cdot \nabla\theta)^2} \nabla\theta$$

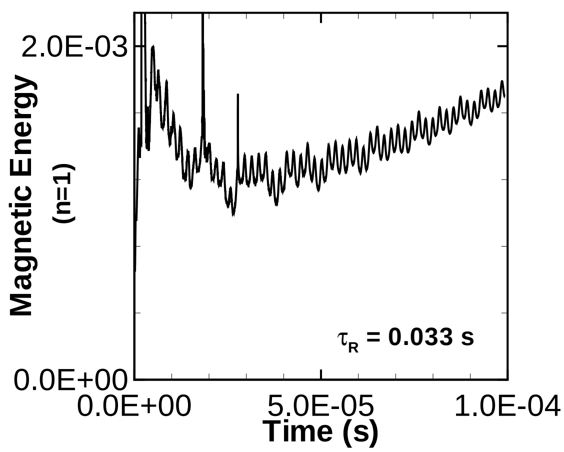
- **Model reproduces poloidal flow damping.**



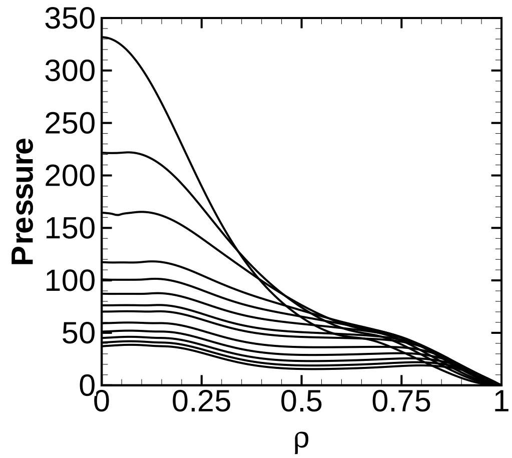


5. “PFD^{θ}” model produces a tearing mode.

- Model produces a tearing instability.



- Generates outward radial flow that produces extreme pressure flattening.



6. “$_{PFD_\theta}$” *Model of stress tensor.*

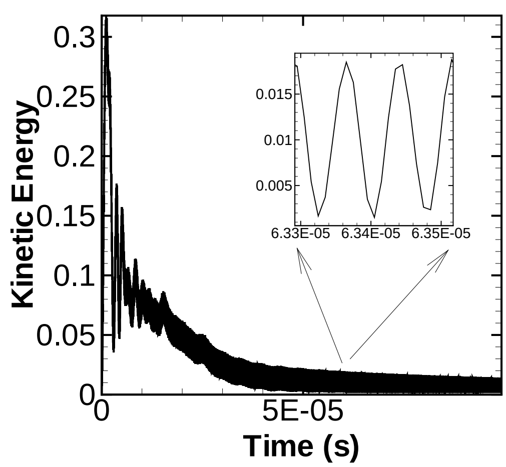
- **The problem with the last form, might be attributed to a non-zero projection in the outward flux-normal.**
- **Ansatz made that**

$$\nabla \cdot \vec{\vec{\Pi}}_\alpha = m_\alpha n_\alpha \mu_\alpha \langle B_0^2 \rangle \frac{\vec{v}_\alpha \cdot \vec{e}_\theta}{(\vec{B}_0 \cdot \vec{e}_\theta)^2} \vec{e}_\theta,$$

where

$$\vec{e}_\theta = \nabla \zeta \times \nabla \psi_0 = \vec{B}_{poloidal}$$

- **Large kinetic energy generation in the n=0 is still present.**



Conclusions

- **FC closure produces an unphysical advection to high-field side.**

- **FO closure has no damping where magnetic pumping vanishes.**

- **PP-CGL variants similar to FO results.**
- **PP-FD variants similar to $\boldsymbol{PFD^{\theta}}$ results.**

- **Hole model can produce an NTM.**

- **Flow damping forms produce large n=0 flows.**

- **The closures are proving less than satisfactory.**