Implementation of Vacuum Region in NIMROD

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Abstract

MHD stability limits in experiments depend on the presence of a vacuum region and on the location of a (geometrically-complicated) conducting wall. Unlike linear codes where one can use Green's functions to predict the response of a fixed plasma-vacuum boundary, the implementation of a vacuum region in a nonlinear initial-value code with a moving plasma-vacuum boundary presents many challenges. We review these challenges and present a practical model for the vaccum region for initial-value codes. We discuss the implementation of this model in the NIMROD code, which was programmed using a finite-element method in order to handle the complicated geometry of modern experiments, and the benchmarking of the NIMROD code to linear analytic results. Of special importance is how to handle the vacuum in the nonlinear regime.
[2]

[2] V.D. Shafranov, Soviet Physics Technical Physics 15 (1970) 175

Implementation of Vacuum Region Needed for Accurate Modelling of Experiments

Obtaining the correct magnetohydrodynamic stability criterion in fusion experiments has long been known to critically depend on the location of the conducting wall and the plasma-vacuum interface (PVI).

- **Linear MHD codes (e.g., PEST, GATO, MARS, ...) have long had vacuum physics.**

  Typically use Green's functions techniques which are relatively easy to implement.

- **Modelling the vacuum region in initial-codes is not as common.**

  Because of moving boundary and nonlinear terms, the use of Green's functions are very difficult.

  Often use destabilized equilibria to study the physics desired (to compensate for stabilizing effect of conducting wall).

- **The goals of the vacuum region in NIMROD are to have a model that:**

  Gives the same results linearly as linear MHD codes which use the Green's function techniques,

  Is computationally tractable with the resources availabe to the fusion community, and

  Is able to be generalized to the nonlinear evolution of MHD instabilities.

Modelling the Vacuum Region for Initial-Value Codes

- **Parameters that are important in modelling the vacuum region are:**

  Density, ρ Want density to be small in vacuum.
  $\rho \rightarrow 0$, $V_A \rightarrow \infty$ $\Rightarrow \Delta t$ to resolve the motions in the vacuum region (which we don't care about). For practicality, $\rho_{vac}/\rho_{core} \sim 10^2$.

  Viscosity, μ Want viscosity to be small in vacuum and at interface.
  Because the viscosity operator is diffusive, it introduces a coupling between motions in the vacuum region and motions in the core region that are undesirable

  Drag, ν Want drag to be large in vacuum.
  We wish to reduce the motions in a vacuum; however, adding a large drag on $\vec{V}$ acts like a wall restricting movement.

  Resistivity, η Want resistivity to be large in vacuum.
  As $\eta \rightarrow \infty, \vec{J} \rightarrow 0$. This has been the most effective method to date.

- **Important consideration is how does one practically determine what is the vacuum region and what is core region.**

- **Introduce a shape variable, S, to track the plasma-vacuum interface.**

  Nonlinearly can determine the interface by evolving the shape variable by a continuity-like equation.

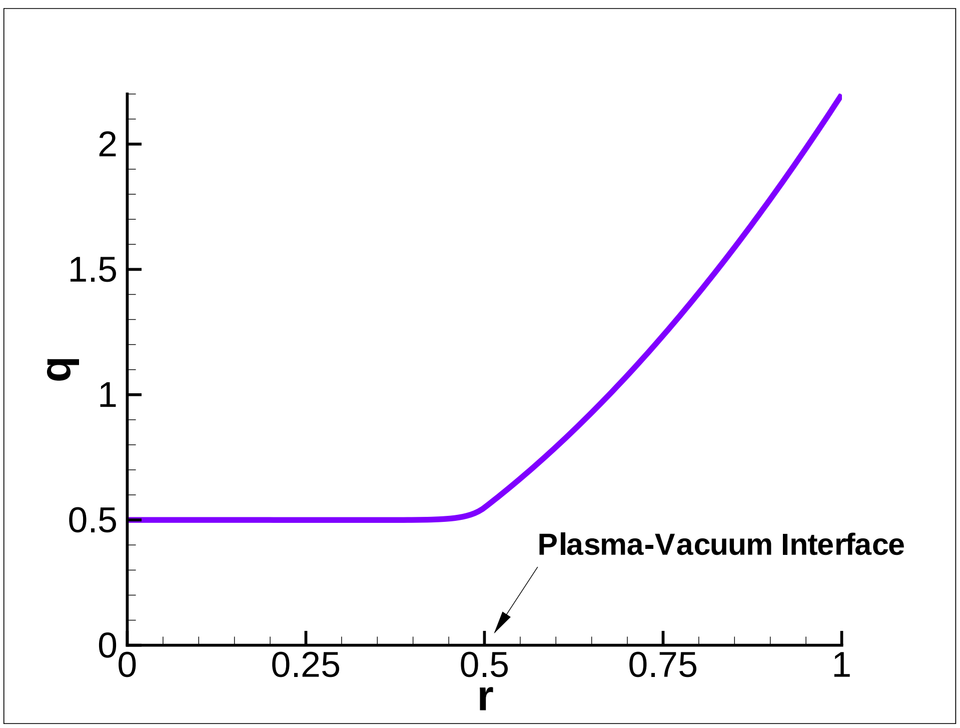
Linear Benchmark Uses Analytic Solutions

- To linearly test the NIMROD vacuum model, use analytic Shafranov's analytic solutions. References:

  V.D. Shafranov, Soviet Physics-Tech. Physics 15 (1970) 175
  –Original reference

  J.A. Wesson, Nuclear Fusion, 23 (1977) 87
  –Better presentation of Shafranov's results

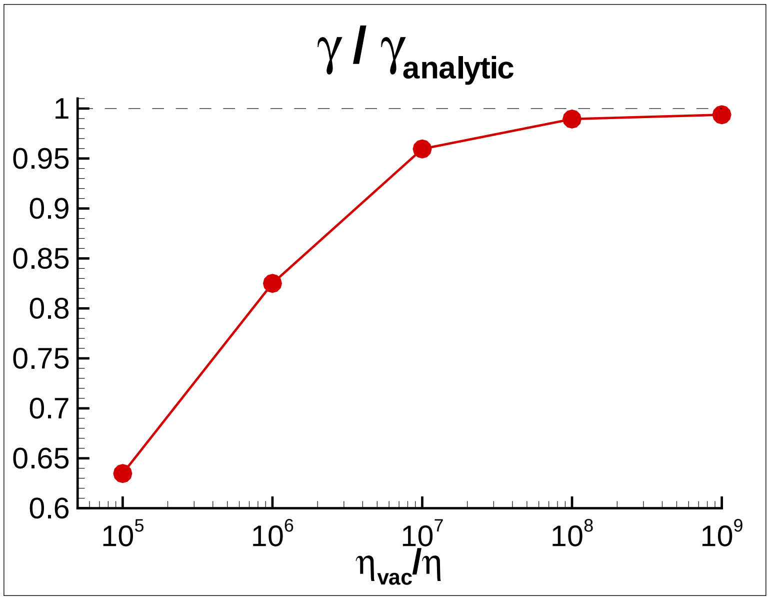
- Equilibria in study chosen has $b/a = 2$.
  It is a zero beta, cylindrical equilibria with a step function like current density profile that produces the q profile:



- Equilibrium is unstable to external kink mode.

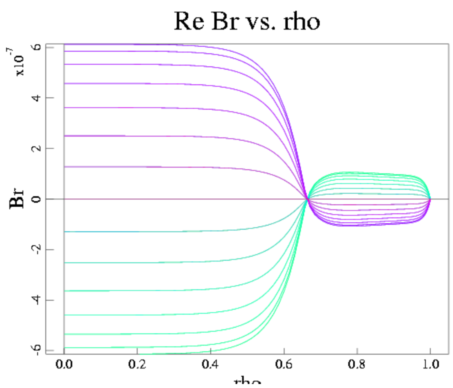
Increasing the Resistivity in Vacuum Works Well for Convergence to Analytic Solution

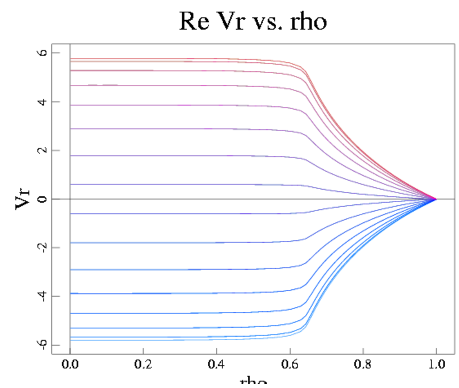
- **The growth rate converges well as the ratio of η_{vac}/η_{core} increases:**



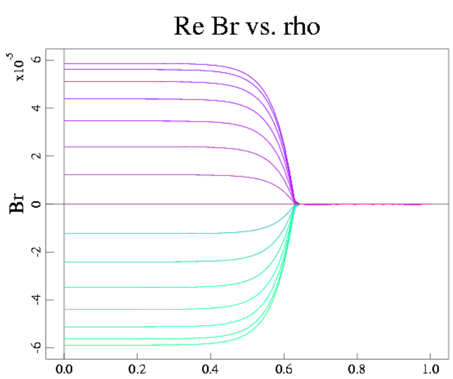
- **Eigenfunctions converge well also:**

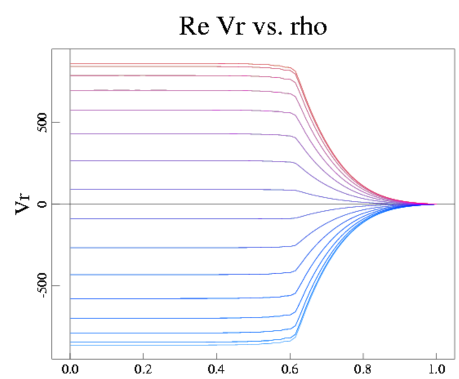
At $\eta_{vac}/\eta_{core} = 10^5$:





At $\eta_{vac}/\eta_{core} = 10^9$:





Matrices for Magnetic Field Advance Become Increasingly Ill-conditioned

- **When the vacuum region has large resistivity, the solution of Ohm's Law asymptotes to the elliptic problem of $\nabla^2(\vec{B}) = 0$ while the core region is predominantly hyperbolic.**

- **This discontinuity of equation types is a classic stiff problem, and leads to ill-conditioned matrices in the temporal advance:**

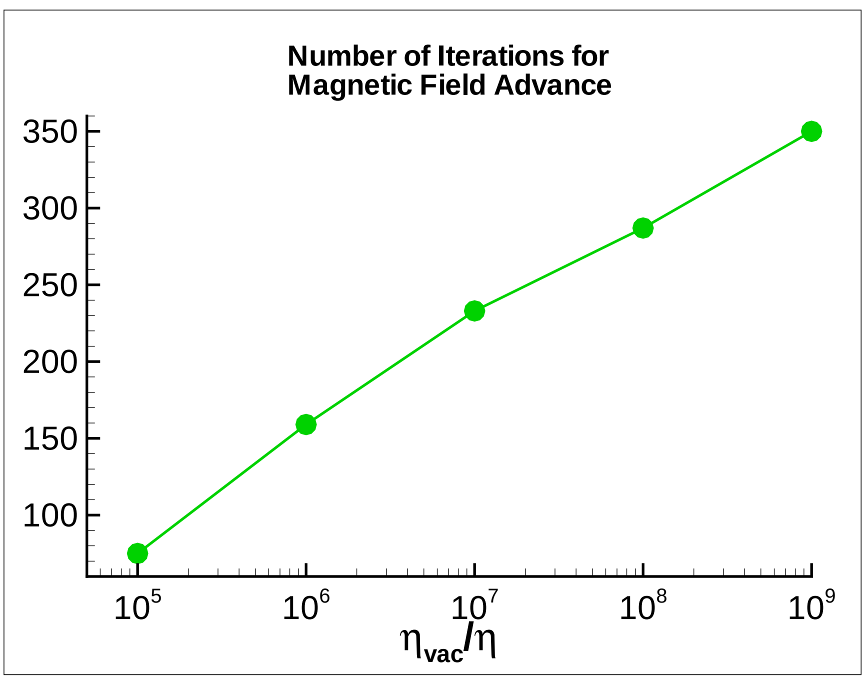
MHD Predictor Step

$$\vec{B}^* \cdot \left[\mathrm{I} + \Delta t \vec{\nabla} \times \eta \vec{\nabla} \times \mathrm{I}\right] = \vec{B}^n + \Delta t \vec{\nabla} \times \left(\vec{V}^{n+1} \times \vec{B}^n\right)$$

MHD Corrector Step

$$\vec{B}^{n+1} \cdot \left[\mathrm{I} + \Delta t \vec{\nabla} \times \eta \vec{\nabla} \times \mathrm{I}\right] = \vec{B}^* + \Delta t \vec{\nabla} \times \left(\vec{V}^{n+1} \times \vec{B}^*\right)$$

This leads to increase in the iteration count of the conjugate-gradient solver:



Nonlinear Tracking of Plasma-Vacuum Interface Uses Shape Variable

- Evolving the resistivity directly can lead to problems as large resistivity region diffuses into the plasma due to the large resistivity gradient at interface.

- Derive resistivity indirectly using a "shape" or "concentration" variable, S.
  Initial condition:

$$S = \begin{cases} 0 & \text{in plasma} \\ 1 & \text{in vacuum} \end{cases}$$

  As S evolves by advection:

$$S < \frac{1}{2} \quad \Rightarrow \quad \text{in plasma}$$
$$S > \frac{1}{2} \quad \Rightarrow \quad \text{in vacuum}$$

- This keeps a sharp plasma-vacuum interface. The resistivity is large ($\sim 10^6$) in vacuum and small outside. Currently no smoothing is done in the region about the plasma-vacuum interface.

- To keep the matrix accurate yet block structured (by toroidal mode number) use a toroidal average for the matrix term:

$$\vec{B}^* \cdot \left[\mathrm{I} + \Delta t \vec{\nabla} \times \langle \eta \rangle_\zeta \vec{\nabla} \times \mathrm{I} \right] = \vec{B}^n + \Delta t \vec{\nabla} \times \left(\vec{V}^{n+1} \times \vec{B}^n \right)$$

Nonlinear Shafranov Case

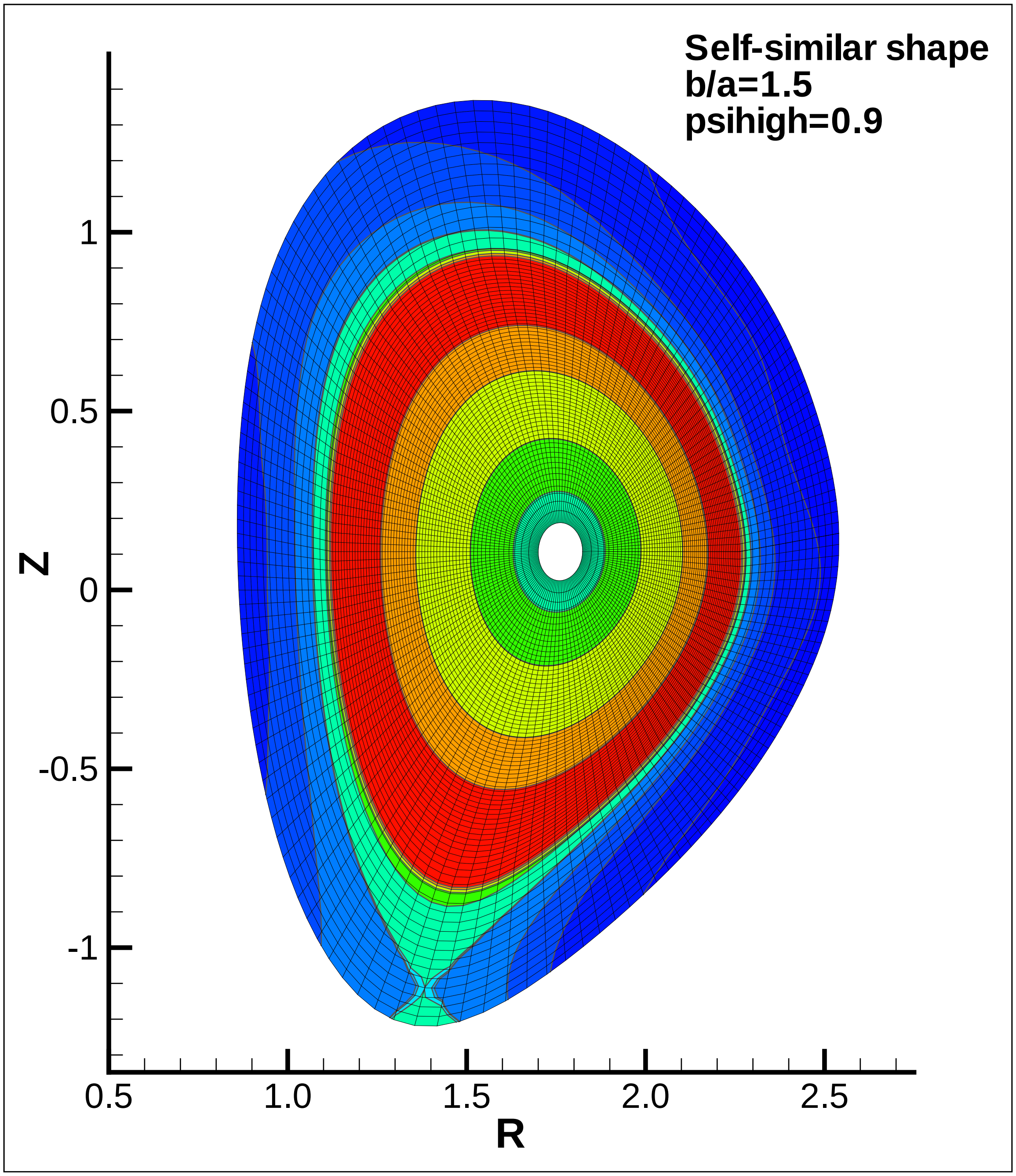
- **At this point, only the advection of the shape variable has been tested. The resistivity is not derived from the shape variable at this point.**
- **Benchmark shows that nonlinear evolution of shape variable is working. The kinking of the plasma is shown:**

Benchmark With GATO Planned In Realistic DIII-D Geometry

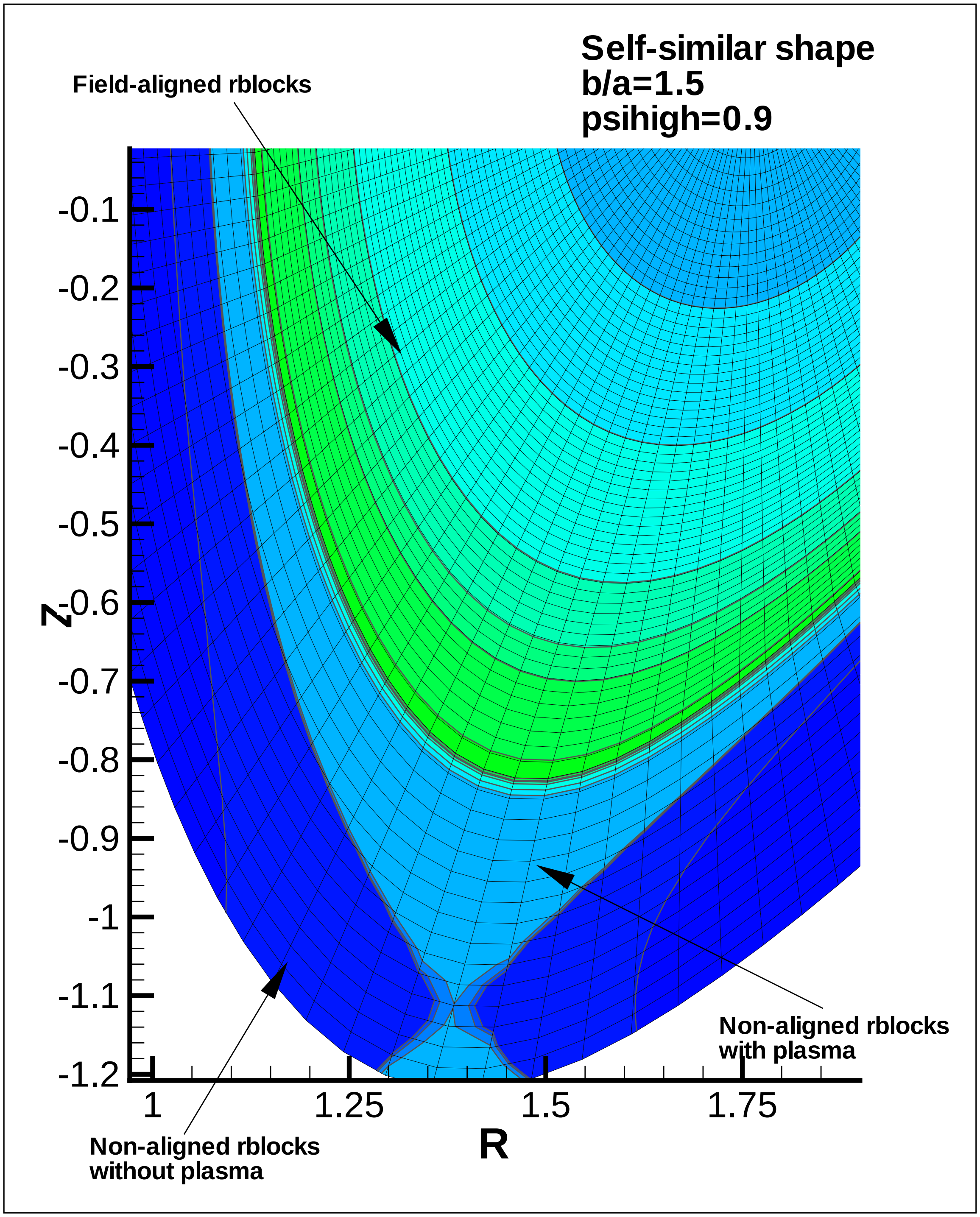
- **NIMROD has two grid types: structured rectangular grid and unstructured triangular grids. The structured grids give better performance so we want to maximize their use.**

- **GATO has option for self-similar and conforming walls. Implement into NIMROD for pure rblock cases. To use full experimental geometry, use rblock/tblock hybrid.**

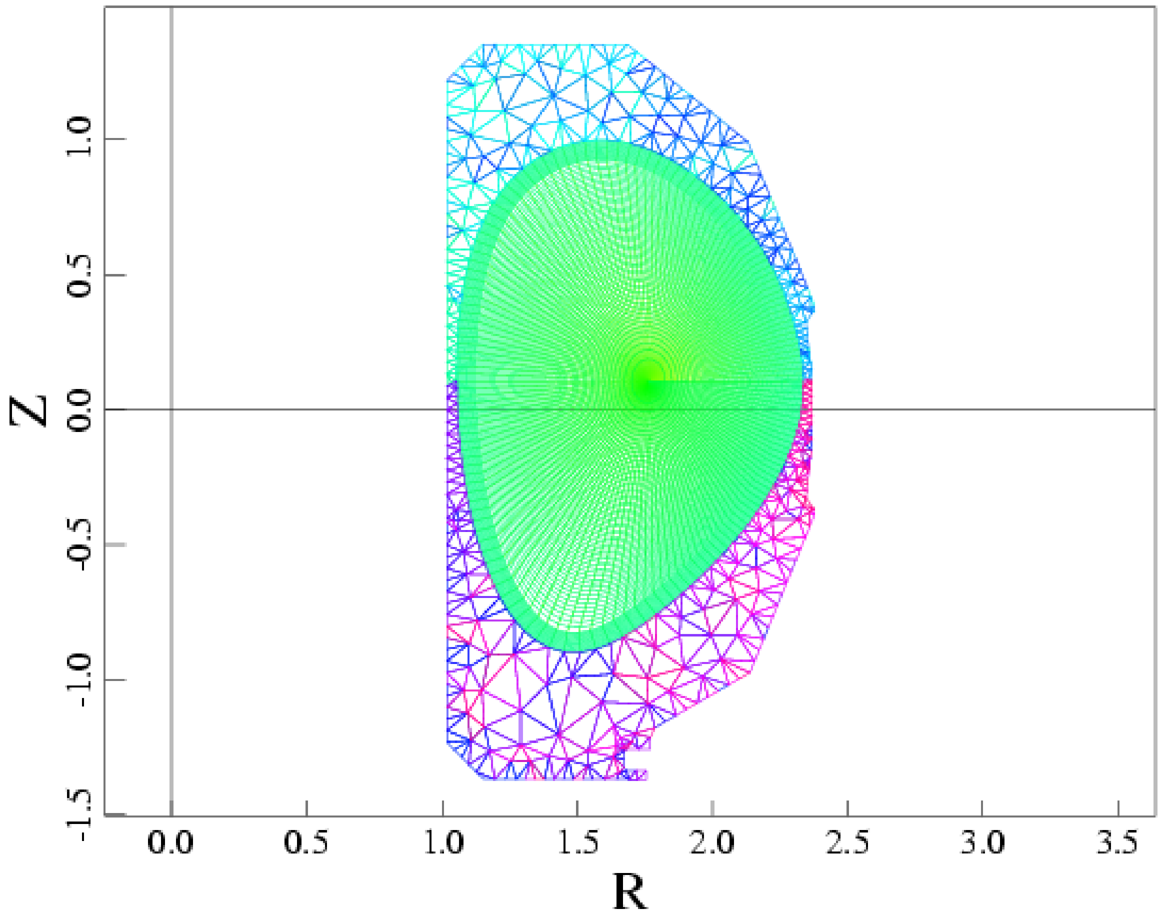
- **Ready to begin linear benchmarks with GATO code. Currently developing good test equilibrium.**



Self-similar shape
b/a=1.5
psihigh=0.9
Z
1
0.5
0
-0.5
-1
0.5
1.0
1.5
2.0
2.5
R


Self-similar shape
b/a=1.5
psihigh=0.9
Field-aligned rblocks
Z
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
1
1.25
1.5
1.75
R
Non-aligned rblocks with plasma
Non-aligned rblocks without plasma

Conformal rblock region w/ triangles



Summary and Future Work

- The vacuum can be effectively modelled as a plasma with a large resistivity with a penalty in the computational cost.

- Nonlinear testing of the code in cylindrical geometry is ongoing.

- Will soon begin linear benchmarking with GATO in DIII-D geometry.

- Experimental comparisons with DIII-D will be performed after benchmarking.

Reprints