MHD Simulations of Fast Magnetic Reconnection

S.E. Kruger, D.D. Schnack, Z. Mikic, R. Lionello

krugers@saic.com

SAIC/CESS
Outline

Conventional Wisdom on Magnetic Reconnection
  Sweet-Parker reconnection
  Petschek reconnection

Topological nonequilibria

Mikic-Lionello Results

NIMROD Results
Properties of Analytic Models

Analytic models generally:

- Assume quasi-steady state
- Quasi-ideal: can only solve “outer-region”
- Done in a subdomain

Two dominant types of reconnection models in 2D systems:

- Y-point models
  - Sweet-Parker, Syrovatskii

- X-point models
  - Petschek-like (Petschek, Sonnerup, ...)

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A Sweet-Parker sheet is characterized by six quantities:

- $B_0$ upstream field.  
- $u_0$ upstream flow.  
- $\delta$ width of the sheet.  
- $\Delta$ length of the sheet.  
- $v_0$ downstream flow.  
- $\eta$ resistivity.

![Diagram of a Sweet-Parker sheet with labels for $B_0$, $u_0$, $\delta$, $\Delta$, $v_0$, and $\eta$.]
Sweet-Parker Model Gives Slow Reconnection Due to Throttling

- Mass conservation: $u_0 \Delta = v_0 \delta$.
- Ohm’s law: $u_0 B_0 = \eta j \approx \eta \frac{B_0}{\delta}$.
- Force balance: $\frac{B_0^2}{2} = p_m - p_0$ and $\frac{v_0^2}{2} = p_m - p_0$.

The reconnection rate is

$$\partial_t \psi = E = u_0 B_0 \sim \eta^{1/2}.$$
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- Ohm's law: \( u_0 B_0 = \eta j \simeq \eta \frac{B_0}{\delta} \).
- Force balance: \( \frac{B_0^2}{2} = p_m - p_0 \) and \( \frac{v_0^2}{2} = p_m - p_0 \).

The reconnection rate is

\[
\partial_t \psi = E = u_0 B_0 \sim \eta^{1/2}.
\]

Too slow!
Petschek’s configuration has an X-point caused by shocks:
Petschek Model Gives Fast Reconnection

Because the outflow is not throttling the reconnection rate, the maximum reconnection rate is proportional $1/\ln S$.

*It’s fast!*
Petschek Model Gives Fast Reconnection

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It’s fast!

Conventional Wisdom (according to Biskamp):

*Nonlinear MHD, Cambridge University Press, 1993*

"...the theory is in error both conceptually and formally. ...it appears quite hopeless to solve the matching problem analytically. ...Various simulations of self-consistent reconnecting systems have been performed... All develop extended current sheets."
“Topological Nonequilibria” are configurations which evolve to configurations with contact discontinuities in the Ideal MHD model.

A magnetic field zero line is a line where $\mathbf{B}_\perp = 0$, where $\perp$ is with respect to a non-zero $B_z$.

The magnetic field remains zero on this line in the presence of ideal MHD flow.

Rosette Problem is Example of Topological Non-Equilibrium

Initial conditions:

\[ \psi = \sum_{i=1,2} \psi_i \exp \left( -\left( \frac{x - x_i}{\sigma} \right)^2 - \left( \frac{y - y_i}{\sigma} \right)^2 \right), \]

\[ A_i = \{0.02, 0.02\}, \quad x_i = \{5.8, 4.2\}, \quad y_i = \{5, 5\}, \]

\[ \sigma = 0.4, \quad B_z = 0.1 \]

Under ideal MHD relaxation, current sheet is formed.
Mikic-Lionello have performed 2D MHD simulations of “rosette” problem.

There is evidence of fast reconnection.

Reconnection rate is independent of $\eta$:
Is There Fast Reconnection?

\[ \max J_z \text{ is inversely proportional to } \eta: \]

Rosette Reconnection

\[ J_z \propto \eta^{1/2} \]

\[ J_z \propto \eta^1 \]

\[ \eta \text{ and } \nu \]
Is There Fast Reconnection?

Magnetic flux and plasma flow:
Caveats to Mikic-Lionello results:

- Results may not be numerically converged
- In doing the numerical convergence studies, it is observed that an increase in resolution leads to slower reconnection

NIMROD has some advantages for benchmarking the problem:

- Slab geometry with flexible boundary conditions:
  Periodic non-periodic
  Doubly-Periodic
  Non-Periodic
- Scales well on MPP systems
Verify With Biskamp Problem

To understand results from the rosette problem, compare with well-verified problem that gives Sweet-Parker scaling. See:

Biskamp, Schwarz, and Drake, *PRL* 84 (1995) 3850

Initial conditions (note exponentials):

\[ \psi = \sum_{i=1,2} \psi_i \exp \left( - \left( \frac{x - x_i}{\sigma} \right)^4 - \left( \frac{y - y_i}{\sigma} \right)^4 \right), \]
Comparison of test cases

Biskamp: doubly-periodic boundary conditions
Comparison of test cases

Rotated Biskamp: conducting wall boundary conditions
Comparison of test cases

Rosette: conducting wall boundary conditions
Preliminary NIMROD Results
Mikic-Lionello possibly see fast reconnection with MHD.

If true, it emphasizes the importance of exterior configuration to understanding reconnection.

The flexibility of NIMROD is being used to benchmark the result and compare to published results.