Integrated Modeling of Spheromak MHD and Energy Transport

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Objectives of the Study

• To develop a comprehensive theoretical understanding of the magnetohydrodynamic (MHD) activity in electrode-driven spheromaks.
• To understand the interaction between thermodynamics and magnetic topology change during externally controlled transients.
• To provide a nonlinear theoretical analysis tool that can be applied to existing spheromak experiments and to optimize potential performance in pulsed and sustained operation.
Outline

• Introduction
  – Primary experimental results
  – Modeling considerations
• Summary of earlier 0-β results \( (\beta \equiv 2\mu_0 p/B^2) \)
  – MHD activity and flux amplification
  – Flux surface formation during decay
• SSPX parameter study at 0-β
• Results considering internal energy
  – Appropriateness of collisional modeling
  – Comparison with SSPX results
  – Profiles in sustained and decaying conditions
  – High-\(n\) MHD during partial sustainment
• Discussion and Conclusions
Poloidal flux amplification and increasing temperatures after the initial drive are the primary experimental results.


- MHD is dominantly $n=1$ with hollow current profiles when driven, becomes quiescent initially in decay, then becomes $n \geq 2$ as profiles become peaked at the magnetic axis [Knox, et al., PRL 56, 842 (1986).]

- During drive, spheromak $T_e < 50$ eV, but during decay or partial drive, $T_e$ can increase well above 100 eV.
Relaxation theory provides insight but not details, so we have turned to numerical simulation.

- The earliest theoretical descriptions have been based on relaxation arguments [Taylor, PRL 33, 1139 (1974); Jarboe, PPCF 36, 945 (1994)].
  - Provide no information on fluctuations or electron force-balance (Ohm’s).
  - Sustainment is described by helicity balance and cascades.
  - ‘Reconnection occurs at small scales as $\eta \to 0$’ has been incorrectly interpreted as ‘MHD activity becomes high-$k$ as $\eta \to 0$.’


- NIMROD simulations at $\beta = 0$ addressed MHD activity underlying formation and sustainment [PRL 85, 4538 (2000), Phys. Plasmas 8, 475 (2001)].
Time-dependent macroscopic behavior can be modeled through nonlinear fluid equations.

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \mathbf{J}) \quad \text{Faraday’s/Ohm’s laws}
\]

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \quad \text{low-\(\omega\) Ampere’s law}
\]

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \nu \rho \nabla \mathbf{V} \quad \text{flow evolution}
\]

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n \quad \text{particle continuity}
\]

\[
\frac{n}{\gamma - 1} \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \nabla \cdot \mathbf{V} + \nabla \cdot n \left[ \alpha ||\hat{\mathbf{b}}\hat{\mathbf{b}} + \chi_\perp (1 - \hat{\mathbf{b}}\hat{\mathbf{b})} \right] + \eta |J|^2 \quad \text{temperature evolution}
\]

\[
\hat{\mathbf{b}} \equiv \mathbf{B}/|\mathbf{B}|
\]

- We solve this system in spheromak-relevant configurations with the NIMROD code (nimrodteam.org & www.cptc.wisc.edu/sovinec_research).

- The artificial particle diffusivity is necessary numerically (including local enhancement where \(n \to 0^+\)) and crudely reproduces effects of non-MHD physics.
Earlier 0-β MHD Simulation Results

- Nonlinear effects from saturation of the \( n=1 \) lead to formation and sustainment.
  - poloidal flux amplification (conversion from toroidal flux)
  - parallel current profile is relaxed
  - MHD dynamo effect sustains toroidal current
  - symmetry-breaking leaves \( B \) chaotic in most sustained conditions

- There are four different classes of nonlinear states in an \( S \)-Voltage phase diagram.

- Closed flux surfaces with net current form during decay.
Three-dimensional computations in can and gun-driven configurations reproduce flux amplification/spheromak formation.

Two-dimensional computations in a can geometry show pinching only.

MHD activity in 3D computations reproduce amplification of the symmetric poloidal flux ($\Psi_0$).
An $n=1$ MHD mode driven by open-field current lies at the heart of the flux conversion.

- This mode can be resistive or ideal depending on how hard the pinch is driven.
- It has been observed during drive in all electrode-driven experiments; thorough measurements were performed on SPHEX [Duck, et al., PPCF 39, 715 (1997)].
Saturation of the $n=1$ leads to several important effects in addition to poloidal flux amplification.

1) The symmetric projection of parallel current density ($\mu_0 a J \cdot B / B^2$) is distributed throughout the volume (relaxation).

Examining 3D distributions, the parallel current density is carried by a helical current channel, reminiscent of the “dough hook” observed in SPHEX. [Duck, PPCF 39, 715 (1997).]
2) When sustained, the fluctuations generate MHD dynamo effects that maintain toroidal current against resistive dissipation.

\[ E_{f||} = \langle -\mathbf{v}_n \times \mathbf{b}_n \rangle \| \text{ from } n=1 \]

\[ \langle -\mathbf{v}_n \times \mathbf{b}_n \rangle \| \text{ from } n=2 \]

- Dynamo was measured in SPHEX, but \( n=1 \) and \( n=2 \) fluctuations were interpreted as separate activity [al-Karkhy, et al., PRL 70, 1814 (1993)].
- NIMROD simulations show \( n>1 \) to be nonlinearly generated by \( n=1 \) during sustainment.
Effects from Saturation, Continued

3) As the MHD fluctuations reconnect \( B \)-field and break the toroidal symmetric of the pinched open-field current column, they ensnarl magnetic field lines throughout the volume.

- The entanglement and reconnection of injected toroidal flux with the bias poloidal flux leads to poloidal flux amplification.
- There is some confinement on the open fields (discussed later).
Nonlinear behavior at different sustained voltages and resistivity values can be summarized in a phase diagram.

Phase diagram of “can” results. Symmetric state exists at low $S$, $V_a$.

$$V_a \equiv \tau_r \int E_a \cdot dL$$

$$S \equiv \frac{\tau_r}{\tau_A}$$

Limit cycle behavior of magnetic fluctuation energies in a sustained “can” configuration at $S=5000$. 
When drive is removed from sustained conditions, closed flux surfaces often form.

- Magnetic fluctuations decay faster than the mean poloidal flux generated by the flux conversion.
- Slow decay of mean poloidal flux acts as an Ohmic drive for toroidal current on the flux surfaces.
- These combined transient effects were proposed as the reason underlying temperature increases during decay in experiments [PoP 8, 475 (2002)].
SSPX Parameter Study at 0-β

- The coil set on SSPX allows for flexibility in setting the initial flux distribution.

- A variety of different flux distributions can be used in NIMROD simulations.

- A preliminary survey of 0-β simulations shows that the poloidal flux amplification is somewhat sensitive to the initial flux distribution.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Flux amp</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>130%</td>
</tr>
<tr>
<td>2</td>
<td>120%</td>
</tr>
<tr>
<td>3</td>
<td>110%</td>
</tr>
</tbody>
</table>
Emulating SSPX Current Waveform

- The boundary conditions have been modified to allow for a current source condition instead of an applied voltage condition used in past simulations.

- The figure to the upper-left shows the applied waveform for SSPX shot #4621.

- This series of simulations has $n=0$ only from $t=0$ to $t\sim0.1\text{ms}$. 
Ejected plasma forms a “line-splayed” pinch

- The initial poloidal field is compressed towards the geometric axis.

- The magnetic configuration is similar to a “line-tied” pinch, but the field lines “splay” out near the top and bottom of the flux conserver.
Resistivity: $0-\beta$ vs. Spitzer $\eta(T_e)$

**0-\beta simulation**

**Finite $\beta$, $\eta(T_e)$ simulation**
left: resistivity
right: temperature
Studies with Integrated MHD / Energy Transport Modeling

- Resistivity is temperature-dependent, \( \eta(T) = \eta_0 \left( \frac{T_0}{T} \right)^{3/2} \)
- Parallel thermal diffusivity is temperature dependent, \( \chi_\parallel(T) = \chi_0 \left( \frac{T}{T_0} \right)^{5/2} \)
- Heating is Ohmic, \( \eta J^2 \).
- Walls are cold, and \( T_{\text{wall}} \) does not determine the computed core temperatures.
- Injector is given a specified current waveform.
- SSPX-relevant physical parameters are used.
- Approximating particle transport may be important.
Validity of Collisional Transport

• 0-\( \beta \) MHD results suggest that open-field transport governs confinement during driven conditions.
  – Although individual magnetic field lines do not fill the volume ergodically, we expect Rechester-Rosenbluth collisional transport if the effective mean-free-path is sufficiently small.
  – We can confirm \textit{a posteriori} that collisionless conditions only exist when and where closed flux surfaces form.

• At \( n=5\times10^{19} \text{ m}^{-3} \) and \( T=1 \text{ eV} \),
  \[
  \nu_{Te} \approx 6\times10^5 \text{ m/s} \\
  \tau_e \approx 7\times10^{-10} \text{ s} \\
  \lambda_e \approx 4\times10^{-4} \text{ m} \ll L
  \]

• Scaling \( \lambda_e \) with \( T^2 \) indicates that \( \lambda_e \) reaches macroscopic scales between 30 and 50 eV.

• From this we infer that anisotropic thermal conduction is a good model for sustained (open field) conditions and for the transition to closed flux during decay.
Finite-$\beta$ simulations of SSPX model the flux conserver and the downstream end of the gun (upside down).

Schematic of SSPX spheromak experiment at LLNL, with contours of reconstructed symmetric poloidal flux.

MROD mesh of cubic finite elements.

Vacuum poloidal flux distribution. (0.020 Wb max)
Parameters of Finite-β SSPX Simulations

<table>
<thead>
<tr>
<th></th>
<th>“Deuterium Simulations”</th>
<th>“Hydrogen Simulations”</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_{</td>
<td></td>
<td>} (T)$ (m²/s)</td>
</tr>
<tr>
<td>$\chi_{\perp} (T,B)$ (m²/s)</td>
<td>21 ($\sim$1eV, 0.15T)</td>
<td>0.5 $T^{1/2}B^{-2}$</td>
</tr>
<tr>
<td>$\eta (T)/\mu_0$ (m²/s)</td>
<td>822 $T^{-3/2}$ ($\sim\eta_{\perp}$)</td>
<td>411 $T^{-3/2}$ ($\sim\eta_{</td>
</tr>
<tr>
<td>$\nu$ (m²/s)</td>
<td>100 and 300</td>
<td>2000</td>
</tr>
<tr>
<td>$D$ (m²/s)</td>
<td>10,000*</td>
<td>2000</td>
</tr>
<tr>
<td>$\Psi_{\text{vacuum}}$ (mWb)</td>
<td>11.</td>
<td>20.</td>
</tr>
<tr>
<td>max($I_{\text{inj}}$) (kA)</td>
<td>300</td>
<td>400</td>
</tr>
</tbody>
</table>

All have $n_0=5\times10^{19}$ m⁻³ and $T_{\text{wall}}=0.1$ eV.

*T- and V-advances in the “Deuterium” simulations use $n_0$ only.
The simulated injector current is programmed to approximate the series of SSPX discharges reported in [McLean, et al., PRL 88, 125004-1 (2002)].

A strongly driven phase is followed by decay and then a second, partial drive. [SSPX Data courtesy of H. S. McLean.]

\[ I_{tor} \text{ resulting from the series of NIMROD simulations is compared with } I_{tor} \text{ from CORSICA equilibrium fits of SSPX data.} \]

- The rate of decay of toroidal current in the simulation is similar to that found by the CORSICA fits [Hooper, et al., NF 39, 863 (1999)] during the partial drive stage.
The simulation results show that some of the energy lost from the magnetic field during decay and partial drive is retained in internal energy.

Simulation and experiment show a drop in magnetic energy after the initial pulse that is later slowed by the partial drive. [SSPX Data courtesy of H. S. McLean.]

Internal energy in the simulation begins to increase shortly after the start of the transient. Partial drive operation improves the gain significantly.

[Solid traces show “Deuterium” series, and dotted traces show ongoing “Hydrogen” series.]
Both simulation and experiment show a quiescent phase when partial drive is applied after a brief period of decay.

Relative poloidal magnetic field fluctuations at the outboard mid-plane position. Simulation results for continuing free decay are also shown ("Deuterium" simulation and SSPX probe).

- Although conditions are not sustained, partial drive forces fluctuations to smaller amplitude, postpones the emergence of \( n > 1 \) modes.

- The more realistic current waveform in the "Hydrogen" case drives the plasma harder but only for \( t < 0.12 \) ms.
Observed and computed temperatures increase to their maximum values during the quiescent phase.

[Data generated by the “H” simulation since the start of the Sherwood conference has been added to this plot.]
Profile Information from the “Deuterium” Simulation and Relation to 0-β Results

Poloidal flux contours show 305% amplification from n=1 MHD activity at $t=0.58$ ms, the end of the simulated sustainment.

Poincaré plots show that there are no large closed-flux surfaces at this point in time.
Despite the open topology, the chaotically scattered magnetic field lines are long enough to allow peak temperatures of \(~40\) eV during full sustainment.

Contours of constant toroidally averaged temperature during sustainment.

With a 3D view, the \(T=37\) eV isosurface shows \(n=1\) deformation.
During free decay, large closed flux surfaces form, and the peak temperature increases with internal energy. ($t=0.63 \text{ ms}, 0.76 \text{ ms}, \text{ and } 0.91 \text{ ms}$ are shown).
With partial drive, the flux surfaces persist longer, and the peak temperature is higher. (86 eV at $t=1.48$ ms vs. 70 eV at $t=0.91$ ms with free decay in the “Deuterium” simulations.)

- The suppression of MHD activity and the reconnection of flux surfaces in decay are enhanced by the decreasing temperatures, hence increasing resistivity, on the outer field lines.
- Qualitatively, the behavior is as expected from the $0$-$\beta$ simulations.
Safety factor and parallel current profiles suggest that the improvement from partial drive results from avoidance of low-order rational surfaces (~Simon Woodruff, recent).

Safety factor profiles from the decay and partial drive computations shown at t=1.01 ms.

Parallel current density profiles shown at t=1.01 ms.
The second “Deuterium” sequence has more toroidal resolution and models more of the gun.

During the formation stage, results from the simulation with $n \leq 5$ are similar to those based on $n \leq 2$ only.
Here, temperatures also increase with the help of the transient, but an $m=4, n=5$ mode limits core confinement, resulting in a lower maximum temperature.

To date, $n=5$ activity has not been detected in SSPX. (Safety factor profiles agree with CORSICA fits of SSPX; absence is possibly 2-fluid effect??)
The impact of continuity evolution is being investigated.

- Heat flux diagnostics show that convection is large and may affect confinement.
- The evolving number density is used in the flow velocity and temperature equations of the "Hydrogen" simulations.
  - $D$ has to be large ($\tau_n \leq \tau_E$-observed).
  - Driven stage is quite violent.
  - Particle transport is one of the least understood aspects of SSPX, according to experimentalists.
- To prevent negative values, $D$ is locally enhanced to $\Delta x^2/\Delta t$ where $<n>$ is less than $0.03n_0$, and amplitudes of nonsymmetric Fourier components are limited.
- Preliminary comparisons do not indicate strong sensitivity to $n$-evolution modeling.

Number density in the $\phi=0$ plane at $t=0.1$ ms of the "Hydrogen" simulation.
Thermal transport changes character from driven to decaying conditions (0.12 ms top row and 0.5 ms bottom row in “H” case).

\[
\text{Note the different exponential scales for the magnitudes of the three heat vectors.}
\]

\[\text{conductive } <q_{\parallel} B_{\text{pol}}>\]

\[\text{conductive } <q_{\perp \text{pol}}>\]

\[\text{convective } <2nTV_{\text{pol}}>\]
Discussion

- The temperature dependence of the parallel thermal conduction is important for providing edge energy confinement during the initial drive.
- An interplay of inductive effects and temperature-dependent transport coefficients produces the low-fluctuation, high-confinement states.
  - The $n=1$ mode of the open-field current channel decays rapidly when the drive is removed—the open-field plasma cools, and the pinch current subsides.
  - Low-resistivity plasma within the hot flux surfaces retains toroidal current associated with the $n=1$-generated poloidal flux.
  - The influence of the MHD activity on the magnetic topology during drive and decay are consistent with earlier $0-\beta$ simulation results (Finn, et al., PRL 85, 4538, 2002 and Sovinec, et al., PoP 8, 475, 2001).
- The realistic parameters and collisional temperature dependencies make the MHD results quantitatively consistent with SSPX results (McLean, et al., PRL 88, 125004, 2002).
  - Temperature profiles—peak values within $\sim$30%.
  - Magnetic fluctuations are $\sim$1 % during partial drive.
  - Magnetic energy decay of $\sim$2-4 MW during partial drive.
Conclusions

• Evolving the complete system with temperature-dependent transport coefficients allows us to assess confinement quality during different stages of spheromak operation.
• Transients play a crucial role; thus, modeling injector current programming impacts the comparison between theory and experiment.
• Relating the 0-β simulation results to the recent results and from the comparison of the experimental observations with the simulations, we see that only the initial phase of the standard two-stage SSPX operation has the characteristics of full sustainment.
• The sustaining $n=1$ MHD mode is not driven during the quiescent phase, which benefits from thermodynamics-assisted decay. The success of partial drive provides optimism for tailoring pulsed spheromak operation.