Two Tokamak Applications of

*NIMROD*:

Ballooning Modes and Secondary Island Generation by Mode Coupling

T.A. Gianakon, S.E. Kruger, C.R. Sovinec, A.H. Glasser, M. Chu
and the NIMROD Team.

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Thesis

- **NIMROD** is a three dimensional initial value fluid code designed for the analysis of advanced fusion experiments. Application of the code to ballooning modes and seed island generation for neoclassical tearing modes are presented.

Outline

- Ballooning Modes
- Mode Coupling
- Neoclassical Closures.
Equilibria for ballooning mode study are based on $TOQ^2$.

- $\langle \vec{J} \cdot \vec{B} \rangle$ held fixed.

- Only magnitude of pressure varied.

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$^2$R.L. Miller and Y.R. Lin-Liu, General Atomics

TAG/Madison — 25 March 1998, p 3
Infinite $n$ ballooning stability of the equilibria analyzed with DCON$^3$. 

\[\frac{\partial}{\partial q} \left( \alpha_1, \text{Infinite n ballooning} \right) \]

\[\beta = 0.005 \quad \beta = 1.0 \quad \beta = 2.0 \quad \beta = 2.19 \quad \beta = 3.00\]

Unstable

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\(^a\) A. Glasser, LANL

TAG/Madison — 25 March 1998, p4
Balloonning mode simulations require significant poloidal mesh.

\[ \tau_A = \frac{R_0^2 q_0 \sqrt{\mu_0 \rho}}{F_0} = 1.1(10)^{-7} s \]

\[ \tau_R = \frac{\mu_0 R_0^2}{\eta} = 5.0(10)^{-3} s \]

\[ S = 4.5(10)^4 \]

\[ P_m = 1 \]

\[ \delta t_m = (10)^{-8}(s) \]

128 Radial Cells

128 Poloidal Cells

\[ R_0 = 2.623 m \]

\[ q_0 = 1.345 \]

\[ F_0 = 11.7 \]
A linear instability with a ballooning structure is indicated.

- Instability has comparable magnetic and kinetic energies.

- Pressure contours illustrate the ballooning structure.

TAG/Madison — 25 March 1998, p 6
Poincare section suggests both kink and island structures.

- The linear eigenfunctions are rescaled until features are detectable in the poincare section.
- Islands appear on some surfaces.
- Kinks appear on other surfaces.
Linear growth rates decrease below beta limit.

- Growth rate is nonzero below the $\beta$ limit probably due to the high resistivity $S = 4.5(10)^4$.

- Results not fully converged:
  
  At $\beta = 0.219$ and $\delta t_m = 10^{-9}$, the growth rate increases to $\gamma \tau_A \simeq 0.4$. Poloidal and radial mesh appear sufficient.
Neoclassical Tearing Modes (NTM’s) can be triggered by a sawtooth crash\(^4\).

\(^4\)personal communication, Rob Lahaye, General Atomics

TAG/Madison — 25 March 1998, p 9
Differential rotation may inhibit reconnection\textsuperscript{5}.  

- $\Delta \omega$ between $q=1$ and $q=3/2$ decreases upon a sawtooth crash.

\textsuperscript{5}personal communication, Rob Lahaye, General Atomics
NIMROD aims to simulate mode coupling trigger of NTM's.

- Equilibrium based on DIIIID shot 86144.02250 without equilibrium flow.

\[
\tau_A = \frac{R_0^2 q_0 \sqrt{\mu_0 \rho}}{F_0} = 1.588(10)^{-7}s \\
S = 10^5
\]

\[
\tau_R = \frac{R_0^2}{\eta} = 1.588(10)^{-2}s \\
P_m = 1
\]

\[
\delta t_m = (10)^{-8}(s)
\]

128 Radial Cells
32 Poloidal Cells

\[R_0 = 1.737m\]
\[q_0 = 0.9802\]
\[F_0 = 2.757\]
Nonlinear NIMROD simulations of 86144.2250 have been conducted.

- Growth of $n=1$ drives $n=0$ and $n=2$ harmonics.
- The $n=1$ is an internal kink: $m/n=1/1$.
Generation of secondary islands is observed.

- Island widths estimated from Poincare plots.


**NIMROD has implemented a neoclassical closure for the parallel stress tensor.**

- Assume a Chew-Goldberger-Low form for $\hat{\Pi}_||$.

\[
\tilde{\pi}_\alpha \simeq \tilde{\Pi}_|| |\alpha| = \left( \frac{\hat{\mathbf{B}} \cdot \hat{\mathbf{B}}}{B^2} - \frac{\hat{I}}{3} \right) (p_|| - p_\perp)\alpha,
\]

where

\[
(p_|| - p_\perp)\alpha = -2m_\alpha n_\alpha \mu_\alpha \frac{\langle B^2 \rangle}{\left[ \frac{\hat{\mathbf{B}} \cdot \nabla B^2}{B^2} \right]^2} \langle \bar{v}_\alpha \cdot \nabla B^2 \rangle,
\]

and $\bar{v}_\alpha$ is the species velocity.

- The viscous damping frequencies are approximated by \(^6\) \(^7\)

\[
\mu_e \simeq \frac{2.3\epsilon^{1/2} \nu_e}{(1 + 1.07\nu_{*e}^{1/2} + 1.02\nu_{*e})(1 + 1.07\nu_{*e}^{3/2})},
\]

- The NIMROD generalized Ohm’s law can include

\[
\tilde{E}_{neo} = -\frac{1}{ne(1 + \nu)} \nabla \cdot \left( \tilde{\pi}_e - \nu \tilde{\pi}_i \right)
\]

\[
\nu = \frac{Z m_e}{m_i}
\]
The linearized closure requires an effective anisotropic pressure.

- Define an effective anisotropic pressure as
  \[ \hat{f} = N_1 \tilde{\nu} \cdot \nabla B^2 + N_2 \tilde{\mathcal{J}} \cdot \nabla B^2, \]
  so that
  \[ \tilde{E}_{neo} = -\frac{1}{ne(1 + \nu)} \nabla \cdot \left\{ \left( \frac{\tilde{B} \tilde{B}}{B^2} - \frac{1}{3} \bar{I} \right) \hat{f} \right\} \]

- Approximate flux average terms with equilibrium quantities
  \[ N_1 = \frac{2\langle B_0^2 \rangle}{\langle [\tilde{B}_0 \cdot \nabla B_0^2]^2 \rangle} n (\nu m_i \mu_i - m_e \mu_e) \]
  \[ N_2 = \frac{2\langle B_0^2 \rangle}{\langle [\tilde{B}_0 \cdot \nabla B_0^2]^2 \rangle} \frac{\nu^2 m_i \mu_i + m_e \mu_e}{e(\nu + 1)} \]

- Linearized closure requires equilibrium and fluctuation for \( \hat{f} \).
  \[ \hat{f}_0 = N_1 \tilde{\nu}_0 \cdot \nabla B_0^2 + N_2 \tilde{\mathcal{J}}_0 \cdot \nabla B_0^2 \]
  \[ \hat{f}_1 = N_1 \tilde{\nu}_0 \cdot \nabla B_1^2 + N_2 \tilde{\mathcal{J}}_0 \cdot \nabla B_1^2 \]
  \[ + N_1 \tilde{\nu}_1 \cdot \nabla B_0^2 + N_2 \tilde{\mathcal{J}}_1 \cdot \nabla B_0^2 \]
  \[ + N_1 \tilde{\nu}_1 \cdot \nabla B_1^2 + N_2 \tilde{\mathcal{J}}_1 \cdot \nabla B_1^2 \]

- Factors of \( \frac{1}{B^2} \simeq \frac{1}{B_0^2} \).
- Factors of \( \nabla(1/B^2) \simeq -\nabla B^2/B_0^4 \).
**Equilibrium portions of the closure.**

- The viscous damping frequencies require the evaluation of the trapped particle fractions.

![Graph showing trapped particle fraction vs. \(\rho\)](image1)

- The factors \(N_1\) and \(N_2\) are singular due to the magnetic pumping terms.

![Graph showing magnetic pumping vs. \(\rho\)](image2)

- The singularity has not yet caused problems in simulations.
Preliminary linear simulations indicate neoclassical term is stabilizing.

- Previous internal kink simulation restarted with neoclassical terms.

- Only expected to be destabilizing in nonlinear simulations.
  
  Equilibration of pressure on perturbed flux surfaces.
  Sufficiently large islands above nonlinear threshold for the mode.
  May require anisotropic heat diffusivities.

- The good news: Apparently no new linear modes introduced.
Summary

- *NIMROD* is rapidly finding application to the analysis of high temperature plasmas.

- Ballooning mode simulations are possible.

- Mode coupling simulations are in progress.

- Neoclassical tearing modes will soon be demonstrated.